Secondary Magazine 118

National Centre for Excellence in the Teaching of Mathematics

Happy New Year! We hope you have had a restful and restorative break as well as a fun one, and we promise not to mention those dreaded post-Christmas units: inches and pounds (either lb or £ - bank balances and waistlines do seem to be inversely proportional at this time of year!). Thank you for the comments and feedback we've had so far on the refreshed Secondary Magazine: we're delighted that you're finding the articles and information interesting and useful. Please do continue to let us know what you think, by email to info@ncetm.org.uk or on Twitter, @NCETMsecondary.

You will have read our first guest article in last month's edition. If you'd like to contribute please get in touch…now there's a New Year Resolution for you!

Contents

Heads Up

Whilst not wanting to define this article as mathematical gossip, it does come close! We've brought together news and current mathematical affairs, all in one place. We do hope it will interest you.

Congratulations to Charlie Stripp, Director of the NCETM and Chief Executive of Mathematics in Education and Industry (MEI), who has received an MBE for services to education in the New Year Honours list.

A date for the diary: the Capita Secondary Maths conference is in London on 23 February. Further information is available on Capita's website.

The marvellous More Or Less has returned to BBC Radio 4 for a new series. The first programme cited some of the newsworthy numbers of 2014; if you want to share with your students a few jaw-droppingly dire examples of statistics being misused, misquoted and misunderstood, listen again or download the podcast. The TES article 2014: A Year of Education In Numbers looks at the numbers behind some critical education events from 2014. Maths and politics – what better to rouse you from any January blues?

Also on Radio 4, on In Our Time, Melvyn Bragg and his guests discussed Renaissance Mathematics. Do you agree that *"Mathematics in the Renaissance was a question of recovering and, if you were very lucky, improving upon Greek ideas. The geometry of Euclid, Appollonius and Ptolemy ruled the day. Yet within two hundred years, European mathematics went from being an art that would unmask the eternal shapes of geometry to a science that could track the manifold movements and changes of the real world"*?

Bolton Local Authority has written a booklet to help its schools develop Mathematical Thinking Across the Curriculum For All Staff. The underlying *"principle [is] that being numerate is a matter of choice. Attitude is important … For some subjects it is easy to see how an understanding of mathematical procedures is integral to the subject being taught. The sciences and business studies are classic examples. For other subjects like English or Religious Education numeracy is often more about mathematical thought processes rather than about number and procedures."* Bolton LA has very kindly offered to share this thinking with all NCETM members: you can download a copy here.

The Ofsted Annual Report 2013-14 was published in December. While not commenting solely on mathematics teaching, the report does contain much of importance and good sense about teaching and learning in general that directly applies to mathematics in particular. The report's discussion of teaching to meet the needs of the highest and lowest attaining pupils is especially worth reading.

If you're missing the ocular stimuli of all the Christmas cards and fairy lights, these twenty-one GIFs will not only excite your visual cortex but they'll also explain mathematical concepts. Watching an ellipse or a parabola being drawn will cement understanding in these areas; and then there are 19 more!

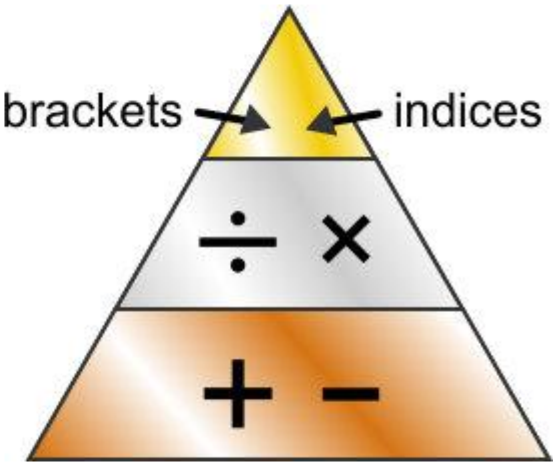
But if you prefer "in your hand" resources to those "in the cloud", you might like to blow the dust off those geoboards at the back of the Maths Store and order a copy of a new ATM book full of rich and interesting activities to explore area and fractions. Batteries not included, or needed!

Image credit
Page header by NS Newsflash (adapted), some rights reserved

Building Bridges

New Year, New Year's resolutions, time for changes (ch … ch … changes) … so let's be contrary and consider things not changing: the fact that 3 + 5 – 5 = 3, and that 3 × 5 ÷ 5 = 3. Let's call these examples of, respectively, additive and multiplicative invariance. Additive invariance is a concept that most secondary pupils have grasped in KS2: it is not a surprise to them that 3 + 5 – 5 = 3. It is, nonetheless, well worth ensuring that your pupils are absolutely confident with less straightforward examples of the form a + b – a and a – b + c + b, and also those of the form a – b + c – d where b + d = a (or c). To confirm that your pupils are fluent with additive invariance, ask them a calculation such as (673 + 356) – (675 + 354): do they reason that the answer must be 0, because a sum of two terms is invariant if one term increases by an amount and the second term decreases by the same amount?

Part of the reason for looking at calculation strings such as these is to challenge your pupils' likely misunderstanding of the unhelpful cliché "BODMAS": they may think (indeed, they may have been told) that "you work out addition before subtraction" and so they think that they **have** to evaluate 3 + 5 – 5 as 8 – 5 = 3, rather than 3 + 0 = 3. The difference in efficiency will be stark when you ask them to evaluate 2368 – 9674 + 1332 + 9674! A better mnemonic for the order of operations is to think of a pyramid or a podium for gold, silver and bronze medals (a mnemonic that will work especially well in 2016!):



They need to be confident when, and why, calculation strings can be re-ordered, for example

- 14 + 6 – 9 = 14 – 9 + 6 = 6 – 9 + 14 etc.
- 8 × 6 ÷ 4 = 8 ÷ 4 × 6 = 6 ÷ 4 × 8 etc.
- 14 + 6 – 9 ≠ either 14 – 6 + 9 or 9 + 6 – 14
- 8 × 6 ÷ 4 ≠ either 8 ÷ 6 × 4 or 6 × 4 ÷ 8
- 30 ÷ 3 ÷ 2 = 30 ÷ 2 ÷ 3, but not 3 ÷ 30 ÷ 2.

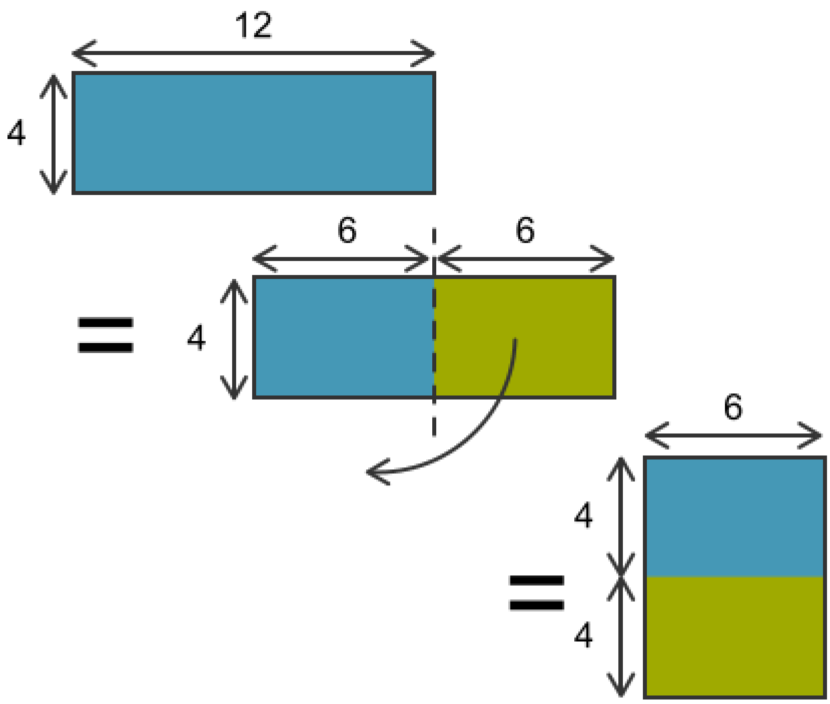
The order of operations is reflected in the medal winners' hierarchy: gold before silver before bronze: 8 + 6 ÷ 2 = 11 not 7, and 5 × 32 = 45 not 225. This podium layout also captures distributivity: pupils see that × is literally written over + and –, and so they recall that multiplication is distributive over addition and subtraction, hence 3 × (a – b + c) ≡ 3a – 3b + 3c. Similarly, they see that division is written over addition and subtraction, and this should help them recall that

$$\frac{12+15}{3} = (12 \div 3) + (15 \div 3) = 4 + 5, \text{ and that } \frac{x-x^2}{x} \equiv (x \div x) - (x^2 \div x) \equiv 1 - x.$$

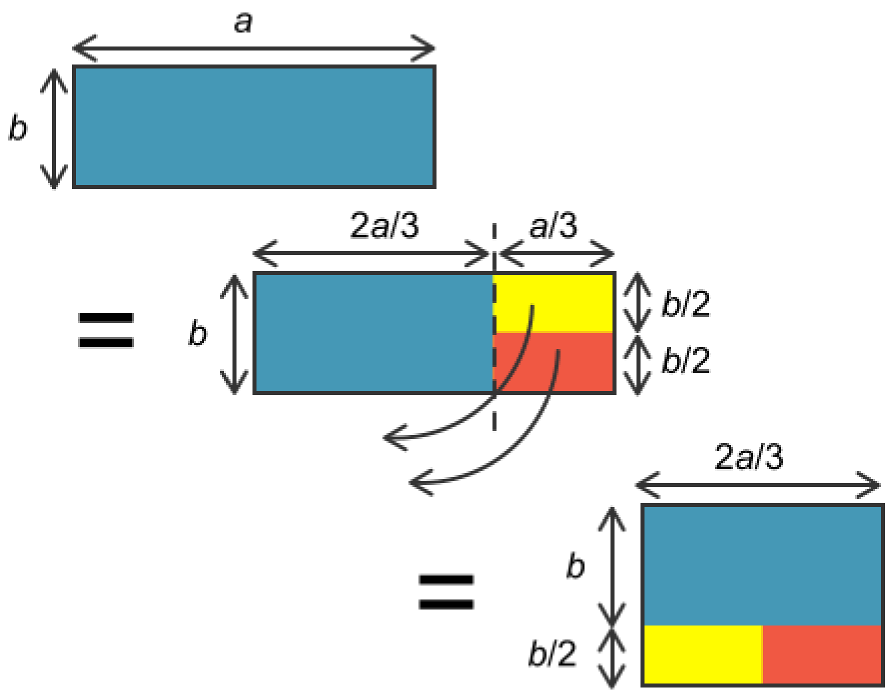
In contrast, $3 \times (4 \times 5) \neq (3 \times 4) \times (3 \times 5)$ because multiplication is not distributive over multiplication: × is not written over ×.

It is likely that your pupils will be less confident with multiplicative invariance, other than in straightforward strings such as $6 \times 4 \div 4$ and $6 \div 4 \times 4$. Calculations such as $7 \div 3 \div 2 \times 24$ and $632 \times 123 - 316 \times 246$ will generate lots of worthwhile discussion and reasoning. Confident mastery of multiplicative invariance will enable your pupils to reasons in contexts where its applicability is not immediately apparent: if the density of metal G is 25% bigger than the density of metal S, by what % will the volume of a block of metal G be smaller than a block of metal S if the two blocks have the same mass?

The area representation of multiplication should help develop pupils' understanding of multiplicative invariance. If your pupils think of the picture of a (real) 12 by 4 rectangle when considering the (abstract) multiplication 12×4, then they can see (with either real or abstract scissors) that doubling the width and halving the length keeps the area the same, i.e. that $12 \times 4 = 6 \times 8$.



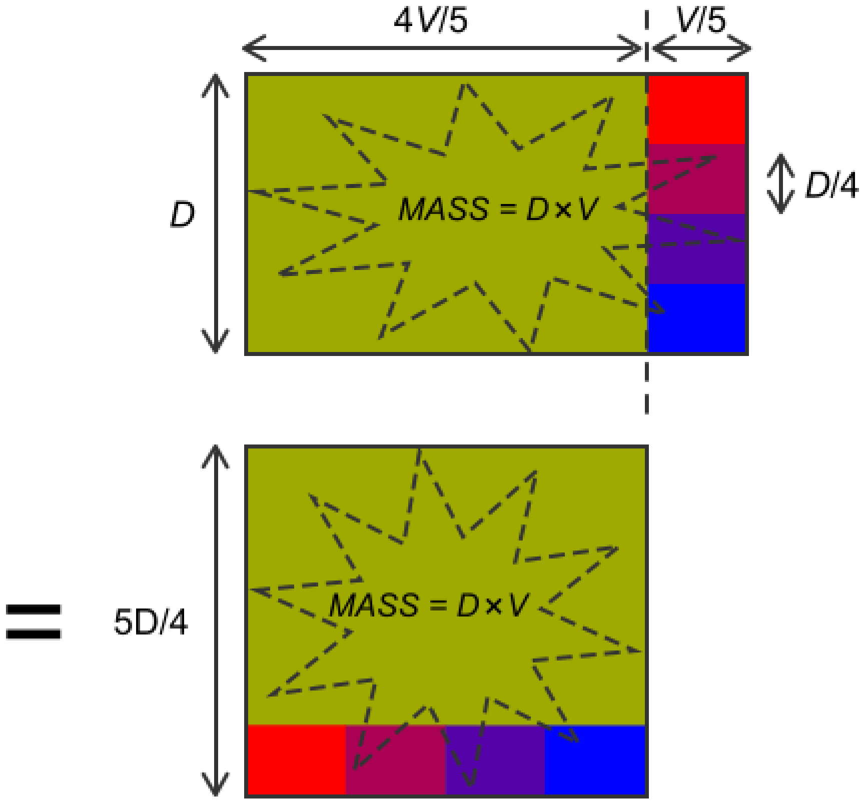
More complicated examples can be tackled either numerically or algebraically:



This experiment shows that if one factor is increased by a half (and so multiplied by a scale factor of 3/2), and the second factor is decreased by a third (and so multiplied by a scale factor of 2/3), then the product of the two factors – the area of the rectangle – will be unchanged. Once pupils have seen this represented pictorially (and, ideally, experienced it in the concrete with paper, scissors and glue), then the identity

$$a \times b \equiv \frac{2}{3}a \times \frac{3}{2}b$$

is easy for pupils to understand and explain. They can then conjecture, and prove, the result needed in the % context cited earlier: increasing the density by 25% is the same as multiplying it by 5/4, and therefore multiplying the volume by 4/5, i.e. decreasing it by 20%, will keep the product, which is the mass, invariant.



In conclusion they can articulate the general result: the product of two factors is invariant whenever one factor is multiplied by a scale factor and the second factor is multiplied by the reciprocal of the same scale factor. The mathematical language is spot on; for song lyrics, though, I'll stick with Bowie.

Image credit
Page header by PauliCarmody (adapted), some rights reserved

Secondary Magazine 118

National Centre for Excellence in the Teaching of Mathematics

Sixth Sense
Making Sense of Probability

The mathematics of probability at Level 3 is often technically quite simple – the heavy artillery of integration is not needed until the later A2 modules – but the language and approaches used often cause problems. Many students bring significant misconceptions from their GCSE, and earlier, study of probability, and so at the start of any work at Level 3 in probability, it is worth asking a range of questions to assess the security of students' fundamental model of probability, before introducing the new ideas such as probability distributions and hypothesis testing. Good questions will also ensure students' familiarity with some key tools that help to represent probability problems, and thereby support their understanding of concepts in probability.

Questions about familiar contexts can help explore students' probabilistic intuition. For example, a question such as

The probability that I roll a multiple of 3 on a fair die is $1/3$. I have rolled the fair die twenty times without getting a multiple of 3. Is the probability that I will roll a multiple of 3 on the twenty-first roll

> *a. considerably greater than $1/3$?*
> *b. approximately equal to $1/3$?*
> *c. considerably less than $1/3$?*

emphasises to students that the probability THEY assign to a situation is the number that THEY choose to represent THEIR degree of belief that something will (or will not) happen: the probability is not "in the die", it's "in the observer's head".

Contrasting this scenario with

I have a list of hard mental arithmetic questions. When I ask my pupils these questions, about a third of the questions are answered correctly. I have asked James twenty of these questions. He has answered none of them correctly. Is the probability that James answers the next question I ask him correctly

> *a. considerably greater than $1/3$?*
> *b. approximately equal to $1/3$?*
> *c. considerably less than $1/3$?*

will ensure that students think about how additional information (either implicit or explicit) alters their degree of belief about a scenario, and hence the probability that they choose to assign to the possible outcomes. These scenarios also help students to refine their understanding of the independence of events.

Similarly, useful and important discussions will be prompted by comparing two statements such as

> *a. "Robin is in my year 9 class. Half the boys in my year 9 class have ginger hair. Therefore the probability that Robin has ginger hair is 0.5"*

and

b. "My birthday is in November. Half the days in November last year were wet. Therefore the probability that my birthday will be wet next year is 0.5".

Discussion of the scenario

Blaise thinks that the probability that a drawing pin will land head-side-down when it is flicked in the air is 1⁄3. He flicks ten drawing pins in the air and six of them land head-side-down.

a. Is Blaise wrong about his assessment of the probability?

Blaise then goes on to flick one thousand drawing pins in the air and five hundred and eighty seven of them land head-side-down.

b. Does this additional evidence change your opinion in a?

René says "The probability of something is a fraction of times it has happened. Therefore the probability of a drawing pin landing head-side-down was 6⁄10; then it changed to 587⁄1000 in the second experiment. It does not make sense to talk about the probability that something will happen because you don't know about the future

will lead to a valuable debate around experimental probability and the use of probability to quantify uncertainty.

Further exploration of probability through traditional questions such as

You are wandering down the street in your local town or city. Place the following events in the order of their probability, from lowest to highest:

a. The next person I see is blonde
b. The next person I see is female
c. The next person I see is both blonde and female
d. The next person I see is either blonde or female (or both)
e. The next person I see is either a blond male or is female

will prompt discussion of mutually exclusive and independent events, and also the probabilities of combined events. This discussion could be facilitated by using a card sequencing activity for the probabilities of the five chosen events. It would also lead naturally to the use of Venn diagrams as a representation tool of probabilities: surely we don't have to wait until September 2017 and the revised AS/A Level Mathematics to revitalise the use of Venn diagrams? It is interesting to note that, according to the Adventures of the Black Square exhibition currently at the Whitechapel Gallery in London, Venn diagrams were banned by the military junta in Argentina in the 1970s because they promoted (it was argued) overlapping ideologies and collaborative working!

Thinking "Venn diagrammatically" might also help students as they consider statements such as

a. The probability it will rain at some time tomorrow is 2⁄5; the probability that it will be dry all day tomorrow is 5⁄8.
b. The probability it will rain at some time tomorrow is 2⁄5; the probability that I will forget to do my maths homework tomorrow is 5⁄8.

c. The probability that it will rain tomorrow is 2⁄5; the probability that there will be torrential rain tomorrow is 5⁄8.

As students become more familiar with the conceptual model that probabilities measure THEIR degree of belief in, or knowledge about, a situation, so it will be easier for them to understand, discuss and resolve some of the classic “pseudo-paradoxes” of probability, and to explain with confidence correct interpretations of risk and likelihood in circumstances such as medical testing and trial by jury. We shall look at these in the next issue.

Image credit

Secondary Magazine 118

National Centre for Excellence in the Teaching of Mathematics

From the Library
Shh! No Talking!

Our regular feature highlighting an article or research paper that will, we hope, have a helpful bearing on your teaching of mathematics

You may feel sated by the Christmas television (all those repeats … just like the cold cuts that seemed to feature in every 'twixmas meal …) but Heather Mendick's article A Mathematician Goes to The Movies persuaded us back in front of the DVD player. In the article, Heather, who is a member of the Educational Research Department, at Lancaster University focuses on four films that have a mathematician as the central character:

- A Beautiful Mind
- Pi
- Good Will Hunting
- Enigma.

She argues that *"the films create gendered pictures of what being a mathematician and doing mathematics mean, and that these pictures have powerful impacts on the ways in which learners construct their relationship with the subject." She concludes that "these stories of mathematicians work to maintain rationality as masculine, and being good at maths as a position that few men and even fewer women can occupy comfortably … they support a key feature of the "nerd" stereotype".*

Robert Wilne, the NCETM's Director for Secondary, is a keen cinephile: he tweets in a personal capacity @FilmCakeMath. He counters to Heather's assessment that *"the first three films are about mental illness, not mathematics: the characters happen to be mathematicians, their profession is incidental to the drama that arises from their malfunctioning brain chemistry. The negative, frightening "nutter" stereotype they perpetrate is far more reprehensible, and dangerous, than any "nerd" stereotype. Enigma is just a Buchanesque tale of derring-do: it's no more a film about mathematics and mathematicians than North By Northwest is about aerial agricultural pest controllers and their magnificent flying machines. These films may well have an impact on how people imagine mathematicians to be, but the impact is not intentional, and it would be unfair to blame the writers and directors. In contrast, it will be very interesting to see how mathematicians are portrayed, and perhaps stereotyped, in the forthcoming deliberately-Maths-Olympiad-set (honestly!) X and Y. Will ASD + genius + hard sums + first love = Xcellent, or Y the XXXX did I pay £10 to see this?"*

Heather's paper is a good stimulus for discussion with your pupils, so that they have a deeper understanding of how portrayals of mathematics in the media could be discouraging them or their peers from further study. What can schools and teachers do to present a more positive mathematical image to our pupils? Let us know what you're trying in your department?

Image credit

It Stands to Reason

Pythagoras Theorem

In this regular feature, an element of the mathematics curriculum is chosen and we collate for you some teaching ideas and resources that we think will help your pupils develop their reasoning skills. If you'd like to suggest a future topic, please do so to info@ncetm.org.uk or @NCETMsecondary.

How do you introduce Pythagoras' Theorem to your pupils? Most adults remember something about a square and that word beginning with h, but few can demonstrate complete mastery of the concept. Other adults know that having a loop of rope with 12 equally spaced knots can be used to find a right angle if it is arranged to make a triangle with sides 3, 4 and 5 … though I've never met anyone who has such a carefully knotted rope to hand when it's needed …

You may like to introduce the ideas of Pythagoras using the NRICH task Tilted Squares. Pupils are encouraged to work systematically to find the areas of squares as they progressively 'tilt' on a square grid:

By considering how to describe the 'tilt' mathematically, the right angle triangle of which these squares are the 'square on the hypotenuse' can be made evident, and this helps pupils to suggest an appropriate connection. To complement this task, you could show a tiling pattern that is based on the theorem:

There are a number of dissections that can demonstrate Pythagoras' Theorem; that of Perigal is on his gravestone! For some pupils, this video clip reaches the parts of their brain that other demonstrations fail to do.

However, pupils need to have deep conceptual understanding of the generality of the theorem: dissections and tiling patterns show a specific example each time. Dynamic Geometry is a good tool for demonstrations which reinforce generality. From this, pupils can develop for themselves a proof of Pythagoras' Theorem. Pythagoras Proofs suggests alternative ways of deriving a proof, which can be scaffolded appropriately so that all pupils can access them.

Following on:

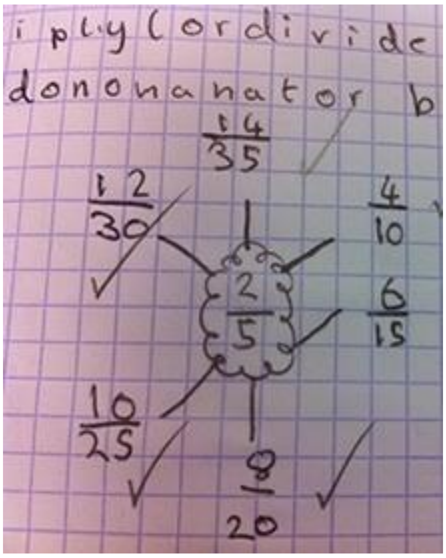
- Triples and quadruples: from Pythagoras to Fermat enables pupils to experiment with generating Pythagorean triples and quadruples;
- Focus on Pythagoras draws together ideas from research and lesson observations;
- The Napkin Problem will liven up the school canteen;
- and, finally, you can display what your pupils have created on a Pinterest board similar to this one.

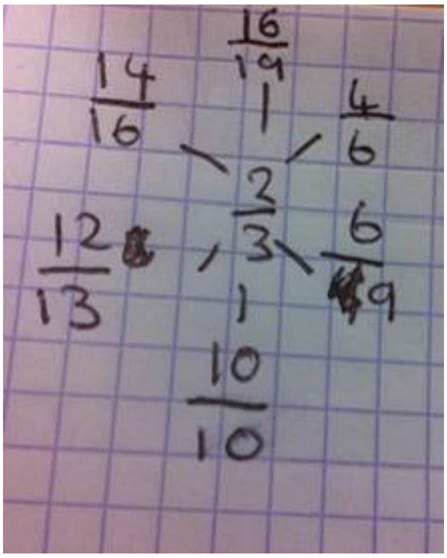
Image credit

Eyes Down

Our monthly picture that you could use with your pupils, or your department, or just by yourself, to make you think about something in a different way

These pictures were taken from the exercise books of two Year 7 pupils. The left hand picture seems to indicate that this pupil has a good understanding of equivalent fractions; what of value can we take from the right hand photograph?





Securely understanding equivalence is crucial if pupils are to be fluent and confident when operating with and on fractions. Using these or similar photographs, you could ask your pupils to

- justify the “ticked” equivalences for 2⁄5;
- suggest a fraction that is equivalent to 2⁄5 but with a constraint, for example the denominator is more than 50, or is odd, or equals 12;
- select the correct equivalences for 2⁄5;
- explain why the incorrect equivalences are wrong (why 2⁄3 ≠ 10⁄10 etc.);
- suggest the (mistaken) thought processes behind the pupil writing the incorrect equivalences;
- write rules to explain how you can and cannot operate on a fraction so that the answer is equivalent to the original fraction;
- devise questions that test the depth of the first pupil’s conceptual understanding. How could the teacher be sure that this pupil really “gets it”?

If you try successfully other ideas, please do share them with us.
If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk, with a note of where and when it was taken, and any comments on it you may have.

Image credit