



Exam season is nearly over, and as we say goodbye to some of our pupils it's also a good time to start to prepare for next year's teaching – in particular, to think and discuss with our colleagues how we're planning to meet the aims and challenges of the new KS3 and KS4 curricula. With this in mind, this issue not only suggests some ideas for the remaining weeks of the summer term but also prompts some thinking for September. As always your views are very welcome, by email to <a href="mailto:info@ncetm.org.uk">info@ncetm.org.uk</a> or on Twitter, <a href="mailto:@NCETMsecondary">@NCETMsecondary</a>.

#### **Contents**

#### **Heads Up**

Here you will find a checklist of some of the recent, or still current, mathematical events featured in the news, by the media or on the internet: if you want a "heads up" on what to read, watch or do in the next couple of weeks or so, it's here. This month we've included an update on the Ofqual sample assessment materials review, the latest blog from the NCETM Director, new research about post-16 participation, and a new online discussion group.

#### **Building Bridges**

The regular feature in which discussion of secondary mathematics topics draws out the inter-connectivity of the topics with preceding, succeeding or surrounding topics, in ways that will support and enrich your teaching in KS3 and KS4. This month: surds.

#### Sixth Sense

Stimulate your thinking about teaching and learning A level Maths. This month: an approach to integration, from first examples up to integration by parts.

#### From the Library

Want to draw on maths research in your teaching but don't have time to hunker down in the library? Don't worry, we've hunkered for you: in this issue we flag up a recent paper from Jo Boaler, in which she discusses fluency within maths, in particular rapid and reliable recall of core number facts.

#### It Stands to Reason

Developing pupils' reasoning is a key aim of the new national curriculum Programmes of Study, and this monthly feature shares ideas how to do so. In this issue we look at a rich geometrical problem, and suggest how to help pupils tackle it and reason about it.

#### Eyes Down

A picture to give you an idea: "eyes down" for inspiration. This month it's "The Maths Mr Men" series.

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### **Heads Up**

Whilst not wanting to define this article as mathematical gossip, it does come close! We've brought together news and current mathematical affairs, all in one place. We do hope it will interest you.



On 21 May, Ofqual released the long-awaited and much anticipated <u>conclusions of their review</u> into specimen GCSE assessment materials, and the headline is that all four awarding bodies have significant work to do. Three out of four Awarding Bodies were told to make some of their papers easier, and one was told to increase the level of difficulty. The video on this <u>Ofqual page</u> gives a helpful summary. Not surprisingly, there has been much counter-balancing comment (of varying quality and value, it must be said) by maths education bloggers and tweeters.



How many NQTs leave the profession within their first year? A misunderstood ATL report has led to the media claim that it is nearly 40 per cent. If true this would be shocking, but the consistently rigorous More or Less programme on BBC Radio 4 unpicked the claim in this recent episode and found it to be incorrect – leading, surprisingly, to a retraction of the claim from the Independent. If you (or your pupils) haven't heard this programme before, you might like to try. It's always an excellent mix of thought-provoking items based on data that have been cited in the media...so often wrongly!



In <u>a new blog post</u> by the NCETM's Director, Charlie Stripp shares his thinking on the philosophy of teaching for mastery in secondary maths. There is also <u>a new area</u> of the NCETM's website, which pulls together all our work, and thinking, in this rich and exciting area



There are some interesting and thought-provoking findings on the issue of GCSE maths (and English) resits at FE colleges in some <u>new research</u> from ACER (the Association of Colleges in the Eastern Region) on behalf of the DfE.

The link contains Case Studies and an extensive resource guide for tutors focusing upon GCSE resits for post-16 students. It highlights a range of tried and tested approaches, highlights the very best practice going on at a college near you, and points you in the direction of the best revision resources. BBC Radio 4's Today programme on 2 June covered this issue; you can listen again <a href="here">here</a> (at about 1h 47).



FRESH is a family of courses specifically designed for experienced teachers of mathematics by Mathematics in Education and Industry (MEI). It is MEI's response to such teachers' requests for short and refreshing professional development in specific areas of secondary mathematics teaching. Apply for current courses via the MEI website.



The brilliant American mathematician and economist John Nash died on 23 May in a tragic accident. He was a winner both of a Nobel Prize and an Abel Prize (the maths equivalent of the Nobel Prize, which interestingly isn't awarded for mathematics). You can read his fascinating story on the <u>Guardian website</u>.





New statistics from the Campaign for Science and Engineering, which encourages girls to study Maths and Physics for A level, sets out the challenge: in A level Maths there are seven girls to every ten boys, and in Further Maths there are only four girls to every ten boys. Ofsted are now publishing national statistics on gender imbalance, and so this issue will increasingly have paid to it the attention it needs if it is to be addressed. Research into tackling the imbalance has been undertaken by the Further Maths Support Programme.



<u>#slowmathchat</u> is gathering momentum on Twitter. Devised and hosted by <u>reasonandwonder.com</u>, it differs from most other webchats in that there is no specific time - it gathers ideas and momentum across the week, with coding to identify what the questions have been, and the answers (so far) that have been posted.

How does #slowmathchat work?

- Every Sunday at 5pm (Pacific Standard Time, GMT-8hrs), the topic for the week is posted on reasonandwonder.com and shared on Twitter
- There are five questions, one for each day (Monday to Friday)
- The daily question is posted on Twitter at 7 am and 7 pm (Eastern Standard Time, GMT -5hrs)
- At the end of each week, an archive of the conversation is posted on reasonandwonder.com and shared on Twitter.

Participation, therefore, is not time zone dependent, and so you have no story time/dinner time/gym time/Game of Thrones time excuse not to join in!

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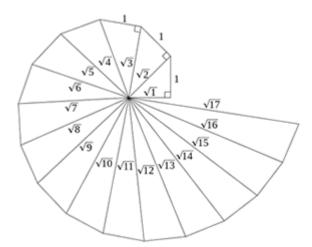


### **Building Bridges**

Surds is a topic which always raises a titter on Twitter and some clamour in the classroom; I'm sure my pupils are chuckling because it rhymes with nerd and not for reason of any other aural similarity! Building pupils' knowledge and conceptual understanding as they make their journey through square numbers and square roots, estimation and accuracy and the efficiency of surd form, and on to rational and irrational numbers, requires that they follow a route that needs to be logical and meaningful; otherwise they stumble around this odd topic and at best learn some procedural rules, but are left still questioning "why am I working in surds?" and "what even is a surd?".

Knowledge of square numbers and square roots, identifying square factors and use of a calculator are all pre-requisite skills, but how should we develop deeper understanding and not just reasonably confident procedural manipulation of surd form?

One starting point is to investigate Pythagoras' Theorem and the understanding of square root form. Using the NRICH <u>Tilted Squares</u> session is one way to deepen understanding of squares and square roots. This is part of a series called "Dotty Grids - an opportunity for exploration" from NRICH which does exactly what is says on the html tin. A recent find which looks to be a very rich resource area is <u>Geogebra Tube</u>. Search for "Pythagoras Theorem" and you will be deluged with resources and activities to try out.



A favourite activity of mine has to be the Spiral of Theodorus, which builds up a seashell-like shape using square roots. This shape can then be used in various art works and thereby could help pupils develop a love of and understanding of the maths in nature. It's well worth spending time on a search for Spiral of Theodorus and incorporating this into some classroom display. The artwork that your pupils create could be very original and clever in design.

Locating square numbers on a number line is a worthwhile activity: by first identifying where the squares for  $1^2$ ,  $2^2$ ,  $3^2$ ,  $4^2$  lie on a number line, pupils can then work forwards and backwards to estimate where  $1.5^2$  lies, 2.8² lies, etc. With a calculator, pupils can compare their estimates of the squares with the true values, and also vice versa: ask them what they notice when they work to one decimal place, comparing e.g.  $1.2^2$  and  $\sqrt{1.4}$  both rounded to one decimal place. An investigation of this kind will make clear the value – and efficiency – of the surd form,



The manipulation of surd form can be very well practised with Tarsia puzzles. There are many good lesson ideas at <u>resourceaholic.com</u> too, and within the NCETM <u>Departmental Workshop</u> on surds there are suggested activities from the NCETM, including the history of surd use, and suggested activities.

Once your pupils become more proficient at manipulating surds, they could respond to this <u>Inquiry Prompt</u> from Rachel Horsman. The mathematical notes on the website provide plenty of rich and challenging surd questions to the point where your pupils will be doing a triple back somersault with confidence and grace!

And if ever there's a moan that surds are dull, show your pupils the highly addictive <u>Angry Surds</u> (love the pun!) game.

You can find previous Building Bridges features here.

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#### Sixth Sense

#### Sixth Sense 121

In my last article, I touched on the strategy of guess and check to integrate functions:

Question: What did I differentiate to get  $16x(x^2-5)^3$ ?

Guess:  $\left(x^2 - 5\right)^4$ 

Check: The derivative of this is  $4(x^2-5)^3 \times 2x$ 

Thought: So I need to multiply this guess by 2 to get the answer I'm looking for

Therefore:  $\int 16x(x^2-5)^3 dx = 2(x^2-5)^4 + c$ 

Developing this thought process as early as possible in your students is well worth doing, because they can return to it time and again during their A level studies. One can start immediately after teaching differentiation:

Question 1: What did I differentiate to get  $x^3$ ? Guess: The power of 3 makes me think of  $x^4$ Check: When I differentiate  $x^4$ , I get  $4x^3$ 

Thought: This is 4 multiplied by the answer I was hoping for

New guess: I'll divide by first guess by 4, to get  $\frac{x^4}{4}$ 

Check: The derivative of this is  $\frac{1}{4} \times 4x^3$ 

Therefore: The answer to the question is  $\frac{\chi^4}{4}$ 

[with constants still to be discussed!]

Question 2: What did I differentiate to get  $5x^3$ ?

Guess: Given what I've just done, I'll try  $\frac{5x^4}{4}$ 

Check: If I differentiate, I get  $\frac{5}{4} \times 4x^3$ 

Therefore: Right first time!

Having practised lots of AS-level problems of the form  $\int ax^b dx$ , your students are now ready for the types they'll meet at A2, such as the one at the top of the article. In time, most of the method should take place in the students' heads – they need to develop procedural fluency, and not have to write a short script every time they integrate - but they must always be able to vocalise the process when asked, so that you can check that they are developing conceptual understanding.

Question:  $\int (x-1)(x^2-2x+7)^4 dx = ?$ 

Guess: I see a power of 4, so I'll try  $(x^2 - 2x + 7)^5$ 



Check: This differentiates to  $5(x^2-2x+7)^4 \times (2x-2)$ 

Thought: Ah-ha: if I factorise the second bracket, this is 10 times what I want, so I

need to divide my guess by 10.

Therefore:  $\int (x-1)(x^2-2x+7)^4 dx = \frac{(x^2-2x+7)^5}{10} + c$ 

Now, once the existence and significance of the function  $e^x$  have been discussed, and your students are happy and confident with the result (as explored last time)

that  $\frac{d}{dx} \Big( e^{\text{function}} \Big) = e^{\text{function}} \times \text{function differentiated}$ , then they can approach integrals such as  $\int 2e^{7x} dx$ 

and  $\int 5xe^{4x^2-3}dx$  using the *guess and check* structure:

Guess:  $e^{7x}$ 

Check: This differentiates to  $e^{7x} \times 7$ , so I need to divide by 7 and multiply by 2.

Therefore:  $\int 2e^{7x} dx = \frac{2e^{7x}}{7} + c$ 

Guess:  $e^{4x^2-3}$ 

Check: This differentiates to  $e^{4x^2-3} \times 8x$ , so the guess is 8 times too big.

Therefore:  $\int 5xe^{4x^2-3}dx = \frac{5e^{4x^2-3}}{8} + c$ 

It is, of course, important to see that this method has its limitations – in the same way that the "rule" for differentiating  $(function)^6$  does not apply to  $x^2(4x-1)^6$ , the *guess and check* procedure will not work easily for certain integrals:

Question:  $\int x^2 (x^2 - 8)^7 dx = ?$ 

Guess: I see a power of 7, so I'll try  $(x^2 - 8)^8$ 

Check: The derivative of this is  $8(x^2 - 8)^7 \times (2x)$ 

Thought: I need to divide by 16 and multiply by *x* 

New guess:  $\frac{x(x^2-8)^8}{16}$ 

Check: Hang on a minute, I can't differentiate this without expanding, or using the product rule...



Therefore: ... I'm stuck

Teacher: For the time being ...

However, having seen further examples of the chain rule, students can successfully guess and check increasingly complicated integrals:

$$\int 3\sin 8x dx = ?$$

$$\int 4\sin 2x \cos^3 2x dx = ?$$

(and it's interesting to ask them to compare this with  $\int \cos^3 2x dx$ )

Now for the payoff! Every time I teach this next bit I am struck by how quickly most students see what's happening – they pick up the idea much faster than I did when I was at school and was taught from the textbook "the formal method".

So, guessing and checking for grown-ups, as my first Head of Department called it (and I'm sure he took it from his previous Head of Department!), works like this (and happens in the scheme **after** the product rule has been explored and mastered!):

Question:  $\int x \sin 3x dx = ?$ 

Guess: I'll try xcos 3x and see what happens

Check: The derivative of this is  $-3x\sin 3x + \cos 3x$ 

Thought: Hmmm. Tricky. One step at a time. Let's sort the first term out.

New guess:  $-\frac{x\cos 3x}{3}$ 

Check: This differentiates to  $x \sin 3x - \frac{\cos 3x}{3}$ 

Thought: Now I need to include in my guess an extra term which, when

differentiated, cancels the second term, which I don't want

Guess again:  $-\frac{x\cos 3x}{3} + \frac{\sin 3x}{9}$ 

Check: This now differentiates to  $x \sin 3x - \frac{\cos 3x}{3} + \frac{\cos 3x}{3}$ 

Therefore:  $\int x \sin 3x dx = -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} + c$ 

Getting the first guess may take a while, and it is important to praise, and explore, the alternative suggestion:

Question:  $\int x \sin 3x dx = ?$ 

Guess: I'll try  $\frac{x^2 \sin 3x}{2}$  and see what happens

Check:  $x\sin 3x + \frac{3x^2\cos 3x}{2}$ 



Thought: Now I need to subtract something but I have no idea what, because

 $\int \frac{3x^2 \cos 3x}{2} dx$  is a harder question than the one I started with...

Therefore: Back to the drawing board!

By the time the students have tried two or three of these they should be able to conclude that, with this type of question, there are two "natural" guesses, but one ends up with a second integral that is *easier* than the original, whereas the other produces a second integral that is *harder* than the original: clearly the skill is in picking the better first guess.

Once they're feeling confident, do get them to try  $\int x^2 e^{4x} dx$ , and  $\int \ln x dx$  is an important example to cover at this point.

By the second or third lesson on this method, having also looked at definite integrals, students need to generalise:

Question:  $\int uv' dx$ Guess: uv

Check: This has derivative u'v + uv'

Thought: This is what I want, with an extra term, so I need to subtract from the guess

a term which, when differentiated, gives u'v

New guess:  $uv - \int u'vdx$ Check: u'v + uv' - u'v

Therefore:  $\int uv' dx = uv - \int u' v dx + c$ 

Hence, informal guessing and checking for grown-ups becomes traditional integration by parts, but specialising before generalising helps enormously. Repeatedly applying an "out of the blue" formula will probably give students procedural fluency, but their conceptual understanding is very unlikely to develop too. I'm reasonably sure that the so-called explanation which my sixth form self was given started with the last line of the above argument, and it took me a long while to understand what on earth was happening!

You can find previous Sixth Sense features here.

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## From the Library

# Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts

Jo Boaler, Professor of Mathematics Education, co-founder youcubed, with the help of Cathy Williams, co-founder youcubed, & Amanda Confer, Stanford University

Jo Boaler, alongside Dylan Wiliam and John Hattie, probably have the highest ratios of "research papers that people assertively quote": "research papers that people have actually read" amongst current educational researchers. This article is a good place to start to lower that ratio back to (at least!) 1:1. It's a summary of Boaler's research, and although it's published on her public-facing website rather than as an academic paper in a journal, there are references at the end well worth following up.

#### Introduction

<u>The article</u> starts with a story about Stephen Byers, a Government Minister in 1998. Being interviewed on BBC Radio Five about government plans to improve numeracy in schools, he was asked to multiply eight by seven. "Fifty-four," said the minister, whose job, ironically, was to raise standards in the classroom for reading, writing and arithmetic.

National ridicule occurred. "It is one of those character-forming events," he said. In contrast, David Cameron and Nicky Morgan both recently avoided answering times tables questions, with the implication that the public backlash and ridicule would be more than they were willing to stand. Boaler's thesis is that it is time to rethink radically the way in which pupils acquire factual knowledge, especially now there is a greater emphasis upon memory and recall within both the primary and secondary curricula.

Boaler's core principle is that "maths is a creative subject that is, at its core, about visualising patterns and creating solution paths". She gives clear and concrete ideas of how to do this with a range of exercises and activity that, she believes, build mathematical understanding. By downloading the full pdf you can get full access to the activities. For example, asking pupils to identify how many ways there are of completing a calculation and then comparing them for efficiency. Activities such as this exemplify the value – and not insuperable challenge – of developing mathematical understanding through engaging activities rather than focusing upon mathematical rote learning.

#### **Fluency**

Developing pupils' fluency is one of the core aims of the new National Curriculum; however, there is a spectrum of fluency, from automaton-like recall (think The Terminator does times tables ... oh actually he did, in 1990's Kindergarten Cop) to deep conceptual understanding and number sense (which Robert Wilne discusses in a <u>short piece</u> for SSAT). In the article Boaler challenges the rote learning camp, and highlights what she identifies as the damage caused by such practice. Given the emphasis in the new GCSE curriculum that pupils develop "fluent knowledge, skills and understanding", and the new directive in primary for the recall of times tables to 12 by the age of 9, it couldn't be more timely that we take the time to understand, and reflect critically, on the arguments and evidence collated here.

You can find previous *From the Library* features here.

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# It Stands to Reason Geometry and Depth

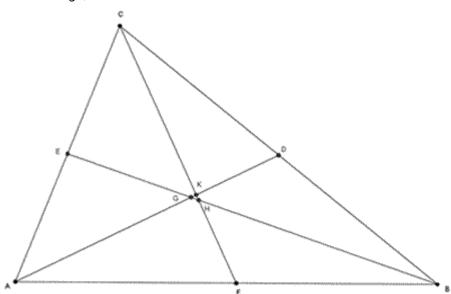
How do we provide opportunities for our pupils to develop both fluent technical skills and deep conceptual understanding and also the connections between these (as is demanded by the new National Curriculum for Mathematics), alongside developing their mathematical reasoning and problem solving confidence and resilience? This month we consider one possible activity related to the geometry of triangles that provides such an opportunity to go deeper, developing the skills and habits described explicitly in the National Curriculum: the activity entails pupils following a line of enquiry, conjecturing relationships and generalisations, and then developing an argument, justification or proof using mathematical language which they apply to this non-routine problem. To do so, they will break down the problem into a series of simpler steps, and will (hopefully!) persevere in seeking a solution.

So what is the activity? It's a classic of triangle geometry: **demonstrate that the three medians of a triangle intersect at a common point, where they divide each other in the ratio 2:1**.

#### Part 1 – Illustrate the problem

All geometrical problems need a visualisation. This could be done by hand or with the assistance of graphing/geometry software. A good picture gives pupils the opportunity to develop a visual representation of the problem and also the notation needed to help both description and later reasoning (Fig 1).

Fig 1 (click to enlarge)



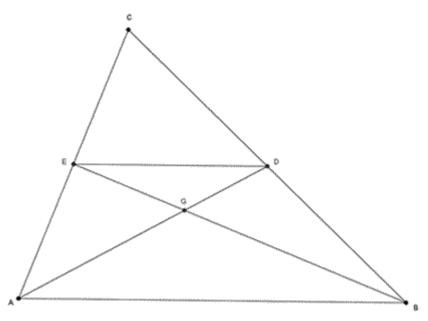
The triangle has been labelled formally ABC as have the midpoints D, E and F of each edge, and each corner has been joined to the opposite midpoint to make the medians. Here, of course, we have not assumed that the three medians are in fact concurrent at G.



#### Part 2 – Simplify the problem

To deduce the result using their reasoning skills from already known and proven results (like similar triangles or properties of 2D shapes) your pupils probably need to simplify the set up first. This could be attempted in a number of different ways, usually simplifying the problem to initially considering only two of the medians at once (Fig 2).

Fig 2 (click to enlarge)



#### Part 3 - Solve the problem

Pupils should already know about and be confident with reasoning with similar triangles before going deeper with a problem proof like this. Once they do, this will probably be a rich challenge for them. They may need some scaffolding/structure (like this) to get started:

Stage 1: Show that triangle CAB and triangle CED are similar triangles (and identify the correspondence and the scale factor).

Stage 2: Deduce that AB is parallel to ED and that AB = 2ED.

Stage 3: Show that triangle AGB is similar to triangle DGE (and identify the correspondence and the scale factor).

Stage 4: Deduce that AG = 2GD and BG = 2GE.

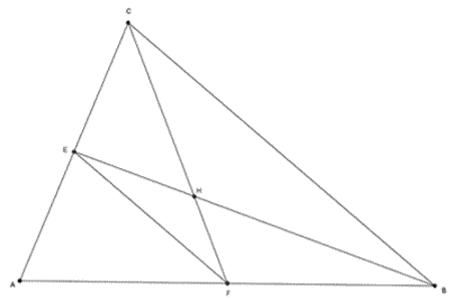
Stage 5: Interpret Stage 4 in relation to what we are trying to prove.

By this stage, they have established that G is the point of intersection of two of the medians and that it divides them in the ratio 2:1, but they will need to do more to justify that all three medians are concurrent at G. So, they pick another pair of medians (Fig 3). This is the bit that develops resilience!





Fig 3 (click to enlarge)



Notice that, again, they must not assume that the new medians intersect at G; we have given the new intersection the name H. Scaffolding should now be less likely to be required, if it was provided at the beginning.

Stage 6: Repeat Stages 1 to 5 for the new pair of medians, being careful to adapt notation accordingly.

Stage 7: Interpret Stage 6 in relation to what we are trying to prove.

Stage 8: Deduce that the three medians all intersect at a single point G, and that G divides each median in the ratio 2:1 as required.

Stage 9: Smiles and "phew!" all round.

Throughout, an emphasis will need to be placed on what your pupils record, how they record it, and the order in which they record it. One idea is to give them the appropriate formal steps but not in the right order for Stages 1 to 5 and ask them to put them in the correct logical order, and then get them to replicate this argument themselves in Steps 6 to 8.

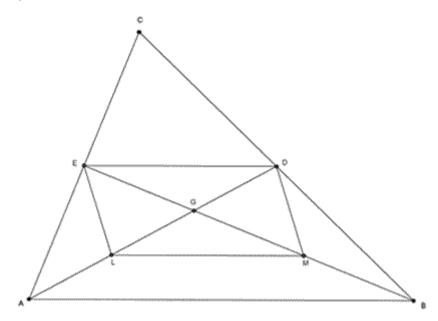
#### Part 4 – An Alternative Approach

As an alternative, some pupils may prefer/come up with an alternative approach using both similar triangles and properties of other shapes (Fig 4).





Fig 4 (click to enlarge)



Here the scaffolding might look like this, with the option to dispense with some or all of it and/or to provide the formal written steps for some or all stages but out of logical order for your pupils to reorder correctly.

Stage 1: Show that triangle CAB and triangle CED are similar triangles (and identify the correspondence and the scale factor) – as before.

Stage 2: Deduce that AB is parallel to ED and that AB = 2ED - as before.

Stage 3: Mark in L and M, the midpoints of AG and BG. Deduce that DELM is a parallelogram (opposite sides are equal and parallel) and so G bisects its diagonals (ideally, this "sub-theorem" would be proved too).

Stage 4: Further deduce that G divides AD and BE in the ratio 2:1.

Stage 5: Interpret Stage 4 in relation to what we are trying to prove.

Stage 6: Repeat Stages 1 to 5 (resilience!) for a new pair of medians being careful to adapt notation accordingly.

Stage 7: Interpret Stage 6 in relation to what we are trying to prove.

Stage 8: Deduce that the three medians all intersect at a single point G, and that G divides each median in the ratio 2:1 as required.

You might at this point want to "tick off" the curriculum points that this activity addresses. I'd suggest that it covers at least:

#### Developing fluency

• use language and properties precisely to analyse 2-D shapes

#### Reasoning mathematically

- begin to reason deductively in geometry, including using geometrical constructions
- extend and formalise knowledge of ratio and proportion in working with geometry, and in formulating proportional relations

#### Solving problems

develop mathematical knowledge including multi-step problems



- develop use of formal mathematical knowledge to interpret and solve problems
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems

#### Ratio, proportion and rates of change

- express the division of a quantity into two parts as a ratio
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- relate the language of ratios and the associated calculations to the arithmetic of fractions

#### Geometry and measures

- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines
- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence and similarity of triangles
- derive and illustrate properties of triangles [for example, equal lengths and angles] using appropriate language and technologies
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- derive and apply formulae to calculate and solve problems involving area of triangles
- understand and use the relationship between parallel lines and alternate and corresponding angles
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides and use known results to obtain simple proofs.

Not bad for one activity!

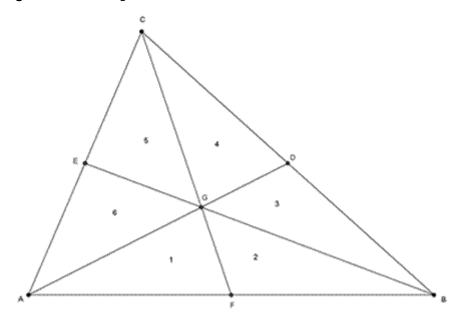
#### Part 5 - A Possible Extension

What else could you do with this triangle and its medians now that you have established concurrence and position of the intersection? One suggestion is to start to look at the areas of the triangles formed by the medians (Fig 5) and ask your pupils to establish relationships between the areas of the various triangles using the standard results that: 1) triangles on the same base with the same height have the same area, and 2) triangles on different bases with the same height have areas in the ratio of the bases. Both of these, especially the second, are worthwhile for your pupils to prove; again, they may need some scaffolding.





Fig 5 (click to enlarge)



The goal is that the triangles 1 to 6 are fully justified as having equal areas. This then leads to investigating relationships between the areas of the any of the six small triangles and the areas of any of the larger triangles, including the original whole triangle ABC (which is six times larger than any of triangles 1-6).

#### **Part 6 - More Advanced Links**

Ceva's Theorem (not generally taught in most schools or colleges these days, regrettably) provides a more general result about concurrent lines and can itself be justified using a more technical 'if and only if' style proof. It can be applied to demonstrate concurrence of the medians of a triangle very quickly, and also to prove similar results for the concurrence of the altitudes and angle bisectors of any triangle.

Those who teach Centres of Mass at KS5 (and find that their students have all but forgotten their basic geometry and geometrical reasoning by Y12/13 in the midst of the algebraic geometry approach now prevalent in AS/A Level Mathematics and Further Mathematics) will also find this activity a useful reminder for students when justifying the location of the centroid/centre of mass/centre of gravity of a uniform triangular lamina.

However or whenever you use this activity, it will certainly provide a rich opportunity for your students to further and hone their geometrical reasoning skills.

Read previous It Stands to Reason features <u>here</u>.

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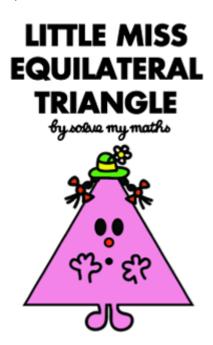


## Eyes Down – The Maths Mr Men

What could you do with a resource like this?

This delightful set of Maths Mr Men has been created by the talented Ed Southall. In total there are 41 of the little critters in the Poster Pack provided via Ed's website (44 if you include the American versions presumably because they have math and we have maths!?!). In the pack there is a picture and then an example of the maths in action.

Use them as a classroom maths display...



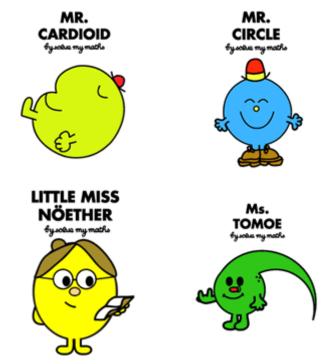
Or...

Encourage your pupils to design their own mathematical Mr Men, like these creations from Y7 pupils at Haygrove School in Somerset (*click each individual image to see the pupils' work; right-click to rotate 'Mr Circle'*):









If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at <a href="mailto:info@ncetm.org.uk">info@ncetm.org.uk</a>, with a note of where and when it was taken, and any comments on it you may have.

Read previous Eyes Down features here

#### **Image credit**

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