MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.30 Composition and calculation: numbers up to 10,000,000

Teacher guide | Year 6

Teaching point 1:

Patterns seen in other powers of ten can be extended to the unit 1,000,000.

Teaching point 2:

Seven-digit numbers can be written, read and ordered by identifying the number of millions, the number of thousands and the number of hundreds, tens and ones.

Teaching point 3:

The digits in a number indicate its structure so it can be composed and decomposed.

Teaching point 4:

Knowledge of crossing thousands boundaries can be used to work to and across millions boundaries.

Teaching point 5:

Sometimes numbers are rounded as approximations to eliminate an unnecessary level of detail; rounded numbers are also used to give an estimate or average. At other times, precise readings are useful.

Teaching point 6:

Fluent calculation requires the flexibility to move between mental and written methods according to the specific numbers in a calculation.

Summer 2018 pilot

Overview of learning

In this segment children will:

- learn to read and write seven-digit numbers and understand the value of each digit
- learn how to order numbers by comparing digits
- explore partitioning larger numbers in different ways and link this to additive calculations
- examine why it is helpful to round numbers and how we might use this
- investigate mental and written strategies and how the appropriateness of a mental or written strategy is determined by the numbers themselves
- solve routine and non-routine problems involving larger numbers.

In segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*, children learnt about multiples of 1,000 up to 1,000,000 (e.g. 342,000). The focus just on these numbers, rather than numbers with six significant digits (e.g. 342,654) was so that children would really make the link between the properties of 342 thousand and the properties of 342. This segment extends the number set that children work with to all six- and seven-digit numbers, and also makes links to numbers less than one. It is recommended that you read segment *1.26* to understand the foundations children have in place prior to starting this segment.

As with other segments with a strong place-value element, such as segments *1.22 Composition and calculation: 1,000 and four-digit numbers* and *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*, this segment starts by spending time learning about the new unit (here, 1,000,000) and its composition. The segment then moves on to reading and writing large numbers up to 10,000,000, as well as ordering, composing and decomposing them. Additive and place-value elements are integrated throughout the segment; for example, expressing the structure of the numbers with additive equations.

Children will then explore the reasons for rounding. They will learn to round large numbers to the nearest 1,000,000 or 100,000, before tying this together with previous work on rounding to round any number to any degree of accuracy.

By the end of the segment, children will have the knowledge and confidence to calculate with large numbers, moving between mental and written methods as appropriate. Throughout the segment, links are also made to smaller numbers, including decimals. However, much of the practice focuses on the new learning, that is, numbers between 1,000,000 and 10,000,000. As you provide practice for your class on working in additive contexts and solving additive problems, make sure that you also include practice with different sized numbers so that the children are confident working with numerals with any number of digits.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Patterns seen in other powers of ten can be extended to the unit 1,000,000.

Steps in learning

	Guidance	**Representations**

1:1

In Year 5, children touched on the unit 1,000,000 (see segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*). They learnt how to write it and that it can be composed of ten groups of 100,000 but did little more than this. Begin this segment by showing them 1,000,000 written as a numeral, both with and without the place-value headings.

1,000,000 is such a large number that it is difficult to imagine it:

- 1,000,000 is about the number of people who live in Birmingham, the second largest city in the UK.
- There are 1,000,000 cubic centimetres in one cubic metre (100 × 100 × 100 = 1,000,000).

The latter provides a useful representation of 1,000,000. A 1 m × 1 m × 1 m cube frame can be made using 12 metre-sticks. Small Dienes cubes are 1 cm^3 so it would take 1,000,000 of them to fill a 1 m^3 frame.

1,000,000 is also made up of 1,000 groups of 1,000. You might ask children to imagine 1,000 jigsaws, with 1,000 pieces in each. As there are 1,000 mm in 1 m, there are also 1,000 × 1,000 mm^2 (i.e. 1,000,000 mm^2) in 1 m^2.

Unitise to make the connection between 999,000 (999 thousands) and 1,000,000 (1,000 thousands). As a class, repeat the generalised statement: ***'One million is one thousand thousands.'***

Whereas we can count up to 100 in a minute or so, it would take about a month to count to 1,000,000 if we counted in ones.

Display a number line with intervals of 1,000,000. Use dual labelling, as shown below, to make sure children are confident with both ways of writing millions. Count up from 0 to 10,000,000 in steps of 1,000,000, tapping each 1,000,000 on the line as you go.

Place-value in 1,000,000:

1,000,000

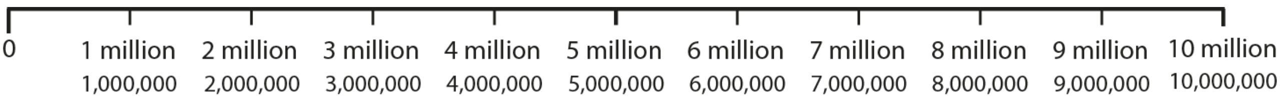
Millions			**Thousands**			**Ones**		
100s	**10s**	**1s**	**100s**	**10s**	**1s**	**100s**	**10s**	**1s**
		1	0	0	0	0	0	0

Unitising:

999,000 (999 thousands)

1,000,000 (1,000 thousands)

Number line:



1:2 Now, on a place-value chart, look at 1,000,000 and 10,000,000 alongside other powers of ten that the children have met previously, including the decimal units one tenth and one hundredth.

When the place-value chart is removed, it is harder to read the numbers, so explain that we use commas and decimal points to support this. The use of commas will be covered further in *Teaching point 2*. For now, practise reading and writing different powers of ten, for example:

- Write numbers as numerals (e.g. 10,000, 0.1 and 1,000,000) for the children and ask them to read out the numbers. Note that *'zero-point-one'* is one way to read out *'0.1'*; however, prompting the children to also read it as *'one tenth'* will help them to associate a value with the numeral, just as when we see *'100'* we say *'one hundred'* rather than *'one-zero-zero'*.
- Say a number (for example *'one million'*) and ask the children to write it in digits on their mini whiteboards. Focus particularly on the new units: one million and ten million.

Place-value table:

Millions			**Thousands**			**Ones**			**-ths**	
100s	**10s**	**1s**	**100s**	**10s**	**1s**	**100s**	**10s**	**1s**	$\frac{1}{10}$	$\frac{1}{100}$
								0 •	0	1
								0 •	1	
								1		
							1	0		
						1	0	0		
					1	0	0	0		
				1	0	0	0	0		
			1	0	0	0	0	0		
		1	0	0	0	0	0	0		
	1	0	0	0	0	0	0	0		

Reading and writing numbers:

0.01	one hundredth
0.1	one tenth
1	one
10	ten
100	one hundred
1,000	one thousand
10,000	ten thousand
100,000	one hundred thousand
1,000,000	one million
10,000,000	ten million

1:3 Once children are comfortable with powers of ten, move on to looking at multiples of these. Show them the Gattegno chart and look along the new rows. It is helpful for children to have a copy of the Gattegno chart in pairs to refer to so they can see the numbers clearly.

- Choose a row and count along it, asking the children to point to each number as you say it. As before, count *'one tenth, two tenths…'* and *'one hundredth, two hundredths…'* for the decimal rows.
- Say a number (for example *'five million'*) and ask the children point to it. Make sure that the children can also write these numbers on their mini whiteboards, moving to doing this without the chart as support.
- Point to a number on the class Gattegno chart and ask the children say it.

When the children are answering confidently, work up and down the columns asking, for example:

- *'Which number is ten times bigger than four million?'*
- *'Which number is one hundred times smaller than six?'*

Gattegno chart:

10,000,000	20,000,000	30,000,000	40,000,000	50,000,000	60,000,000	70,000,000	80,000,000	90,000,000
1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	70	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

1:4 By now, the children should be very confident with breaking down powers of ten into two, four, five and ten equal parts. These are significant multiplicative divisions, because they give the most common intervals used on scales in graphing and measures.

By displaying bar models, briefly recap how 1,000 can be composed multiplicatively from 500, 250, 200 and 100 from segment *1.22 Composition and calculation: 1,000 and four-digit numbers*.

Bar models – multiplicative compositions of 1,000:

<table>
<tr><td colspan="2">1,000</td></tr>
<tr><td>500</td><td>500</td></tr>
</table>

<table>
<tr><td colspan="4">1,000</td></tr>
<tr><td>250</td><td>250</td><td>250</td><td>250</td></tr>
</table>

<table>
<tr><td colspan="5">1,000</td></tr>
<tr><td>200</td><td>200</td><td>200</td><td>200</td><td>200</td></tr>
</table>

<table>
<tr><td colspan="10">1,000</td></tr>
<tr><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td></tr>
</table>

1:5 Extend this now to 1,000,000, using bar models to represent the divisions into equal parts. Unpick the related multiplication and division equations, as shown below.

Use unitising to make the link back to children's knowledge of 1,000, for example:

- *'We know that one quarter of one thousand is two hundred and fifty.'*
- *'So one quarter of one thousand thousands is two hundred and fifty thousands.'*
- *'That is, one quarter of one million is two hundred and fifty thousand.'*

Encouraging children to construct sentences such as these will ensure they are confident describing the numbers in words as well as writing them as numerals.

Bar models – multiplicative compositions of 1,000,000:

<table>
<tr><td colspan="2">1,000,000</td></tr>
<tr><td>500,000</td><td>500,000</td></tr>
</table>

$1{,}000{,}000 \div 2 = 500{,}000$

$1{,}000{,}000 \div 500{,}000 = 2$

$\frac{1}{2} \times 1{,}000{,}000 = 500{,}000$

$1{,}000{,}000 \times \frac{1}{2} = 500{,}000$

$2 \times 500{,}000 = 1{,}000{,}000$

$500{,}000 \times 2 = 1{,}000{,}000$

<table>
<tr><td colspan="4">1,000,000</td></tr>
<tr><td>250,000</td><td>250,000</td><td>250,000</td><td>250,000</td></tr>
</table>

$1{,}000{,}000 \div 4 = 250{,}000$

$1{,}000{,}000 \div 250{,}000 = 4$

$\frac{1}{4} \times 1{,}000{,}000 = 250{,}000$

$1{,}000{,}000 \times \frac{1}{4} = 250{,}000$

$4 \times 250{,}000 = 1{,}000{,}000$

$250{,}000 \times 4 = 1{,}000{,}000$

<table>
<tr><td colspan="5">1,000,000</td></tr>
<tr><td>200,000</td><td>200,000</td><td>200,000</td><td>200,000</td><td>200,000</td></tr>
</table>

$1{,}000{,}000 \div 5 = 200{,}000$

$1{,}000{,}000 \div 200{,}000 = 5$

$\frac{1}{5} \times 1{,}000{,}000 = 200{,}000$

$1{,}000{,}000 \times \frac{1}{5} = 200{,}000$

5 × 200,000 = 1,000,000

200,000 × 5 = 1,000,000

1,000,000									
100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000

1,000,000 ÷ 10 = 100,000

1,000,000 ÷ 100,000 = 10

$\frac{1}{10} \times 1,000,000 = 100,000$

$1,000,000 \times \frac{1}{10} = 100,000$

10 × 100,000 = 1,000,000

100,000 × 10 = 1,000,000

1:6 Again by displaying bar models, briefly recap how 10,000 can be composed multiplicatively from 5,000, 2,500, 2,000 and 1,000 from segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000.*

Bar models – multiplicative compositions of 10,000:

10,000	
5,000	5,000

10,000			
2,500	2,500	2,500	2,500

10,000				
2,000	2,000	2,000	2,000	2,000

10,000									
1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

1:7

Extend this now to 10,000,000. Just as in step *1:5*, use bar models to represent the divisions into equal parts and then unpick the related multiplication and division equations.

Use unitising to make the link back to children's knowledge of 10,000, for example:

- *'We know that one quarter of ten thousand is two thousand five hundred.'*
- *'So one quarter of ten thousand thousands is two thousand five hundred thousands.'*
- *'That is, one quarter of ten million is two million five hundred thousand.'*

Bar models – multiplicative compositions of 10,000,000:

10,000,000	
5,000,000	5,000,000

$10{,}000{,}000 \div 2 = 5{,}000{,}000$

$10{,}000{,}000 \div 5{,}000{,}000 = 2$

$\frac{1}{2} \times 10{,}000{,}000 = 5{,}000{,}000$

$10{,}000{,}000 \times \frac{1}{2} = 5{,}000{,}000$

$2 \times 5{,}000{,}000 = 10{,}000{,}000$

$5{,}000{,}000 \times 2 = 10{,}000{,}000$

10,000,000			
2,500,000	2,500,000	2,500,000	2,500,000

$10{,}000{,}000 \div 4 = 2{,}500{,}000$

$10{,}000{,}000 \div 2{,}500{,}000 = 4$

$\frac{1}{4} \times 10{,}000{,}000 = 2{,}500{,}000$

$10{,}000{,}000 \times \frac{1}{4} = 2{,}500{,}000$

$4 \times 2{,}500{,}000 = 10{,}000{,}000$

$2{,}500{,}000 \times 4 = 10{,}000{,}000$

10,000,000				
2,000,000	2,000,000	2,000,000	2,000,000	2,000,000

$10{,}000{,}000 \div 5 = 2{,}000{,}000$

$10{,}000{,}000 \div 2{,}000{,}000 = 5$

$\frac{1}{5} \times 10{,}000{,}000 = 2{,}000{,}000$

$10{,}000{,}000 \times \frac{1}{5} = 2{,}000{,}000$

$5 \times 2{,}000{,}000 = 10{,}000{,}000$

$2{,}000{,}000 \times 5 = 10{,}000{,}000$

10,000,000									
1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000

$10{,}000{,}000 \div 10 = 1{,}000{,}000$

$10{,}000{,}000 \div 1{,}000{,}000 = 10$

$\frac{1}{10} \times 10{,}000{,}000 = 1{,}000{,}000$

$10{,}000{,}000 \times \frac{1}{10} = 1{,}000{,}000$

$10 \times 1{,}000{,}000 = 10{,}000{,}000$

$1{,}000{,}000 \times 10 = 10{,}000{,}000$

1:8 Finally, provide varied practice for breaking different powers of ten into two, four, five and ten equal parts in a range of contexts, including graphing and measures, such as those shown below. Each time, encourage the children to identify the number of parts each interval is divided into.

- *'My cat weighed 4 kg. She has been poorly and has lost* $\frac{1}{4}$ *kg of weight. How much does she weigh now?'*
- *'About 1 million people live in Birmingham. About* $\frac{1}{5}$ *of the population is over 60 years old. Approximately how many over 60s live in Birmingham?'*
- *'A builder orders 1,000 kg of sand. She has about 100 kg left. What fraction of the total amount is left?'*
- *'It is about 10,000 km from Manchester to Cape Town. A plane is* $\frac{1}{4}$ *of the way through the flight. How many kilometres of the journey are left?'*

Bar models:

'Fill in the missing numbers.'

<table>
<tr><td colspan="4">10,000,000</td></tr>
<tr><td>?</td><td>?</td><td>?</td><td>?</td></tr>
</table>

<table>
<tr><td colspan="10">1,000</td></tr>
<tr><td>?</td><td>?</td><td>?</td><td>?</td><td>?</td><td>?</td><td>?</td><td>?</td><td>?</td><td>?</td></tr>
</table>

<table>
<tr><td colspan="4">?</td></tr>
<tr><td>250,000</td><td>250,000</td><td>250,000</td><td>250,000</td></tr>
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<table>
<tr><td colspan="2">0.1</td></tr>
<tr><td>?</td><td>?</td></tr>
</table>

<table>
<tr><td colspan="5">1,000,000</td></tr>
<tr><td>?</td><td>?</td><td>?</td><td>?</td><td>?</td></tr>
</table>

<table>
<tr><td colspan="2">?</td></tr>
<tr><td>0.5</td><td>0.5</td></tr>
</table>

Missing-number sequences:

'Fill in the missing numbers.'

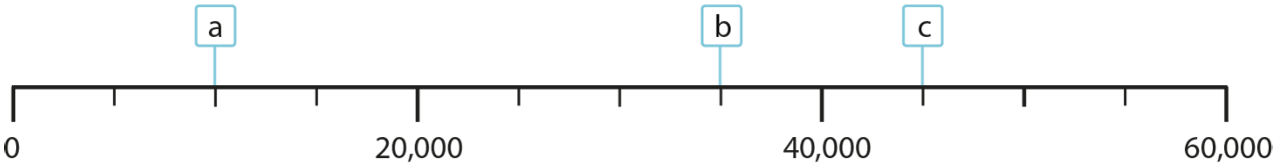
0			0.6	0.8		

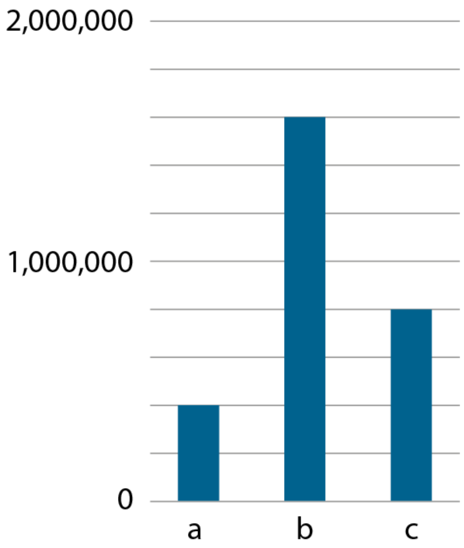
	400,000	425,000			500,000

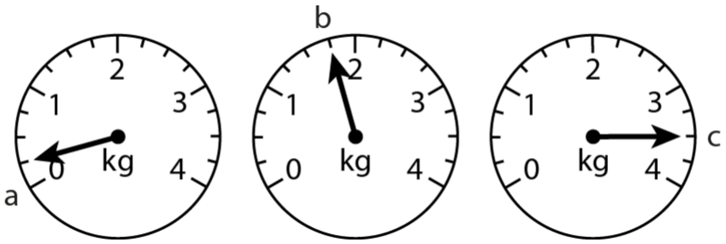
990,000			1,020,000		

Reading scales (including graphing and measures):

'What numbers are shown by the letters a, b *and* c*?'*







Equations:

'Fill in the missing numbers.'

$10{,}000{,}000 \div 4 = \boxed{}$

$20{,}000 = 100{,}000 \div \boxed{}$

$\frac{1}{10}$ of $1{,}000{,}000 = \boxed{}$

$\boxed{} \times 2{,}500 = 10{,}000$

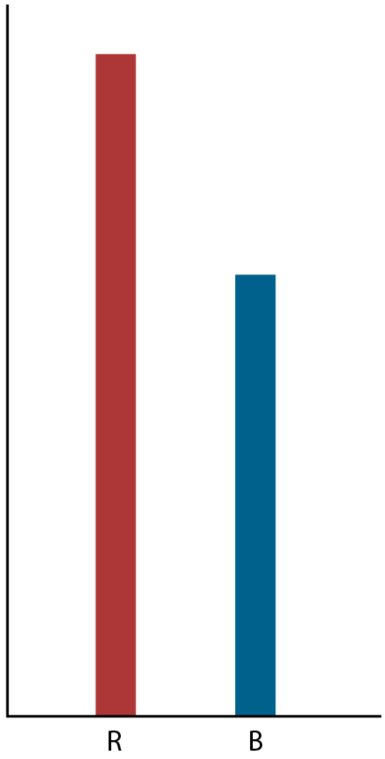
$\boxed{} \times 10{,}000 = 2{,}500$

Dòng nǎo jīn:

'The red bar has a value of 8,250,000. The blue bar has a value of 5,500,000.

Draw bars with values of approximately:

- *7,000,000*
- *5,750,000*
- *1,200,000'*



Teaching point 2:

Seven-digit numbers can be written, read and ordered by identifying the number of millions, the number of thousands and the number of hundreds, tens and ones.

Steps in learning

	Guidance	Representations

2:1

In segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*, children learnt about five- and six-digit multiples of 1,000. Check that they are confident reading and writing these numbers. Ask:

- *'Can you read these numbers?'*
  512,000 65,000 408,000…
- *'Write these numbers:*
  - *Three hundred and sixty-seven thousand*
  - *Seven hundred and nineteen thousand…'*

They should also be confident writing and reading four-digit numbers from segment *1.22 Composition and calculation: 1,000 and four-digit numbers*. Show them a four-digit number (e.g. 1,937) on a place-value chart, then add a ten thousands digit (e.g. 51,937) and then a hundred thousands digit (e.g. 451,937), each time asking the children to read the new number.

Now teach the structure of larger numbers by pointing out how the digits are grouped; that is, they are arranged in groups of three starting from the right-hand side. Build on previous learning, starting with how numbers beyond hundreds, tens and ones means we are dealing with thousands.

Show how the thousands digits represent one thousands, ten thousands and hundred thousands.

Place-value chart:

Millions			Thousands			Ones		
100s	**10s**	**1s**	**100s**	**10s**	**1s**	**100s**	**10s**	**1s**
					1	9	3	7
				5	1	9	3	7
			4	5	1	9	3	7

2:2

Add one further digit to the front of the number and together read the number that is now written. If necessary, refer back to the number 5,000,000, which the children are familiar with from *Teaching point 1*.

A place-value chart supports identifying the value of digits. The use of commas as millions and thousands separators was touched upon in *Teaching point 1* but give this more attention now, pointing out that the commas are where we say the words *'million'* or *'thousand'*.

Give the children practice reading and writing numbers up to 10,000,000, for example:

- say a number and ask the children write it on their mini whiteboards

- show the children seven-digit numbers written with numerals and read them out both as a class and choosing individual children.

Place-value chart:

Millions			**Thousands**			**Ones**		
100s	**10s**	**1s**	**100s**	**10s**	**1s**	**100s**	**10s**	**1s**
					1	9	3	7
				5	1	9	3	7
			4	5	1	9	3	7
		5	4	5	1	9	3	7

1**,**937 (one **thousand**, nine hundred and thirty-seven)

51**,**937 (fifty-one **thousand**, nine hundred and thirty-seven)

451**,**937 (four hundred and fifty-one **thousand**, nine hundred and thirty-seven)

5**,**451**,**937 (five **million**, four hundred and fifty-one **thousand**, nine hundred and thirty-seven)

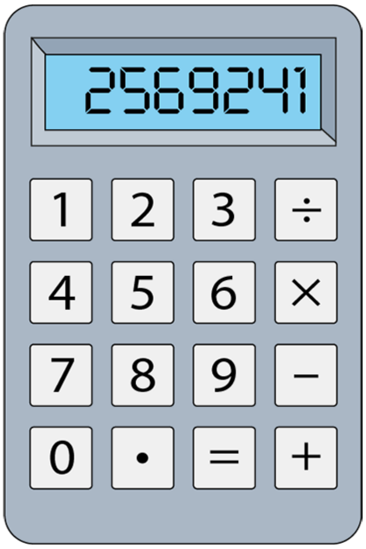
2:3

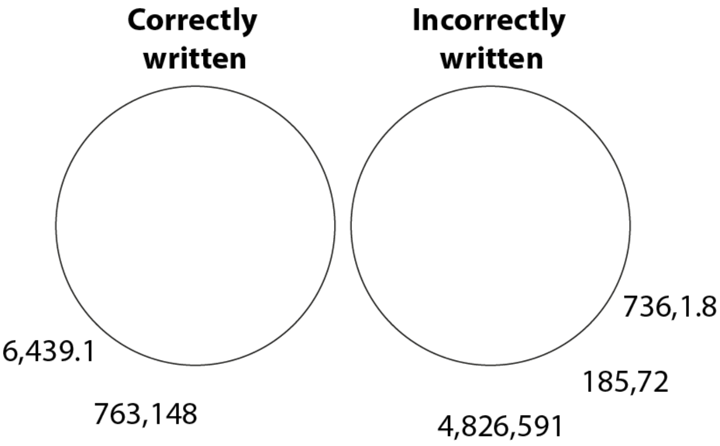
Now display some seven-digit numbers that have spaces instead of commas as millions and thousands separators so that children become confident reading the different formats.

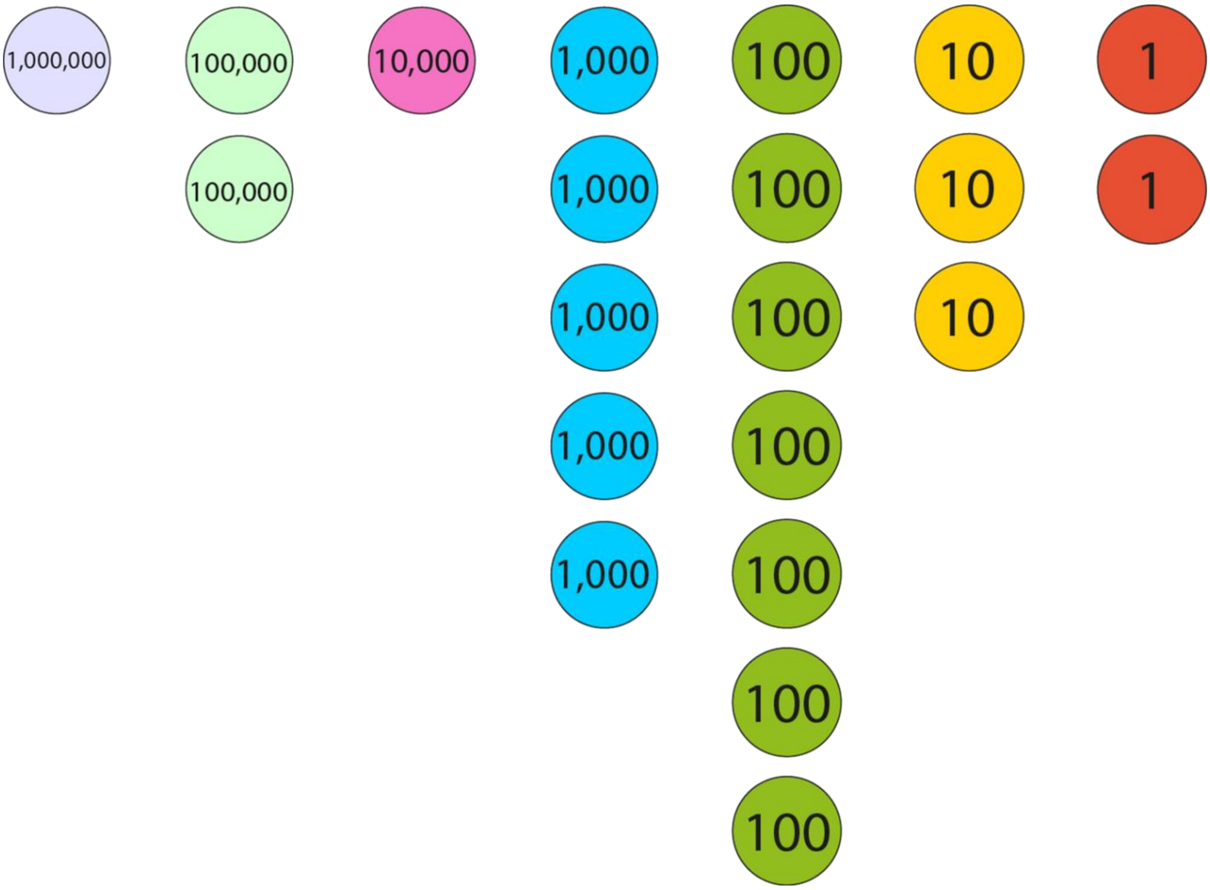
Also display some numbers that have no separators at all, such as on a calculator display. Show the children how to write these correctly with separator commas, starting from the right-hand side (e.g. 2,569,241).

Point out the common error of working from the left-hand side (e.g. 256,924,1). Explore why this is incorrect, drawing attention to the fact that the last three digits represent the hundreds, tens and ones and the commas link to the separations of the place-value chart (into ones, thousands and millions as in step *1:2*). Explain that it is therefore necessary to work from right to left, identifying the value of the final three digits first.

Numbers with no millions and thousands separators:

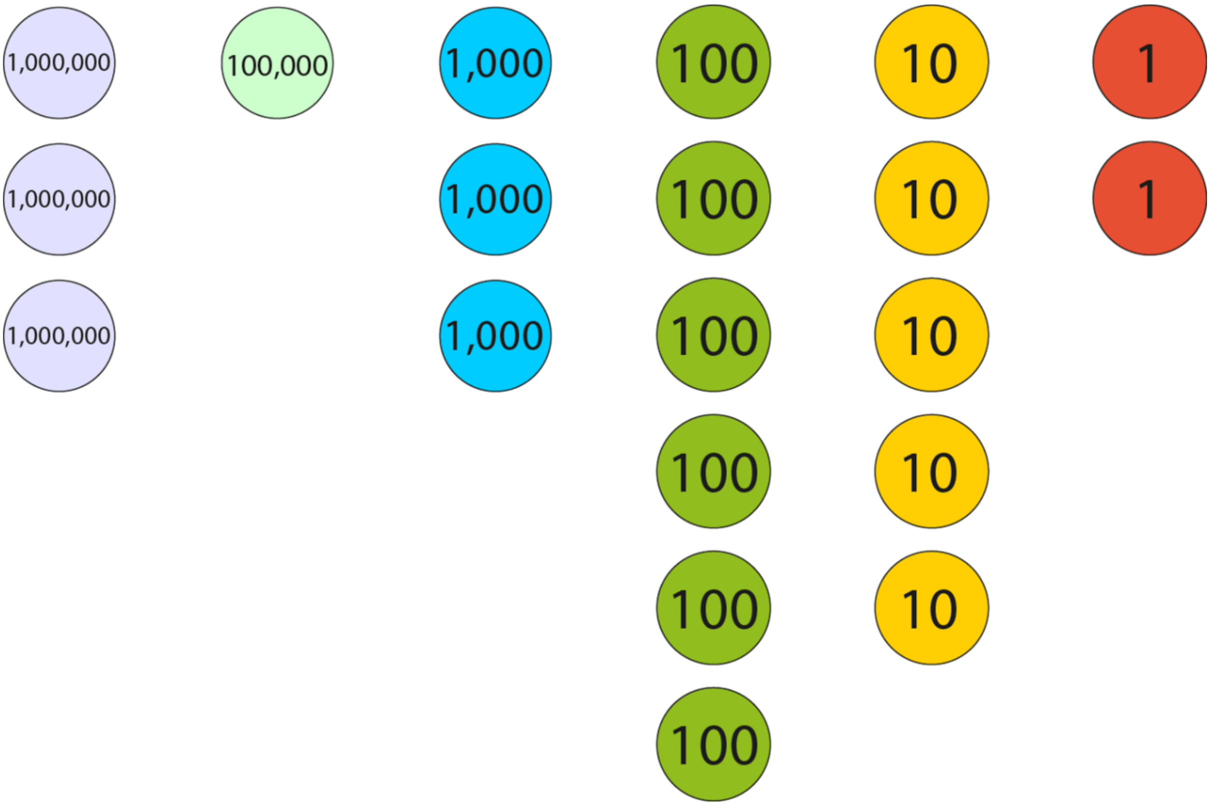


<table>
<tr>
<td></td>
<td>Sort correctly and incorrectly written numbers into sorting circles.
Give the children a range of calculations to solve on a calculator that result in five-, six- and seven-digit answers. Ask them to write the answers including the thousands and millions separator commas.</td>
<td>Sorting:

Calculator work:
‘Work out the following using a calculator. You do not type the comma into the calculator.’
$3{,}500 \times 782 = \boxed{}$</td>
</tr>
<tr>
<td>2:4</td>
<td colspan="2">Now move on to using place-value counters to represent different numbers up to 10,000,000 for children to read and write.
Spend time practising and consolidating this.</td>
</tr>
</table>

	Place-value counters: • *'What number is represented?'*  • *'Now can you represent the number four hundred and thirty-two thousand, one hundred and fifty-six?'*
2:5	Continue the activity form the previous step, but now draw careful attention to the use of zero as a place holder. This builds on key learning about place-holding zeros in the context of four-digit numbers from segment *1.22 Composition and calculation: 1,000 and four-digit numbers*.

Place-value counters:

- *'What number is represented?'*

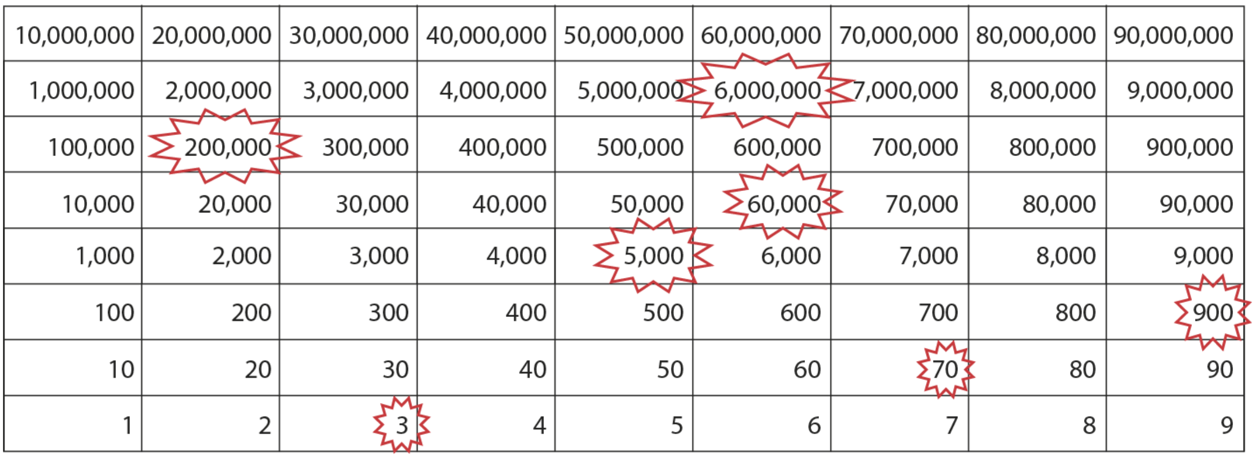


- *'Now can you represent the number six hundred and three thousand and forty-one?'*

2:6 For further practice and consolidation, use the Gattegno chart. Highlight or point to the different parts of a number, as shown below, and ask the children to say or write the whole number. Then give them numbers to represent in the same way. Continue to give opportunities to recognise and create numbers that contain place-holding zeros.

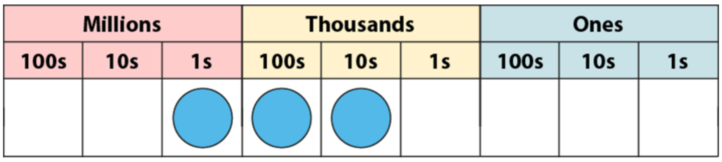
Also give children practice moving between the Gattegno chart, place-value counters, written numerals (e.g. 6,265,973) and spoken number names, (for example, *'six million, two hundred and sixty-five thousand, nine hundred and seventy-three'*).

Gattegno chart:



10,000,000	20,000,000	30,000,000	40,000,000	50,000,000	60,000,000	70,000,000	80,000,000	90,000,000
1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

2:7	Once children are secure in the representation of numbers, develop this further by asking them to determine the value of each digit in numbers up to 10,000,000, without the use of place-value charts. This is essential before moving on to *Teaching point 3*, in which numbers are partitioned in different ways. Use the stem sentences: - ***'The ___ represents ___.'*** - ***'The value of the ___ is ___'.***	Place value: *'What is the value of the red digit?'* **9,378,912** - *'The "8" represents eight thousand.'* - *'The value of the "8" is eight thousand.'*

2:8	It is important to vary the questions you present to children to promote conceptual understanding.
	Provide opportunities for the children to reason and explain. For example:
	- *'Jack has written the number three million, forty-two thousand, five hundred and sixty-four as "3,42,564". Is he correct? Explain.'*
	- *'What is the value of the digit "5" in each of these numbers?*
	720,541
	5,876,023
	1,587,900
	651,920
	905,389
	2,120,806.5
	8,002,345
	701,003.15'
	- *'Write a seven-digit number where the "5" is worth:*
	  - *five million*
	  - *five thousand*
	  - *five hundred*
	  - *fifty thousand*
	  - *five hundred thousand.'*
		Dòng nǎo jīn:
		'Walid has a place-value chart and three counters. Here, he has represented the number 1,110,000.'
		
		
		- *'Find two different numbers that Walid could represent that add up to 2,002,002.'*
		- *'Now find another pair and another… How many can you find?'*
		- *'Find two different numbers that Walid could make that:*
		  - *have a difference of nine hundred thousand*
		  - *have a difference of ninety thousand*
		  - *have a difference of nine thousand.'*
		- *'Find two different numbers that Walid could make so that:*
		  - *one number is one thousandth of the other number*
		  - *one number is one hundred times larger than the other number.'*

2:9	Now that children understand the value of each digit, the next step is to compare numbers up to 10,000,000 using the symbols <, > and =. You should be able to move through this fairly quickly, as it will be familiar to	

	children from their earlier work (for example, comparing five- and six-digit numbers in segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*). Begin by comparing numerals with different numbers of digits, as shown by the first example opposite. Progress to comparing similar numerals that start with the same digits, as shown by the second example opposite. In this example, draw attention to the largest (place-value) digit that is different, in order to decide which is the larger number. Show the children how to compare digits moving from left to right. Incorporate opportunities for the children to reason and explain using place value.	Comparing numbers: *'Fill in the missing symbols (< > or =).'* 34,601 ○ 341,670 45,301 ○ 45,261 Comparing numbers with reasoning: *'What is the missing symbol (< >* or *=)? Explain why using place value.'* 6,352,783 ○ 6,357,283 206,312 ○ 206,052
2:10	Complete this teaching point by providing varied practice comparing numbers up to 10,000,000. Include: • comparisons without millions or thousands separators (comma or space) • comparisons where the learning is contextualised, for example: *'A postman delivers 100,312 letters in his first week and 100,052 letters in his second week. In which week did he deliver more letters?'* • opportunities for the children to articulate their understanding using place value, for example: *'Look at this set of numbers.* *5,963,931 782,548* *828,804 4,060,942* *Ezra says 828,804 is the largest because it starts with the highest digit. Do you agree? Explain.'* • ordering sets of numbers. If children struggle with ordering sets of numbers, prompt them to look at the place-value of each digit and then	Comparing numbers: *'Fill in the missing symbols (< or >).'* 7,142,294 ○ 7,124,294 6,090,100 ○ 690,100 589,940 ○ 1,010,222 99,000 ○ 600,000 1300610 ○ 140017 Ordering numbers: *'Put these numbers in order from smallest to largest.'* 8,102,304 8,021,403 843,021 8,043,021

	compare the digits with the same place value (for example, comparing the hundred thousands digits), working from left to right.	Dòng nǎo jīn: *'How many ways can you arrange these digit cards so that the inequality is true?'* 1 3 4 7 4 ☐ 3 ☐ 2 0 0 < ☐ 6 ☐ 2 1 0 0

Teaching point 3:

The digits in a number indicate its structure so it can be composed and decomposed.

Steps in learning

Guidance | **Representations**

3:1 Once children are secure in reading and writing large numbers, move them on to partitioning numbers.

Begin this teaching point by displaying a seven-digit number (e.g. 5,381,492) and, as a class, partitioning each digit. A place-value chart or Gattegno chart would be ideal for this.

Also write the partitioned addition equation, as shown below.

Partitioning:

5,381,492

- Place-value chart

Millions			**Thousands**			**Ones**		
100s	**10s**	**1s**	**100s**	**10s**	**1s**	**100s**	**10s**	**1s**
		5	3	8	1	4	9	2

- Gattegno chart

10,000,000	20,000,000	30,000,000	40,000,000	50,000,000	60,000,000	70,000,000	80,000,000	90,000,000
1,000,000	2,000,000	3,000,000	4,000,000	**5,000,000**	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	**300,000**	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	**80,000**	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	**400**	500	600	700	800	900
10	20	30	40	50	60	70	80	**90**
1	**2**	3	4	5	6	7	8	9

- Equation

5,381,492 = 5,000,000 + 300,000 + 80,000 + 1,000 + 400 + 90 + 2

3:2 Repeat step *3:1* with other large numbers, varying the representations you use. For example:

- present the number as a numeral, as in the previous step
- represent the number with place-value counters, as shown below.

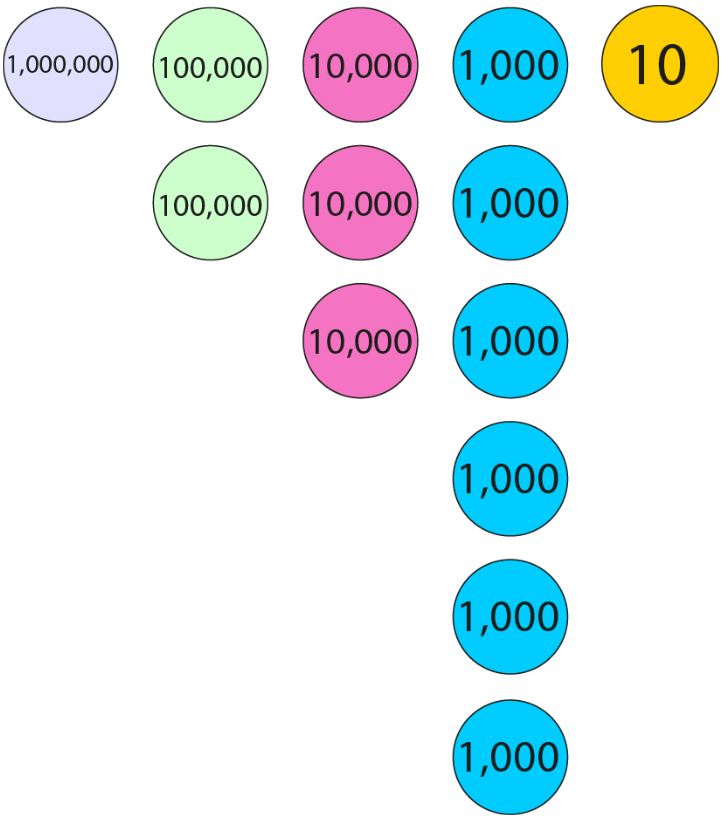
As well as presenting place-value counters in place-value order, present randomly arranged mixed groups to the children or ask them to pick, for example, six, eight or ten counters at random and then write an addition equation to show the total they have.

For further practice, give problems such as:

- *'You have place-value counters for each power of ten from one up to and including one million. You pick six counters.*
    - *What is the largest number you can make?*
    - *What is the smallest number you can make?*
    - *Make a number as near to five hundred thousand as possible.*
    - *Make a number as near to nine hundred thousand as possible.'*
- Dòng nǎo jīn:
  'You have some place-value counters. Some show "1,000,000", some show "1,000" and some show "1". You choose any three counters. Write all the different possible numbers you can make.'

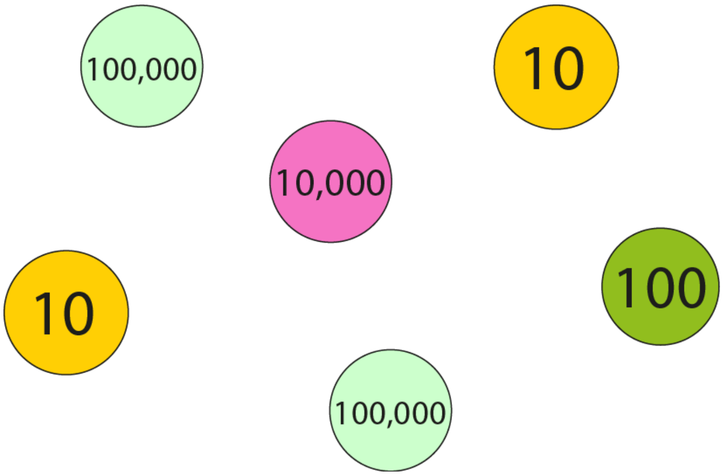
Place-value counters:

- Grouped by value and in place-value order



1,000,000 + 200,000 + 30,000 + 6,000 + 10 = 1,236,010

- Randomly arranged mixed group



200,000 + 10,000 + 100 + 20 = 210,120

3:3 Develop this concept further by setting missing-number addition problems. This will challenge the children’s thinking and their understanding of how to partition numbers in different ways.

Link this to what children learnt about comparing numbers in *Teaching point 2* by asking them to compare numbers that involve adding parts.

Missing-number problems:

‘Fill in the missing numbers.’

761,902 = 700,000 + ☐ + 1,000 + 900 + 2

2,124,003 = ☐ + 3

4,800,672 = 4,000,000 + 800,000 + ☐ + 70 + ☐

923,516 = ☐ + 3,000 + 500 + 10 + 6

5,062,100 = 5,000,000 + ☐ + 100

Comparing numbers up to 10,000,000:

‘Fill in the missing symbols (< or >).’

5,320,812 ○ 5,000,000 + 300,000 + 40,000 + 10

459,013 ○ 200,000 + 500 + 200,000 + 60,000

3:4

To develop fluency, activities could include *'zap the digit'*, where you ask the children to change a particular digit to zero by subtracting its value. For example: *'Write a subtraction calculation to zap the digit "6" from the number 3,561,301.'*

Doing this on calculators will allow the children to check their answers as they go, as well as give them practice in decoding larger numbers presented without separator commas.

Practise zapping digits of different values, including decimal numbers. For example: *'Write a subtraction calculation to zap the digit "2" from the number 539.25.'*

3:5

The children should now be ready to learn how to add and subtract mentally where the calculation *does not bridge a boundary*. Begin by adding and subtracting thousands, then ten thousands and, finally, hundred thousands, so that only one digit is changed at a time.

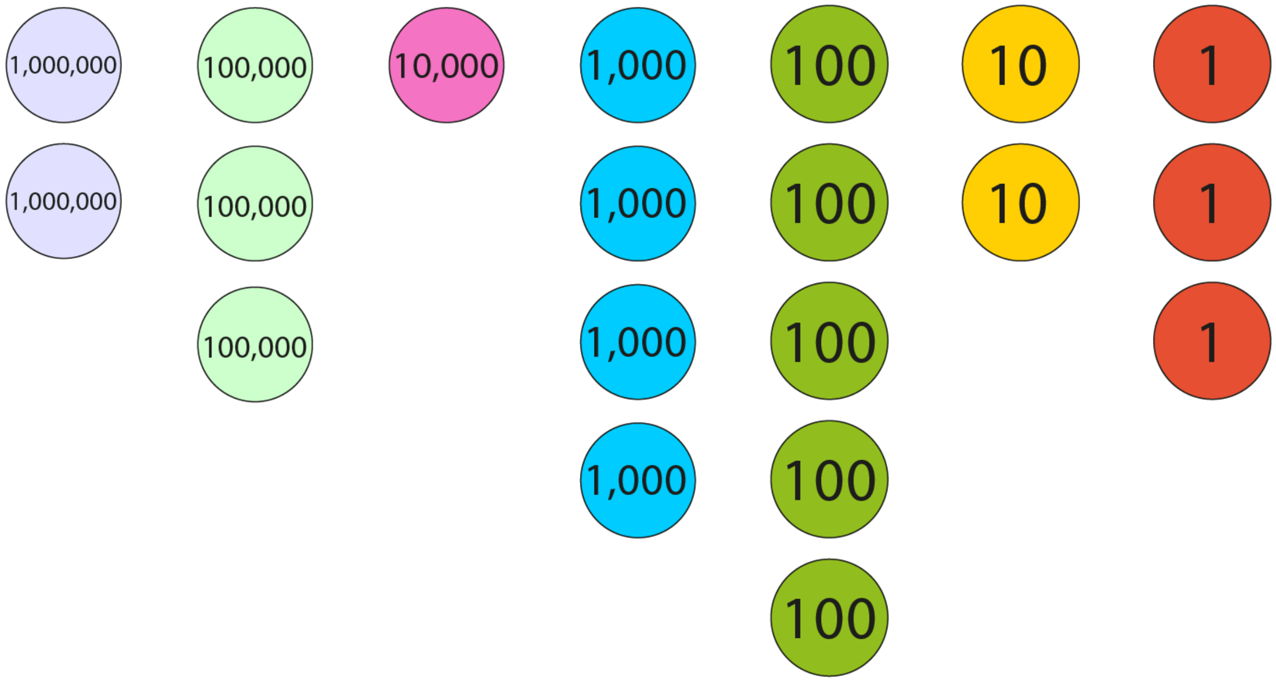
Return briefly to the use of place-value counters and place-value charts, but be mindful of the use of these as a temporary scaffold to be removed once independence is achieved. From previous segments, the children have had lots of practice subtracting parts of numbers in this way and you may find that they can move to working without these scaffolds quite quickly.

Here it is helpful to explore sets of additions and subtractions using the same digits. Aim to make small steps between the example questions so that connections can be seen more easily.

For the dòng nǎo jīn question (shown below), prompt the children to solve the calculations by thinking about the relative difference between the number being added and the number being subtracted. For example, for the first calculation, they should work out that the answer must be 100,000 more than the starting number.

Addition and subtraction – without bridging, changing one digit at a time:

- *'What is 2,314,523 + 30,000?'*



- *'Subtract 20,000 from 523,810.'*

Millions			Thousands			Ones		
100s	**10s**	**1s**	**100s**	**10s**	**1s**	**100s**	**10s**	**1s**
			5	2	3	8	1	0

- *'Fill in the missing numbers.*

3,514,203 + 4,000,000 = ☐

3,514,203 + 400,000 = ☐

3,514,203 + 40,000 = ☐

3,514,203 + 4,000 = ☐

124,362 – 20,000 = ☐

124,362 – 2,000 = ☐

124,362 – 200 = ☐

124,362 – 2 = ☐

☐ + 6,453,128 = 9,453,128

☐ + 6,453,128 = 6,953,128

☐ + 6,453,128 = 6,463,128

☐ + 6,453,128 = 6,459,128

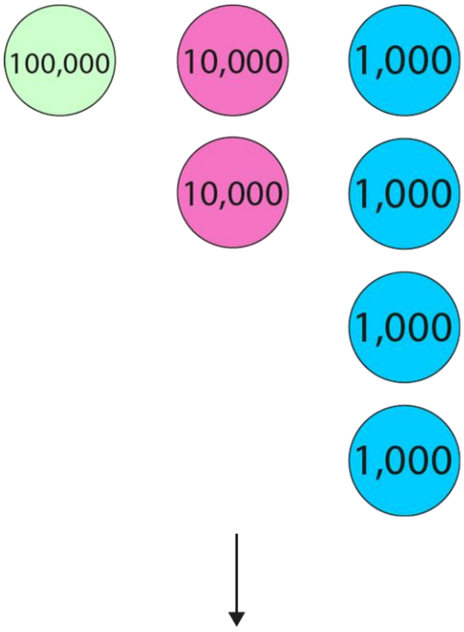
☐ + 6,453,128 = 6,453,828

☐ + 6,453,128 = 6,453,158

	1,234,567 – ☐ = 1,134,567 1,234,567 – ☐ = 1,234,267 1,234,567 – ☐ = 1,234,562 ☐ – 300,000 = 4,543,654 ☐ – 2,000 = 4,543,654 ☐ – 40 = 4,543,654 Dòng nǎo jīn: *'Fill in the missing numbers.'* 3,514,203 + 900,000 – 800,000 = ☐ 3,514,203 – 700,000 + 400,000 = ☐ 3,514,203 + 8,000 – ☐ = 3,517,203 3,514,203 – 6,000 + ☐ = 3,516,203
3:6	Move on to addition and subtraction involving changes to more than one digit, but still with no bridging across a boundary. Again, use place-value counters and place-value charts as an initial scaffold only. Once children are confident to move away from concrete and pictorial representations, secure this concept by providing further practice.

Addition and subtraction – without bridging, changing more than one digit at a time:

- *'What is 124,000 + 362?'*



Millions			**Thousands**			**Ones**		
100s	**10s**	**1s**	**100s**	**10s**	**1s**	**100s**	**10s**	**1s**
			1	2	4	0	0	0
						3	6	2

- *'Fill in the missing numbers.*
  Use the answer to each calculation to help you solve the next one in the set.'

630,421 + 130,000 = ☐

630,421 + 136,000 = ☐

630,421 + 136,200 = ☐

630,421 + 136,205 = ☐

7,125,364 – 24,000 = ☐

7,125,364 – 24,100 = ☐

7,125,364 – 24,130 = ☐

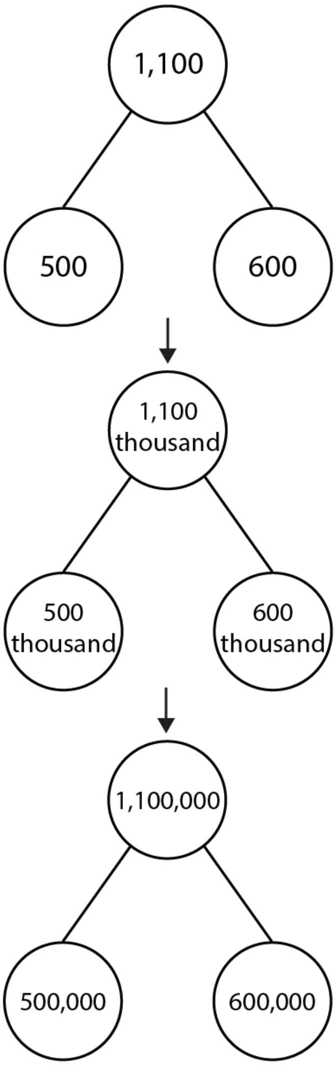
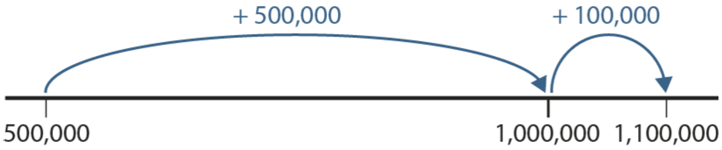
7,125,364 – 24,013 = ☐

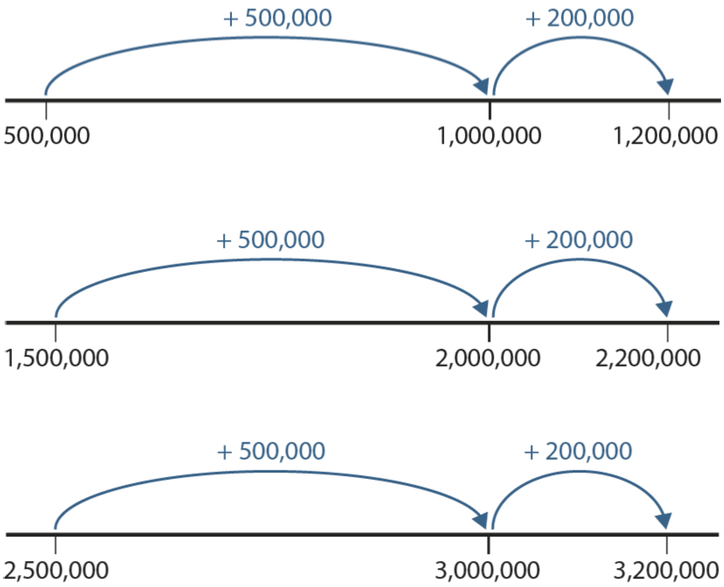
Teaching point 4:

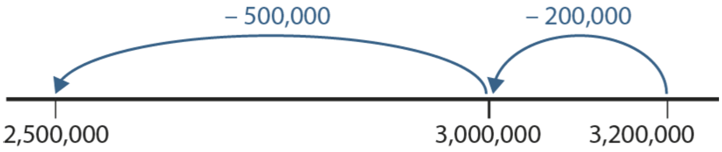
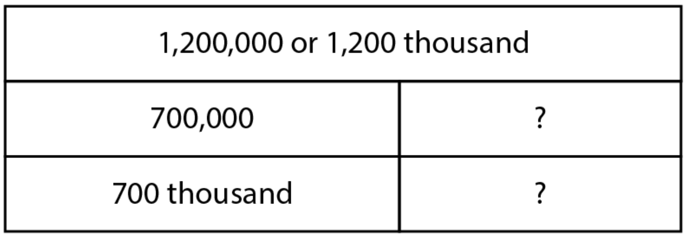
Knowledge of crossing thousands boundaries can be used to work to and across millions boundaries.

Steps in learning

	Guidance	Representations
4:1	Now that children are confident adding and subtracting numbers up to 10,000,000 without bridging, this teaching point moves them on to calculations with bridging across the millions boundary. Often, once bridging is involved in seven-digit numbers, a written method will be the most efficient method of calculation (e.g. *4,349,287 + 12,345*). However, some calculations can best be performed mentally. In segment *1.22 Composition and calculation: 1,000 and four-digit numbers*, the children learnt to add multiples of 100 across the thousands boundary. By unitising, make the link from this to adding multiples of 100,000 across the millions boundary. Use a progression like the one shown opposite. Children can use part–part–whole models to help them with unitising.	Addition – bridging across 1,000 and 1,000,000: *'Fill in the missing numbers.'* 500 + 300 = ☐ 500,000 + 300,000 = ☐ 500 + 400 = ☐ 500,000 + 400,000 = ☐ 500 + 500 = ☐ 500,000 + 500,000 = ☐ 500 + 600 = ☐ 500,000 + 600,000 = ☐ 500 + 700 = ☐ 500,000 + 700,000 = ☐

		Part–part–whole models: 
4:2	Explore different methods for bridging the millions boundary, for example, by representing it on a number line, as shown opposite. Alternatively, *500,000* + *600,000* can be solved by using near doubles, a strategy familiar to children from their work with smaller numbers in segment *1.19 Securing mental strategies: calculation up to 999*: 500,000 + 500,000 + 100,000 = 1,000,000 + 100,000 = 1,100,000 Make sure that you celebrate and encourage sensible alternative	Number line: 

	methods to calculations that the children suggest.	
4:3	Once children are confident bridging the millions boundary, extend this to other millions boundaries using progressions such as the one shown opposite. Again, use number lines to make the link between these chains of calculations. Discuss the pattern with the children and ask them to apply it to other calculations (e.g. *6,500,000 + 700,000*). If necessary, you can make the link back to bridging thousands boundaries, as in step *4:1* (e.g. *6,500 + 700 = 7,200)*.	Addition – bridging across millions boundaries: *'Fill in the missing numbers.'* 500,000 + 700,000 = ☐ 1,500,000 + 700,000 = ☐ 2,500,000 + 700,000 = ☐ 3,500,000 + 700,000 = ☐ Number lines: 
4:4	Once the children are confident with addition across millions boundaries, extend this to the inverse: subtracting multiples of 100,000 across millions boundaries. Continue to represent the calculations on number lines. If children struggle with subtractions, it may be helpful to continue unitising, linking to subtracting over thousands boundaries. These subtractions can be represented on part–part–whole	Subtraction – bridging across millions boundaries: *'Fill in the missing numbers.'* 1,200,000 – 700,000 = ☐ 2,200,000 – 700,000 = ☐ 3,200,000 – 700,000 = ☐

	models, as in step *4:1*, or on bar models, as shown opposite.	Number line:  Bar models: 
4:5	Provide further practice of adding and subtracting multiples of 100,000 across millions boundaries, including some calculations where the addend or subtrahend is missing.	Missing-number calculations: *'Fill in the missing numbers.'* 1,800,000 + 600,000 = ☐ 3,700,000 + ☐ = 4,300,000 5,600,000 – 800,000 = ☐ 9,200,000 – ☐ = 8,800,000
4:6	So far, we have focused on adding and subtracting multiples of 100,000 across millions boundaries. Now look at working back from these boundaries in different powers of ten. First, explore the patterns as different powers of ten are subtracted from millions boundaries, e.g.: • 1,000,000 – 100,000 = 900,000 1,000,000 – 10,000 = 990,000 1,000,000 – 1,000 = 999,000 • 5,000,000 – 1,000,000 = 4,000,000 5,000,000 – 100,000 = 4,900,000 5,000,000 – 10,000 = 4,990,000 5,000,000 – 1,000 = 4,999,000	Subtraction – bridging across millions boundaries: *'Fill in the missing numbers.'* 7,000,000 – 1,000,000 = ☐ 7,000,000 – 100,000 = ☐ 7,000,000 – 10,000 = ☐ 7,000,000 – 1,000 = ☐ 9,000,000 – 2,000,000 = ☐ 9,000,000 – 400,000 = ☐

	Provide children with practice, such as the first set of calculations shown opposite. When the children are confident subtracting powers of ten, move them on to subtracting multiples of these, as shown by the second set of calculations opposite.	9,000,000 – 60,000 = ☐ 9,000,000 – 8,000 = ☐
4:7	Now make the link between working back from boundaries, as in the previous step, and other aspects of these part–whole relationships. It may be helpful to start with five- and six-digit numbers, with which the children will already be confident, before moving on to millions boundaries. For example, if we know: 400,000 – 40,000 = 360,000 then we also know: 360,000 + 40,000 = 400,000 400,000 – 360,000 = 40,000	Missing-number calculations: *'Fill in the missing numbers.'* 360,000 + ☐ = 400,000 400,000 – 360,000 = ☐ 2,930,000 + ☐ = 3,000,000 3,000,000 – 2,930,000 = ☐ 4,995,000 + ☐ = 5,000,000 5,000,000 – 4,995,000 = ☐
4:8	Finally in this teaching point, look at counting patterns across millions boundaries. Display a counting pattern, such as the one below, and support children in identifying the part of the number that is changing. In the below example, the numbers get smaller by 100,000 each time. Highlighting the part of the number that is changing will help children to identify the next numbers in the sequence. Working across the boundary will be the biggest challenge for the children, so focus particularly on this. Provide practice with sequences: • in both ascending and descending order • with different sized intervals between terms • with several gaps in the sequence, including the first number of the sequence missing so that children need to use the inverse to identify the missing number.	

Counting:

2,402,159	2,302,159	2,202,159	2,102,159	2,002,159	1,902,159	1,802,159	…

Missing-number sequences:

'Fill in the missing numbers.'

6,361,040	6,371,040	6,381,040	6,391,040	6,401,040	6,411,040		

2,004,567	2,003,567	2,002,567	2,001,567	2,000,567	1,999,567		

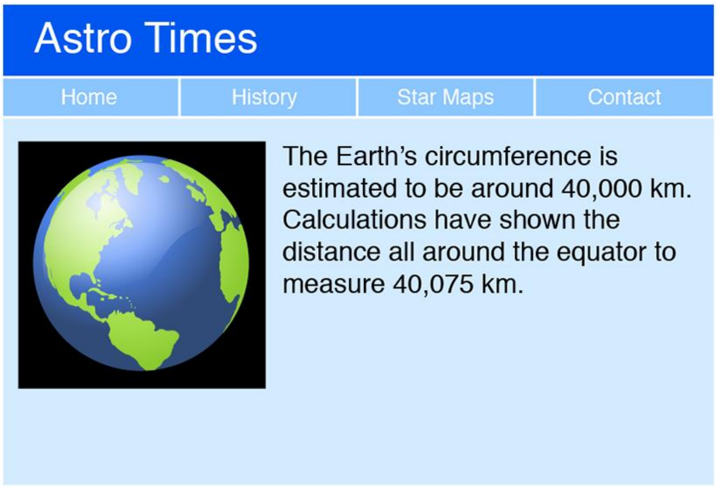
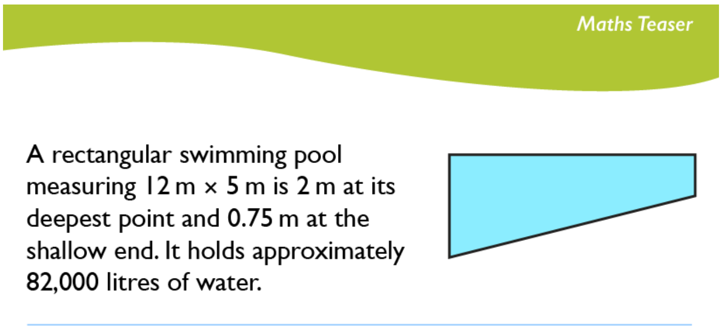
7,730,004		7,930,004	8,030,004		8,230,004		8,430,004

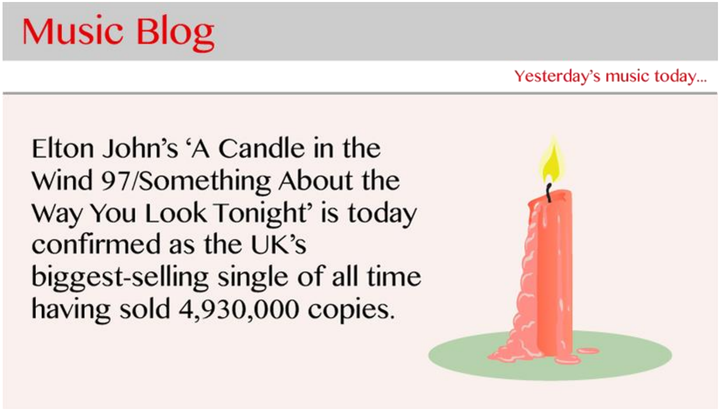
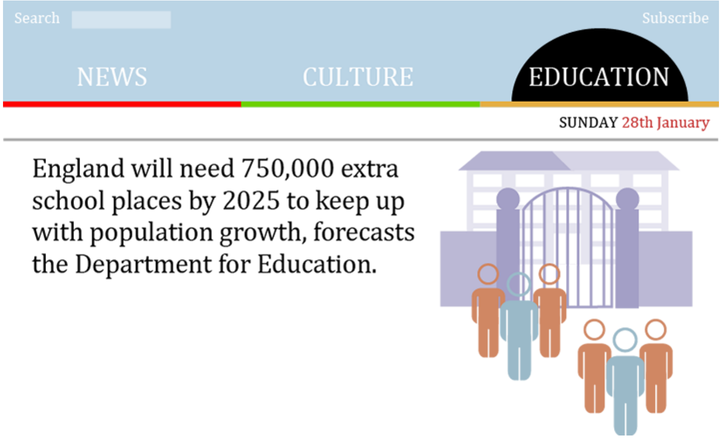
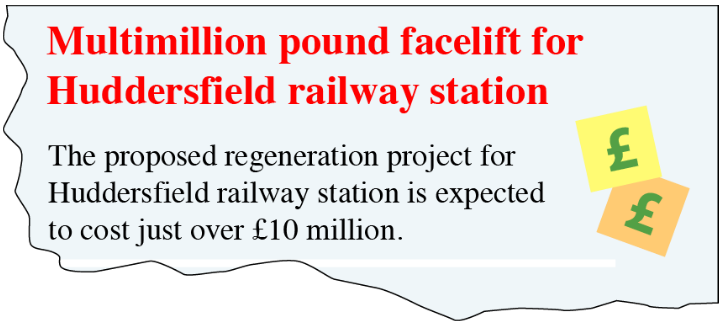
	9,149,301		9,129,301	9,119,301			9,089,301

Teaching point 5:

Sometimes numbers are rounded as approximations to eliminate an unnecessary level of detail; rounded numbers are also used to give an estimate or average. At other times, precise readings are useful.

Steps in learning

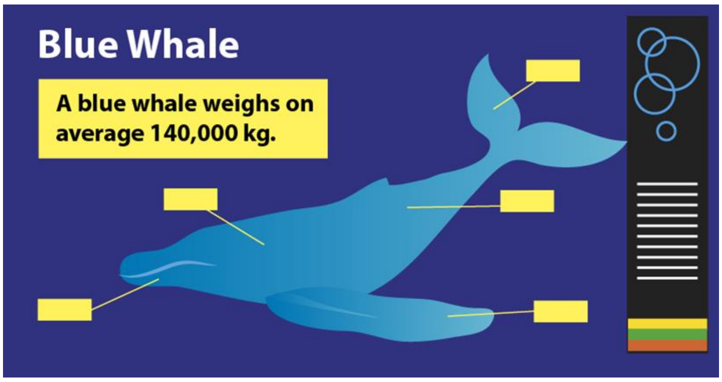
	Guidance	**Representations**
5:1	The children have already met rounding in earlier segments, but now spend a bit of time exploring the different reasons why we round. Start by providing children with headlines and statements in which numbers are rounded to *approximate*. Discuss, for example, why a headline might use a rounded figure and when precise figures are needed. Look at a range of examples, including measures and money contexts.	Rounding – approximations:  **JO BRAND RAISES OVER £1m FOR SPORT RELIEF** Jo Brand has raised a staggering £1,159,220 for Sport Relief by walking 135.7 miles across the width of the country. Source: *Comic Relief*, 2018  Astro Times The Earth's circumference is estimated to be around 40,000 km. Calculations have shown the distance all around the equator to measure 40,075 km.  A rectangular swimming pool measuring 12 m × 5 m is 2 m at its deepest point and 0.75 m at the shallow end. It holds approximately 82,000 litres of water.

		 Music Blog Yesterday's music today... Elton John's 'A Candle in the Wind 97/Something About the Way You Look Tonight' is today confirmed as the UK's biggest-selling single of all time having sold 4,930,000 copies. Source: *The Official Charts Company*, 2018
5:2	Now explain that rounded numbers are also often used for *estimates* when an exact figure is not known. Support this with examples.	Rounding – estimates:  Search Subscribe NEWS CULTURE EDUCATION SUNDAY 28th January England will need 750,000 extra school places by 2025 to keep up with population growth, forecasts the Department for Education.  **Multimillion pound facelift for Huddersfield railway station** The proposed regeneration project for Huddersfield railway station is expected to cost just over £10 million.

5:3

Finally, discuss that a third common context for using rounded numbers is *averages*. Look at examples of these as well.

Rounding – averages:



Source: *Reebok*

5:4

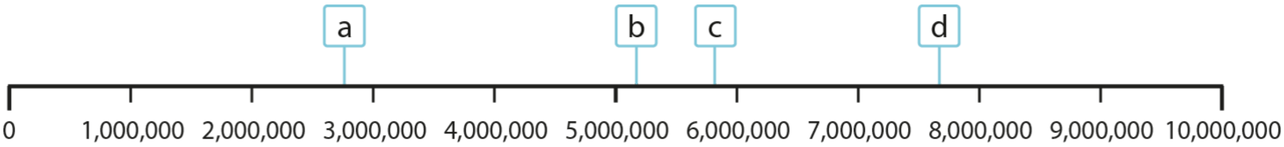
Now that the children have discussed different reasons *why* we round numbers, move on to *how* to round seven-digit numbers.

Children already know how to round four-digit numbers to the nearest 10, 100 and 1,000 (see segment *1.22 Composition and calculation: 1,000 and four-digit numbers*). They also know how to round multiples of 1,000 (up to six-digit numbers) to the nearest 10,000 and 100,000 (see segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*). Furthermore, they know how to round numbers with tenths and hundredths to the nearest whole number (see segments *1.23 Composition and calculation: tenths* and *1.24 Composition and calculation: hundredths and thousandths*). Now, they need to bring this all together to be able to round all the numbers they meet to any degree of accuracy.

Display a 0–10,000,000 number line, with the multiples of 1,000,000 marked. Draw arrows or boxes (labelled *a*, *b*, *c* and *d*) at some points on the line that lie between tick marks, as shown below.

For each point, determine with the children which multiple of 1,000,000 lies previously and which multiple lies next. Note the use of the inequality signs *greater than* and *less than* here.

Number line:



Inequality:

previous multiple of 1,000,000 — **next** multiple of 1,000,000

$$\boxed{2,000,000} < a < \boxed{3,000,000}$$

5:5

Once children are confident identifying the previous and next multiples of 1,000,000, ask them to make a visual judgement about which multiple point *a* from the previous step is nearest to. Use the following stem sentences to reinforce understanding:

- ***'a is between ___ and ___.'***
- ***'The previous multiple of one million is ___. The next multiple of one million is ___.'***
- ***'a is nearest to ___.'***
- ***'a is ___ when rounded to the nearest million.'***

Repeat the process for numbers *b, c* and *d* from the previous step using the stem sentences.

Rounding to the nearest 1,000,000:

previous multiple of 1,000,000 — **next** multiple of 1,000,000

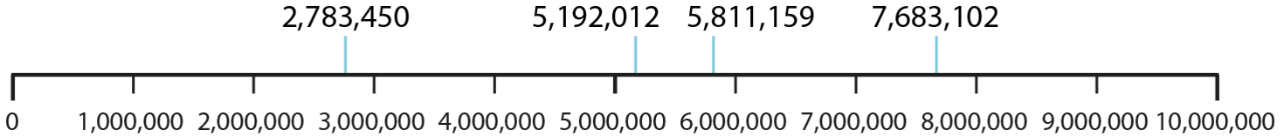
$$\boxed{2,000,000} < a < \boxed{3,000,000}$$

- *'a is between two million and three million.'*
- *'The previous multiple of one million is two million. The next multiple of one million is three million.'*
- *'a is nearest to three million.'*
- *'a is three million when rounded to the nearest million.'*

5:6

Now, instead of letters, show the marked values on the number line from step *5:4*. Repeat the sequence from the previous two steps, still drawing on visual judgement of the previous and next multiples of 1,000,000, and which each marked number is nearest to.

Number line:



	Rounding to the nearest 1,000,000: **previous** multiple of 1,000,000 — **next** multiple of 1,000,000 2,000,000 < 2,783,450 < 3,000,000 • *'2,783,450 is between two million and three million.'* • *'The previous multiple of one million is two million. The next multiple of one million is three million.'* • *'2,783,450 is nearest to three million.'* • *'2,783,450 is three million when rounded to the nearest million.'*
5:7	Look again at each of the four values from the previous step, with their previous and next multiples of 1,000,000. Discuss how the second and third numbers both have five whole millions in them, but one rounds down to 5,000,000 and the other rounds up to 6,000,000. Draw out that this is because 5,192,012 sits *before* the midpoint of 5,000,000 and 6,000,000, while 5,811,159 sits *after* the midpoint. In previous segments, the children learnt that, for example, to round to the nearest 100,000 it is the 10,000s digit that they must consider. Now introduce the following generalisation: ***'When rounding to the nearest million, the hundred thousands digit is the digit to consider. If it is four or less we round down. If it is five or more we round up.'***
	Rounding to the nearest 1,000,000: **previous** multiple of 1,000,000 — **next** multiple of 1,000,000 2,000,000 < 2,783,450 < 3,000,000 5,000,000 < 5,192,012 < 6,000,000 5,000,000 < 5,811,159 < 6,000,000 7,000,000 < 7,683,102 < 8,000,000
5:8	Give children practice rounding a range of seven-digit numbers to the nearest million (e.g. 9,810,000, 9,180,000, 4,500,000, 4,499,999). It is important that they are confident in this before moving them on to rounding to other degrees of accuracy.
5:9	Now look at rounding seven-digit numbers to the nearest 100,000, following the same sequence as above.

You may be able to move through this relatively quickly as children already know how to do this for six-digit numbers from segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*; however, some teaching points to highlight are:

- 3,000,000 is a multiple of 100,000 (children might not realise that all multiples of 1,000,000 are also multiples of 100,000, and indeed of 10,000, 1,000, 100 and 10 as well)
- points *b* and *c* (shown below) both round to 3,400,000 despite one being less than this number and one being more than this number.

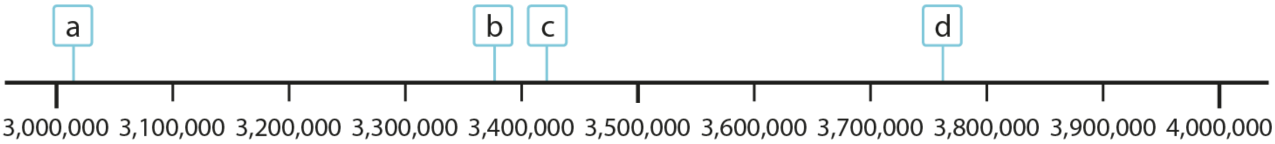
Continue to reinforce understanding with stem sentences, encouraging the children to repeat these for each of the numbers:

- ***'___ is between ___ and ___.'***
- ***'The previous multiple of one hundred thousand is ___. The next multiple of one hundred thousand is ___.'***
- ***'___ is nearest to ___.'***
- ***'___ is ___ when rounded to the nearest one hundred thousand.'***

Once children are confident rounding seven-digit numbers to the nearest 100,000, repeat the generalised statement they met in segment *1.26*: ***'When rounding to the nearest one hundred thousand, the ten thousands digit is the digit to consider. If it is four or less we round down. If it is five or more we round up.'***

At this stage, it can be valuable to make a wider generalisation about rounding: ***'When rounding to a particular degree of accuracy, the digit to the right of the place value you are rounding to is the one that determines whether to round up or down.'***

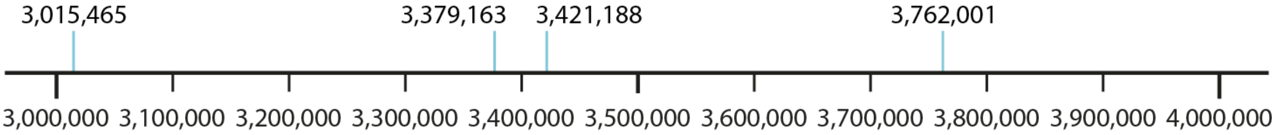
Number line:



Rounding to the nearest 100,000:

previous multiple of 100,000				**next** multiple of 100,000
3,000,000	<	a	<	3,100,000

Number line:



Rounding to the nearest 100,000:

previous multiple of 100,000 | **next** multiple of 100,000

$$\boxed{3{,}000{,}000} < 3{,}0\underline{1}5{,}465 < \boxed{3{,}100{,}000}$$

- *'3,015,465 is between three million and three milion one hundred thousand.'*
- *'The previous multiple of one hundred thousand is three million. The next multiple of one hundred thousand is three million one hundred thousand.'*
- *'3,015,465 is nearest to three million.'*
- *'3,015,465 is three million when rounded to the nearest one hundred thousand.'*

5:10 Give children practice rounding a range of seven-digit numbers to the nearest 100,000 (e.g. 6,349,999, 7,500,000, 5,909,810, 3,051,000). Also include some practice rounding six-digit numbers to the nearest 100,000 (e.g. 549,999, 450,010, 609,898, 353,211), as previous rounding of six-digit numbers (in segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*) was limited to six-digit multiples of 1,000.

Look out for children who make the error of rounding a seven-digit number to a six-digit number. For example, children sometimes say that 5,734,000 rounded to the nearest 100,000 is 700,000 rather than 5,700,000. Making sure that children write out the previous and next multiples of 100,000 for as long as they need to should help avoid this.

5:11 Children have now learnt to round seven-digit numbers to the nearest 1,000,000 and the nearest 100,000. However, they need to be able to draw together everything they have learnt about rounding up to this point to round any number to any specified degree of accuracy.

Take a large number in context and explore how this rounds to the nearest 10, 100, 1,000, 10,000, 100,000 and 1,000,000. For example:

- *'A city library contains exactly 1,756,451 books. Round this number to the nearest:*
  - *10*
  - *100*
  - *1,000*
  - *10,000*
  - *100,000*
  - *1,000,000.'*

It is useful to identify the digits that need to be considered each time, that is, the place value the number is being rounded to and the digit to its right.

Rounding:

Here, when rounding to the nearest 1,000, the hundreds digit is the significant digit.

1 , 7 5 6 , ④ 5 1	
1 , 7 5 6 , 4 5 0	nearest ten
1 , 7 5 6 , 5 0 0	nearest hundred
1 , 7 5 6 , 0 0 0	nearest thousand
1 , 7 6 0 , 0 0 0	nearest ten thousand
1 , 8 0 0 , 0 0 0	nearest hundred thousand
2 , 0 0 0 , 0 0 0	nearest million

5:12 Repeat the previous step for other numbers in context, including money and measures, identifying the significant digits each time. For example:

- *'A charity concert raises £289,910. Round this to the nearest:*
  - *£10*
  - *£100*
  - *£1,000*
  - *£10,000*
  - *£100,000.'*

Draw attention to a couple of potential difficulty points:

- Numbers may give the same rounded answer when rounded to different degrees of accuracy. For example, in the above question, 289,910 is 290,000 when rounded both to the nearest 1,000 and to the nearest 10,000.
- Children need to be aware that a number may not need further rounding. Here, the number 289,910 is already a multiple of ten and so doesn't change when rounded to the nearest ten.

Rounding in context:

£ 2 8 9 , 9 1 0	
£ 2 8 9 , 9 1 0	nearest £10
£ 2 8 9 , 9 0 0	nearest £100
£ 2 9 0 , 0 0 0	nearest £1,000
£ 2 9 0 , 0 0 0	nearest £10,000
£ 3 0 0 , 0 0 0	nearest £100,000

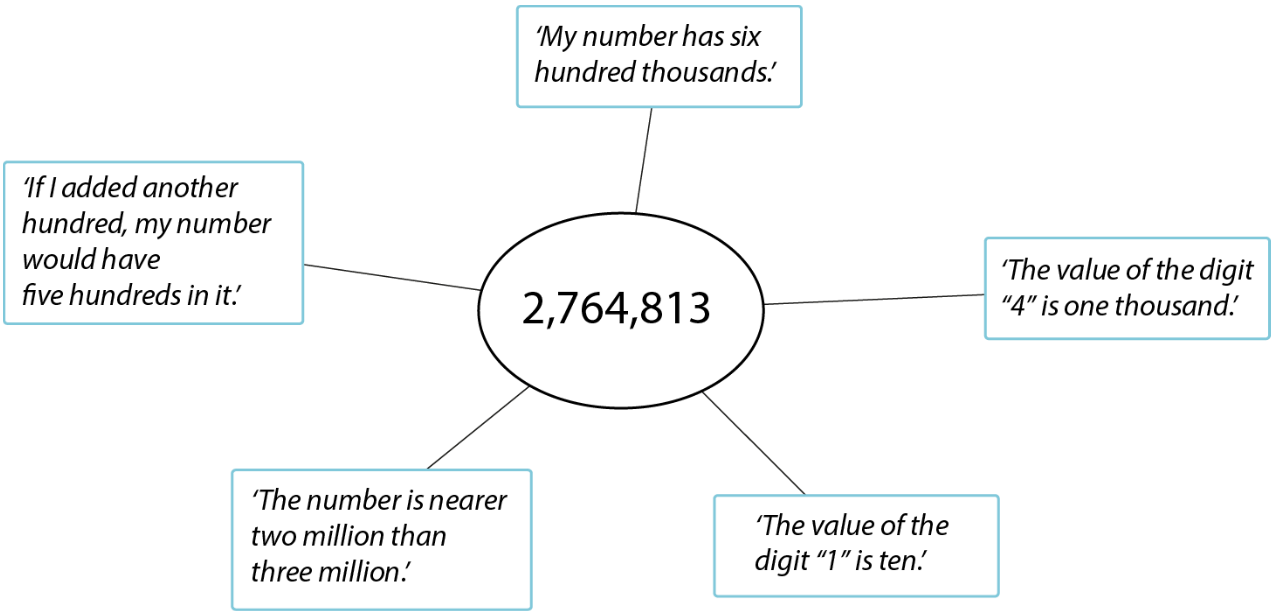
Here, the nearest £1,000 is the same as the nearest £10,000.

5:13 Complete this teaching point by offering varied practice for rounding any number to any given degree of accuracy, as shown below. Include opportunities for the children to reason, explain and draw on their learning from previous teaching points. For example:

- *'Round 8,472,916 to the nearest:*
  - *10*
  - *100*
  - *1,000*
  - *10,000*
  - *100,000*
  - *1,000,000.'*
- Dòng nǎo jīn:
  - *'When I round my number to the nearest ten thousand the answer is two hundred and thirty thousand.*
    When I round my number to the nearest thousand I get the same answer.
    What could my number be? Find all possible answers.'
  - *'Caroline says that 1,439,500 rounded to the nearest ten is 1,439,510. Explain her misunderstanding.'*
  - *'Eduardo says "The population of Kentucky is four million (to the nearest million) and the population of Oregon four-point-one million (to the nearest hundred thousand)."*
    He says, "The population of Oregon must be bigger than the population of Kentucky because four-point-one million is bigger than four million."
    Do you agree with him?'

True or false?

'Is each statement true or false? Explain your answers.'



2,764,813

'My number has six hundred thousands.'

'If I added another hundred, my number would have five hundreds in it.'

'The value of the digit "4" is one thousand.'

'The number is nearer two million than three million.'

'The value of the digit "1" is ten.'

Dòng nǎo jīn:

0 1 2 3 4 5 6 7 8 9

'Use the digit cards zero to nine to create two numbers. ***You may use each card only once.***
The first number rounds to thirteen thousand when rounded to the nearest thousand.'

☐ ☐ ☐ ☐ ☐ → 13,000

'The second number rounds to thirty thousand when rounded to the nearest ten thousand.'

☐ ☐ ☐ ☐ ☐ → 30,000

Teaching point 6:

Fluent calculation requires the flexibility to move between mental and written methods according to the specific numbers in a calculation.

Steps in learning

6:1

Guidance

Begin this teaching point by presenting the children with a set of numbers in context that demonstrate rounded numbers, for example, average mass of different whale species (shown opposite).

Look at the question: *'How much heavier is a sperm whale than a humpback whale?'*

Ask the children to write a calculation they could do to solve this. Discuss their calculations and conclude that both of the following equations express the calculation the children need to carry out:

$$30{,}000 + \boxed{} = 40{,}000$$

$$40{,}000 - 30{,}000 = \boxed{}$$

Show that this can also be represented on the bar model opposite.

Ask the children how they would solve this. Is a mental strategy or written method more suitable? The children should be able to see that this is an easy calculation to solve mentally.

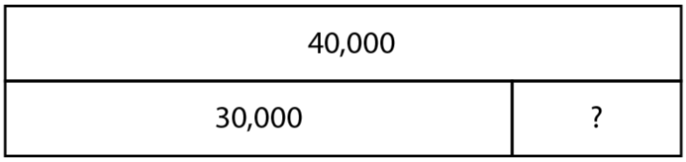
Repeat with similar questions based on the data set.

Representations

Rounded contextual numbers:

Species of whale	Average mass (kg)
Beluga	1,400
Killer	4,000
Humpback	30,000
Sperm	40,000
Blue	140,000

Bar model:



6:2

Guidance

The data set of average whale masses also provides an excellent opportunity to look at additive and multiplicative comparisons. Ask children questions such as:

- *'On average,* how much *heavier is a blue whale than a beluga whale?'*

- *'On average,* how many times *heavier is a blue whale than a beluga whale?'*

When setting questions about real-world contexts to consolidate learning, try to include a mix of additive and multiplicative comparisons. In everyday life, children will have to move between these comparisons, so aim to reflect this in maths lessons.

6:3

Now provide the children with another context, this time with large numbers presented as precise readings, for example, areas of European countries (shown opposite).

Using these figures, pose the following question: *'How much larger is Germany than the United Kingdom?'*

As in step *6:1*, ask the children to write a calculation that they can use to solve the question. Both of the equations below represent the question they are being asked to solve, as does the bar model opposite.

$$243{,}610 + \boxed{} = 357{,}022$$

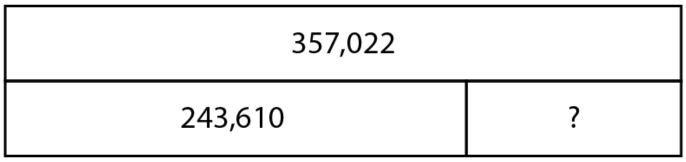
$$357{,}022 - 243{,}610 = \boxed{}$$

Precise contextual numbers:

European country	Area (km²)
France	643,801
Spain	505,370
Sweden	450,295
Germany	357,022
Italy	301,340
United Kingdom	243,610

Source: *The World Factbook* 2018
Washington, DC: Central Intelligence Agency, 2018

Bar model:



6:4

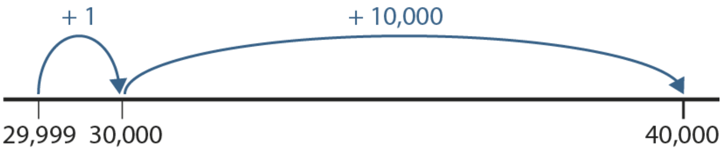
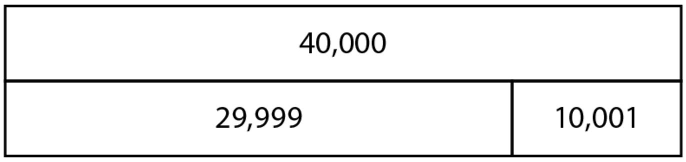
At this stage, ask the children to compare two of the questions considered so far:

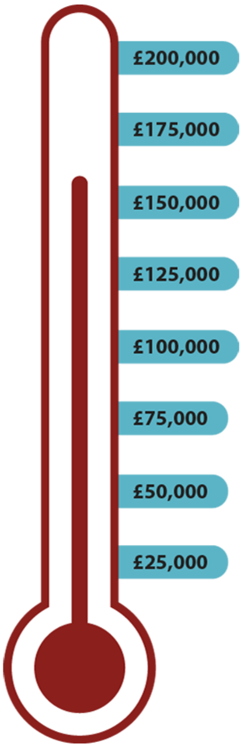
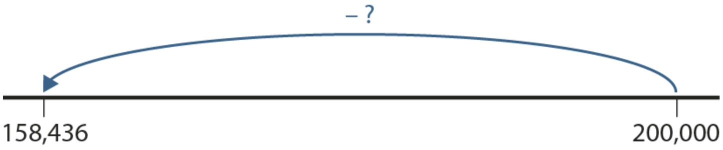
- *'How much heavier is a sperm whale than a humpback whale?'*
- *'How much larger is Germany than the United Kingdom?'*

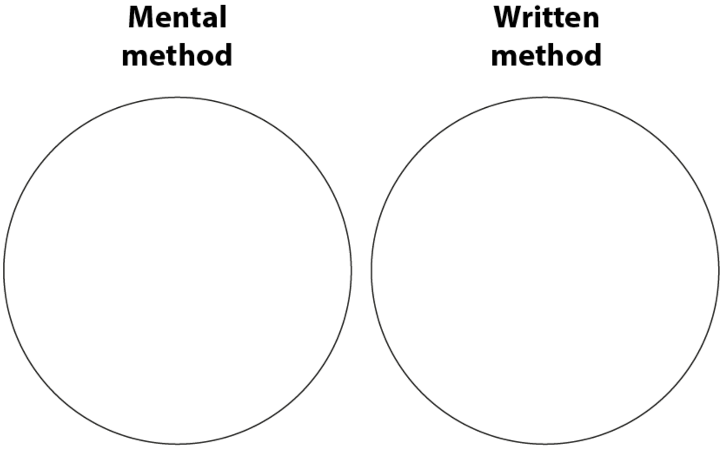
Display the two questions and associated calculations from steps *6:1* and *6:3* alongside each other. Ask:

- *'What is the same about the questions?'*
- *'What is different about the questions?'*

	Some things that are the same: - Both share the same structure: comparing the size of two things. - In both cases, the size of both things is known and the difference is unknown. - Both are in measures contexts. Some things that are different: - One measure is kilograms and one is kilometres squared. - One is comparing rounded averages and one is comparing exact measurements. - One calculation can easily be done mentally. The other calculation is not easy to do mentally and a formal written method should be used.	
6:5	By now, the children will be very confident with column subtraction. Using the subtraction calculation from step *6:3* (*357,022 – 243,610*), model how to extend the method they learnt in segment *1.21 Algorithms: column subtraction* to larger numbers.	Column subtraction: $$\begin{array}{rcccccc} & 3 & 5 & \cancel{7}^{6}, & {}^{1}0 & 2 & 2 \\ - & 2 & 4 & 3, & 6 & 1 & 0 \\ \hline & 1 & 1 & 3, & 4 & 1 & 2 \\ \hline \end{array}$$
6:6	Similarly, children will also be confident with column addition, so now extend the method they learnt in segment *1.20 Algorithms: column addition* to larger numbers. For example, referring to the data set from step *6:3*, ask: '*What is the combined area of France and Spain?*'	Column addition: $$\begin{array}{rccccccc} & & 6 & 4 & 3, & 8 & 0 & 1 \\ + & & 5 & 0 & 5, & 3 & 7 & 0 \\ \hline & 1, & 1 & 4 & 9, & 1 & 7 & 1 \\ \hline & & & & {}_{1} & & & \end{array}$$
6:7	Provide practice for adding and subtracting using formal written methods. As well as adding and subtracting two numbers with the same number of digits, make sure you include the following: - Adding and subtracting numbers with different numbers of digits to check that the children are confident aligning them correctly (e.g. *435,798 + 1,928,791* and *4,868,217 – 965,200*).	

	- For subtraction, exchanging* where there is a zero in the column to the left so the next column needs to be considered (e.g. *680,343 – 471,621*). - Multi-step problems, for which children have to decide which calculation to work out first (e.g. *102,341 – 1,310,230 + 5,620,000*). - For addition, adding more than two numbers (e.g. *1,245,829 + 583,271 + 3,098,612*). - For subtraction, subtracting more than one number, where children may add the subtrahends first (e.g. *8,399,768 – 1,306,008 – 74,641*). * '*Exchanging*' is when there is an insufficient number of a unit to subtract from in a given column, so a unit is exchanged from the column to the left. For example, when one ten is exchanged for ten ones.	
6:8	Throughout this segment, children have looked at plenty of calculations that can be solved mentally. Look at one such calculation now, for example: *'A new car costs £29,999. How much money is left from £40,000?'* Work through this as a class, exploring how the calculation can be represented on a number line or bar model.	Number line:  Bar model: 

6:9	Now introduce a new context; money and calculating change are both ideal choices. For example: *'A charity aims to raise £200,000. So far it has raised £158,436. How much more does the charity need to raise to reach its target amount?'* Discuss how the children would go about solving this. This time, a mental approach is much harder. However, a formal written method is also problematic, as it involves exchanging with so many zeros.	Money context:  Mental method – number line:  Written method – column subtraction: $\begin{array}{r} \cancel{2}^{1}\ {}^{1}\cancel{0}^{9}\ {}^{1}\cancel{0}^{9},\ {}^{1}\cancel{0}^{9}\ {}^{1}\cancel{0}^{9}\ {}^{1}0 \\ -\ 1\ 5\ 8,\ 4\ 3\ 6 \\ \hline 0\ 4\ 1,\ 5\ 6\ 4 \\ \hline \end{array}$
6:10	By this segment, children have already met the strategy of same difference, summarised by the generalised statement: ***'If the minuend and subtrahend both decrease (or increase) by a given amount, the difference remains the same.'*** Now apply this to the calculation from the previous step, decreasing both the minuend and the subtrahend by one to	Same difference: $\begin{array}{rcr} 2\ 0\ 0,\ 0\ 0\ 0 & \xrightarrow{-1} & 1\ 9\ 9,\ 9\ 9\ 9 \\ -\ 1\ 5\ 8,\ 4\ 3\ 6 & \xrightarrow[-1]{} & -\ 1\ 5\ 8,\ 4\ 3\ 5 \\ \hline \\ \hline \end{array}$

	give an equivalent calculation that can be solved using column subtraction without the need for any exchange.	
6:11	Provide practice using equivalent calculation to calculate similar subtractions using the column method, for example: 1,000,000 – 365,912 3,500,000 – 2,548,102 5,900,000 – 265,631	
6:12	Now provide the children with a selection of addition and subtraction calculations out of context. Ask them to discuss and sort these according to which they would solve using a mental strategy and which would be better solved with a written method. For each calculation, ask the children to explain their choice of method. There may not always be a clear 'best' approach.	Sorting circles: *'Sort these calculations into the circles.'* 308,724 – 10,000 20,000 – 9,999 78,921 – 3,000 426 + 784,000 + 3,000,000 9,000,000 – 437,208 43,701 – 34, 067 103,436 + 45,618 1,070,640 – 65,231 

6:13

Finally, end this segment by providing varied practice for solving addition and subtraction calculations involving numbers up to 10,000,000, as shown opposite and below. Include:

- calculations that might be solved with mental methods, formal written methods and equivalent calculations
- equations presented in a range of forms:
  - missing number in different places
  - operators on both sides of the equals sign
  - mixed operations
  - equals sign presented towards the start of the equation
  - problems set in contexts.

Contextual problems:

- *'In the year 2015, 777,165 babies were born in the UK. 776,352 babies were born in 2014. How may more babies were born in 2015 than in 2014?'*

Source: *Office for National Statistics*
Public sector information licensed under the
Open Government Licence v3.0

- *'A school has a budget of £350,000. The total spent for the year is £325,000. How much money does the school have left?'*

Dòng nǎo jīn:

- *'Two numbers have a total of 3,456,789. They have a difference of 987,655. What are the two numbers?'*
- *'Two numbers have a total of* $5\frac{1}{4}$ *million. One number is twice the other number. What are the two numbers?'*
- *'Two numbers have a difference of 789,456. To the nearest million, both numbers are seven million. What could the two numbers be?'*

Solving equations:

'Fill in the missing numbers.'

- Missing addend

4,139,432 = 3,672,980 + ☐

- Missing minuend

☐ – 6,504,873 = 1,999,999

- Subtraction complements

4,310,002 – ☐ = 2,562,038

4,500,000 = 7,000,000 – ☐

- Equivalent calculations

500,000 – 345,678 = 400,000 – ☐

1,546,423 + 5,500,000 = ☐ + 6,500,000

- Multi-step calculations

3 × 2,500,000 – 750,000 = ☐

4,000,000 – 5 × 199,999 = ☐

456,123 + 299,999 – 100,000 = ☐

6,890,021 – 43,607 – 3,051,352 = ☐

- *'Mrs Wong has four cards.*

  2,123,999 2,200,000

  2,000,000 1,924,000

  She gives one card to each child. The children look at their cards and each say a clue:

  - *Anna says, "My number is two million when rounded to the nearest million."*
  - *Bashir says, "The difference between my number and Caris's number is one hundred and ninety-nine thousand, nine hundred and ninety-nine."*
  - *Caris says, "My number is a multiple of one thousand."*
  - *David says, "My number has 2,200 thousands in it."*

  Which card does each child have? Explain your choices.'

For notes on common difficulties on missing addend, missing minuend and subtraction complement problems, see segment *1.26 Composition and calculation: multiples of 1,000 up to 1,000,000*, step 4:6.