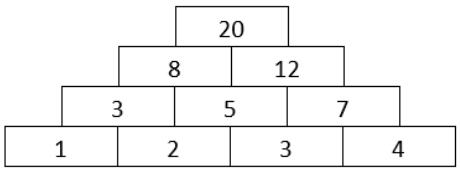
Pyramids

Activity overview

The idea of the activity is that numbers in a lower row of the pyramid are added to generate the numbers in the row above and so on until one value is generated at the top. Pyramids can vary in size. The activity here starts with 4 cells in the bottom row. A pyramid with a bottom row of 3 could be used to introduce the activity if appropriate for particular pupils.



Once pupils are confident with the structure of the pyramid, the activity asks them to consider what would happen if the order of the numbers in the bottom row changes and then investigate to find out.

Consecutive numbers are chosen in this sequence of tasks but a further investigation could be based around other sets of numbers.

Once the order of numbers has been investigated, the tasks introduce different representations to support more abstract thinking which could lead to the use of letters to represent the values in the bottom row in the future.

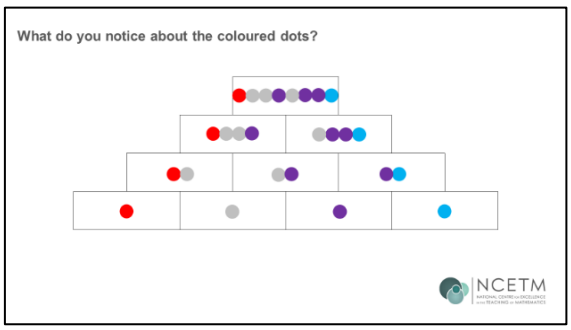
Rationale

Pyramids is a versatile activity designed to allow pupils to make and test hypotheses with numbers, which can then be developed into generalisations. The numbers are chosen to be well within the pupils' mental calculation knowledge so that the emphasis can be on the mathematical thinking, hypothesising and testing.

Language and representations

Sum and *total* should be used when referring to the upper rows of the pyramid.

The pyramid with the coloured dots replacing the values gives a different representation, which draws attention to the number of times each cell in the bottom row is reflected in the final total at the top.



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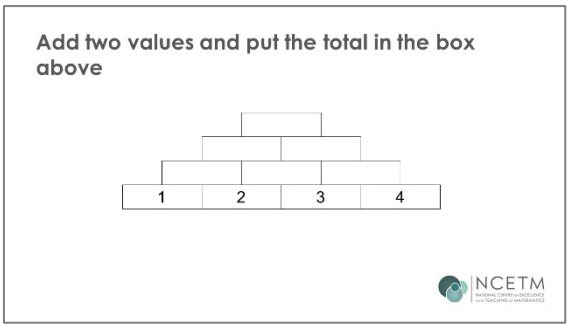
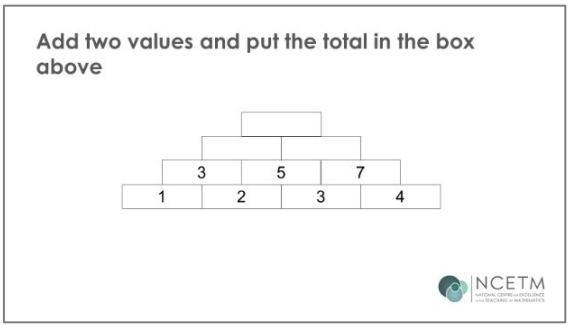
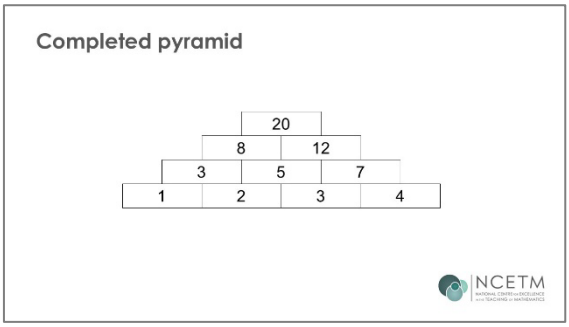
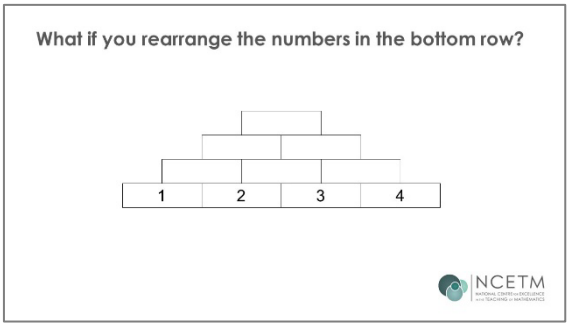
Task 1: Starting with 1, 2, 3, 4

To begin with, pupils investigate a pyramid with a bottom row of four consecutive numbers starting with 1, 2, 3, 4.

Slides 2–8	Commentary
Slide 3  **Slide 4**  **Slide 5**  	The first three slides build up the pyramid gradually. Allow pupils time to explore and complete the pyramid. Invite pupils to ask questions and to try different things, but be wary of the use of very large numbers as this will detract from the thinking. The numbers used in all the examples are consecutive.
Slide 6  	Once they are familiar with the way the pyramid is constructed, Slides 6, 7 and 8 invite pupils to consider if the order of the numbers in the bottom row changes the other rows of the pyramid. Exploring these possibilities may start to reveal the significance of the numbers in the outer and inner boxes on the bottom row. If they do not come up with the idea themselves, ask: *'Does the order of the numbers in the bottom row matter?'* Many pupils will believe that the order does not matter and, once they discover that it does, may not be able to explain why.

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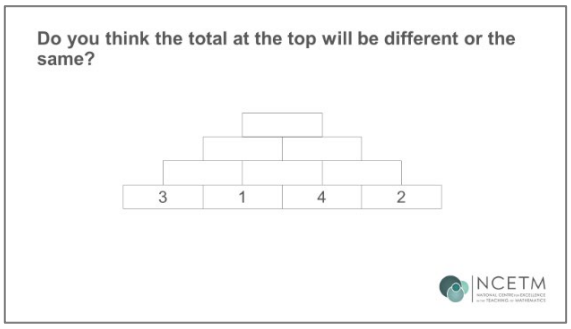
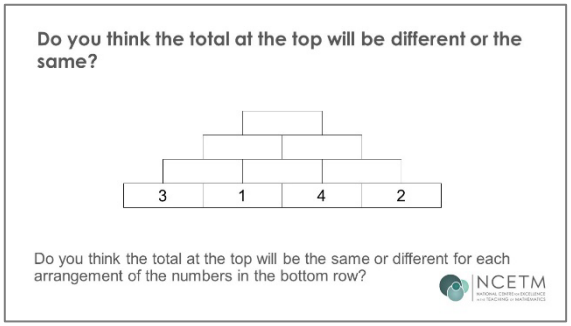
Slide 7



Slide 8



Once they realise that changing the order does change the value in the top box, invite them to ask their own questions.

You might need to prompt to ensure that you get questions like:

'What is the highest value you can get in the top box?'

'What is the lowest value you can get in the top box?'

'How many different values can you get in the top box?'

The aim of this initial activity is to raise awareness of the significance of the order of the numbers in the bottom row and the impact that this has on the top total. Pupils may not yet be able to explain why at this point in the sequence of tasks.

This is an interesting line of enquiry as it leads into Task 2 and considering the significance of the outer two and inner two boxes in the bottom row.

'Is the number of different values in the top box the same as the number of different ways that you can arrange the numbers 1, 2, 3, 4 in the boxes on the bottom row?'

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Task 2: Comparing pyramids

In this task, pupils are asked to compare different pyramids and decide which will give the highest and lowest top row values. They are also asked to consider which arrangements of numbers will give the same top values.

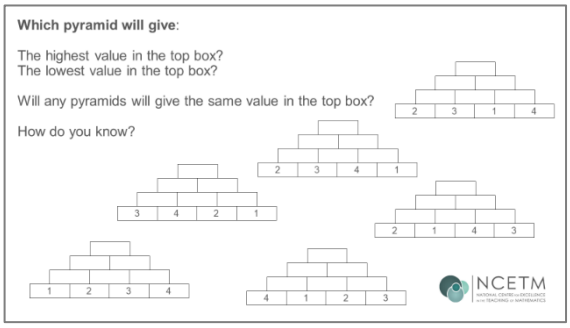
The pyramids can be presented as a set of cards to sort or as a handout for pupils to use as they discuss and make their decisions: see **Worksheet 2**.

The aim of this is for pupils to begin to form an explanation of how they know which pyramids will produce higher and lower top values. This prepares them for the final task where they explore different representations to explain and prove their thinking.

Slides 9–14

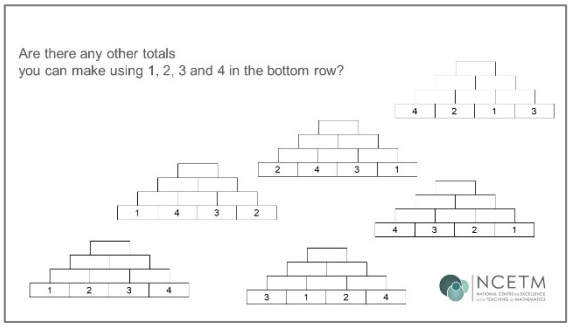
Slide 11

Slide 11 poses particular questions about the pyramids and the top values:



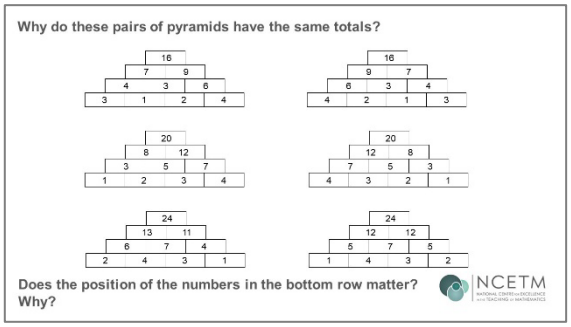
Slide 12

Slide 12 asks if it is possible to use the numbers to create any other totals.



Slide 14

Slide 14 compares pairs of pyramids with the same total at the top.



Commentary

'What do you notice about the pyramids?'

'What is the same and what is different?'

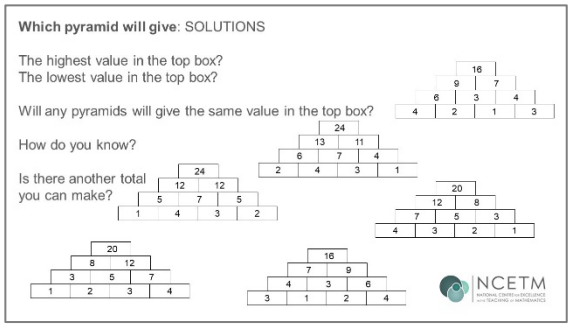
Encourage pupils to explain their thinking.

Where the same two numbers are in the central cells and the outer cells of the bottom row, whatever the order, the total at the top will be the same.

For example 2, 4, 1, 3 will give the same total as 3, 1, 4, 2 (20)

It is possible to make the totals 16, 18, 20, 22 and 24 in the top cell of this pyramid.

The solutions are on hidden **Slide 13**.

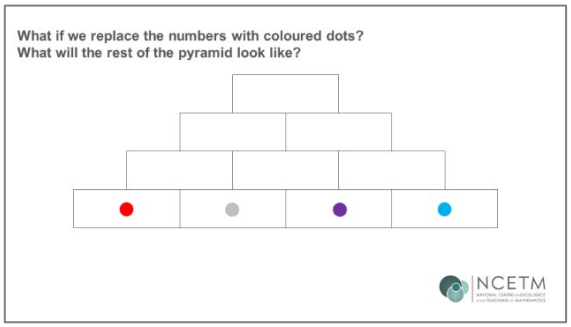
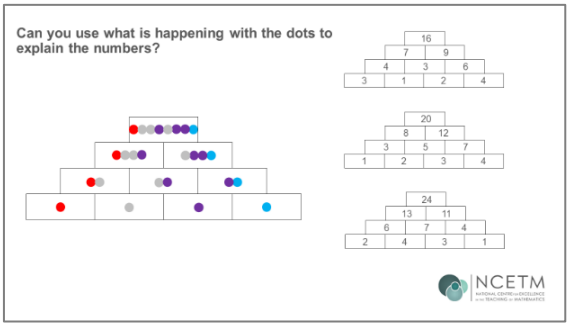
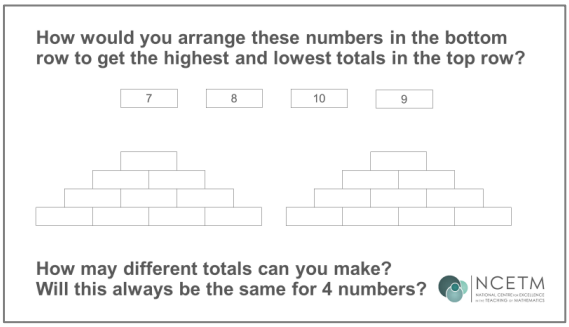


Task 3: Explaining the pyramids

Once the pupils have explored the totals, they can think about using expressions to represent the values of the boxes and focus on how the values are made up from combinations of the numbers in the bottom row.

The use of coloured dots instead of numbers allows pupils to focus on the combinations and to see that there are more grey and purple dots than red and blue as we go up the pyramid.

You could try using coloured counters to represent the numbers, which may highlight that there are more of certain colours at the top than others.

Slides 15–24	Commentary
Slide 16  	*'If we replace the numbers with dots, what will the rest of the pyramid look like?'* The slides build the dots up a row at a time. Pupils can then compare the pyramids with numbers to the pyramids with dots to explain why the totals at the top are larger/smaller. The total at the top is larger when the larger numbers are in the middle two boxes on the bottom row.
Slide 20  	*'What do you notice about the colours?'* Compare the colours with the numbers: • Which colour represents 1 in this image, which represents 2 etc.? • What do they notice about the colours in the top row?
Slide 24  	The final activity in this section gives a different set of four consecutive numbers for the bottom row and asks pupils to arrange them to get the highest and lowest totals in the top row, before going on to work out all the possible totals with these four numbers. There will always be five possible totals with a bottom row of four numbers because two arrangements give the 'middle' total value (20 for 1, 2, 3, 4).

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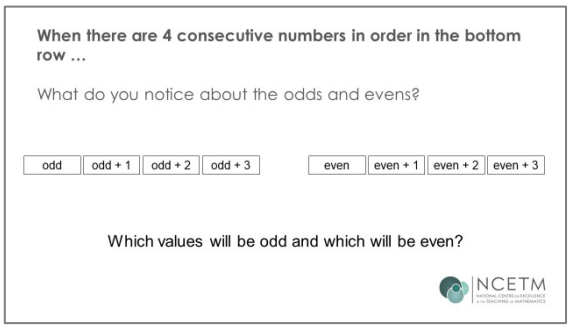
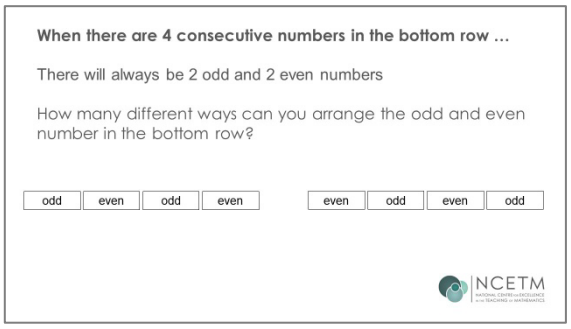
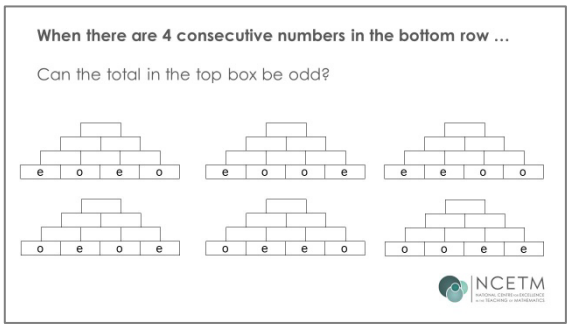
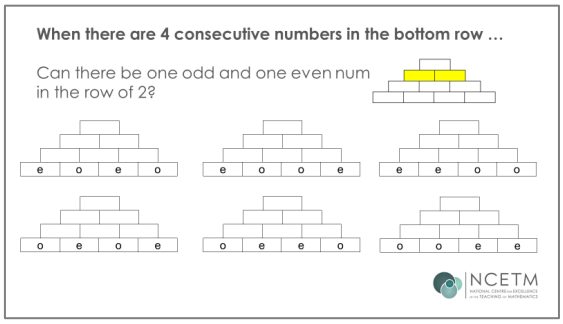
Task 4: Exploring odds and evens

This final task starts by looking at patterns of odd and even numbers in the totals. It gives the opportunity for pupils to generalise about sequences of odd and even numbers, and about the sum when odd and even, odd and odd or even and even numbers are added.

Worksheet 3 is a version of Slide 28 to support discussion.

Slides 26–31	Commentary
Slide 26  **Slide 27**  	Slides 26 and 27 look at the fact that we can express consecutive numbers (starting with the smallest) as: odd, odd + 1, odd + 2, odd +3 or even, even + 1, even + 2, even + 3 This can be simplified to: odd, even, odd, even or even, odd, even, odd Pupils may need to be reminded of the generalisations when adding odd and even numbers to support the discussion.
Slide 28  	Pupils then explore the different arrangements of odd and even values in the bottom row of the pyramid, before going on to complete the pyramids to find out if the top value can ever be odd.
Slide 29  	Next, pupils look at the row of 2 (the third row), and the odd and even properties of the two values. Through discussion of the patterns of odd and even numbers in the pyramids formed from consecutive numbers, the following statements can be made: • The second row (the row of 3) is always symmetrical. • The third row (the row of 2) can't have one odd and one even value. • The top total has to be even because e + e and o + o always gives an even total.

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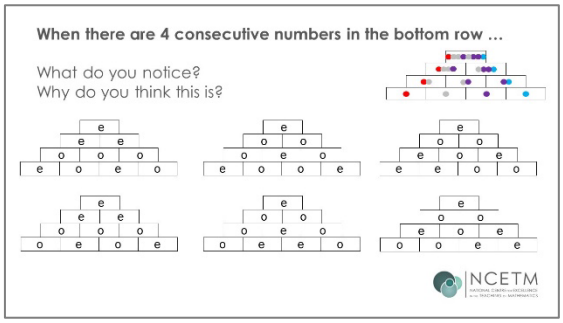
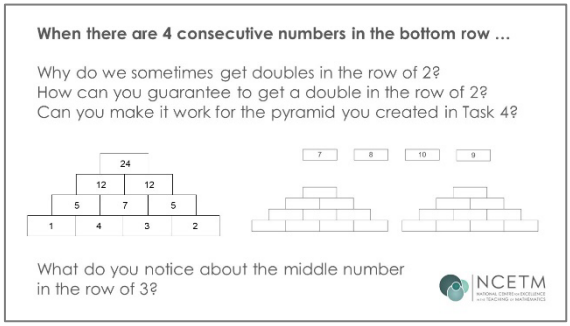
	Looking at the coloured dots, you see that there is always an even number of odd numbers, however you arrange the numbers.
Slide 30  	There is then an opportunity to look at the pyramids completed with o and e to represent odds and evens and to say what they notice and why they think this is. The coloured dot representation is included on Slide 30, as this allows pupils to see that there is always an even number of odd values in each arrangement, meaning that the total will always be even.
Slide 31  	Slide 31 focuses on the arrangements in the bottom row which lead to doubles in the row of two (the third row). Pupils finish by using a different set of four numbers and arranging them to create the double number in the third row.

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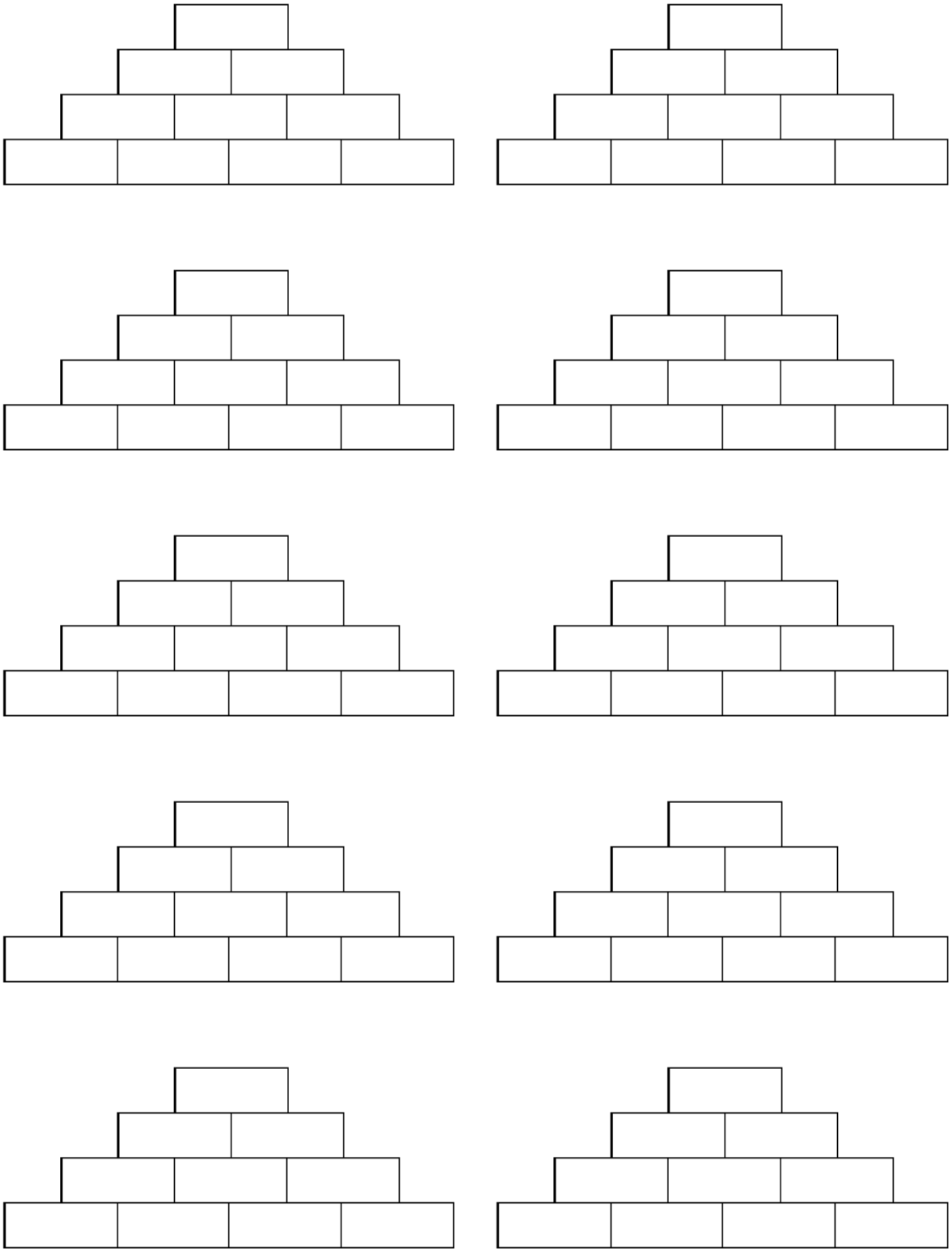
Worksheet 1

Blank pyramids



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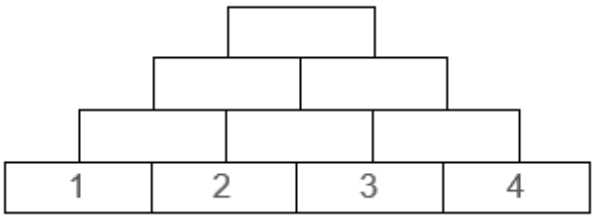
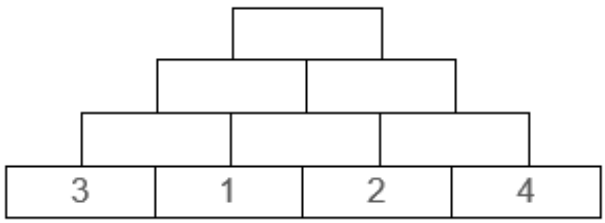
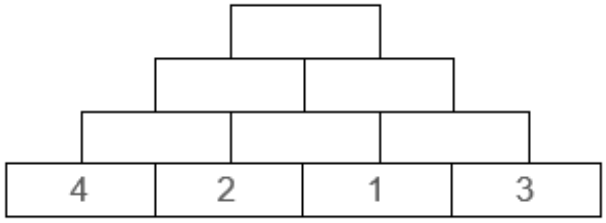
Worksheet 2

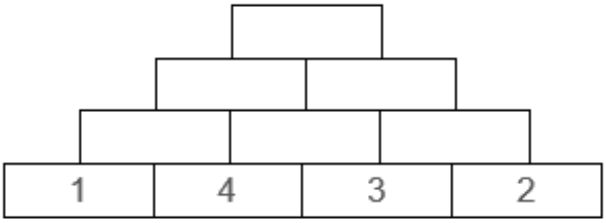
Card images of Slide 9

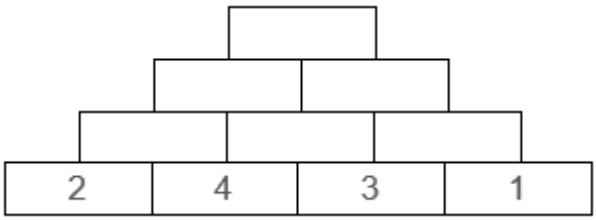
What do you notice about these pyramids? What is the same and what is different?

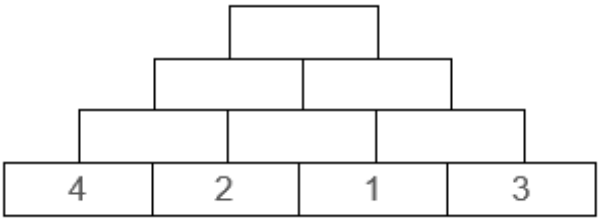












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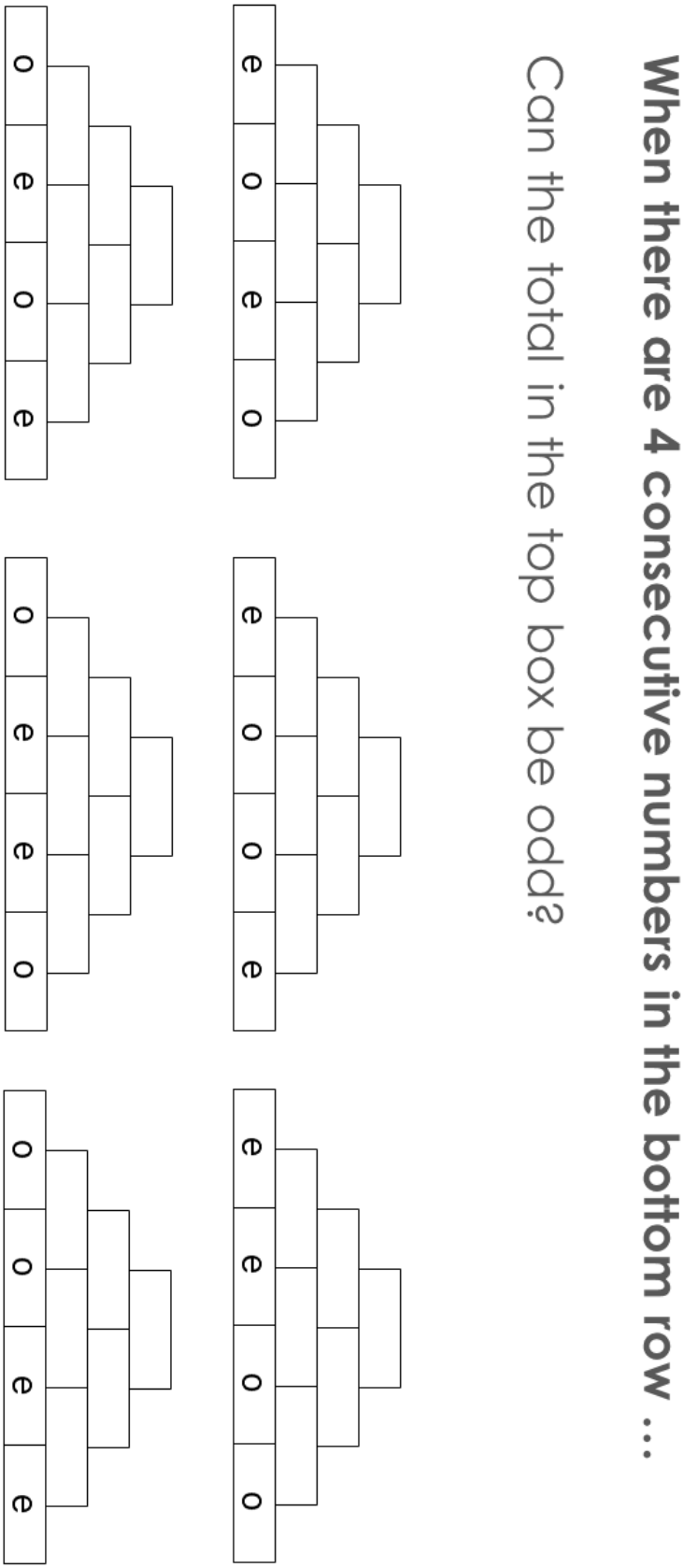
Worksheet 3

Card images of Slide 28