National Centre for Excellence in the Teaching of Mathematics

KS3 Progression Map: Number

This progression map expands upon the statements of subject content in the DfE document *Mathematics programmes of study: Key Stage 3* published September 2013. Suggested allocation of material to Years 7, 8 and 9 is given as starting points for writing schemes of work, but the implicit chronology is not intended to be prescriptive or restrictive; indeed, the programme of study is explicit that "Decisions about progression should be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content in preparation for key stage 4. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on". The NCETM fully endorses these principles, and will be developing further this progression map to help teachers achieve them.

Furthermore, although the map is organised by content, this is only for ease of reference and use. In the classroom, links between topics on the map, and between different maps, should be looked for and explored at every opportunity, so that "by the end of Key Stage 3, pupils … know, apply and understand the matters, skills and processes specified". Throughout Y7-9 pupils should have regular and opportunity and developmental feedback that helps them to **develop fluency**, to

- consolidate their numerical and mathematical capability from Key Stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]

- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics;

to **reason mathematically**, to

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally;

and to **solve problems**, to

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

The NCETM will be developing further resources to support the development and embedding of these skills.

Year 7	Year 8	Year 9
Structure		
understand and use place value for decimals, measures and integers of any size	state the multiplicative relationship between the numbers represented by any two digits in any number	state in the form $A \times 10^n$ (n any positive or negative integer) the multiplicative relationship between the numbers represented by any two digits in any number
order positive and negative integers, decimals and fractions	order positive and negative integers, decimals, fractions and numbers given in the form √n	order positive and negative integers, decimals, fractions and numbers given in the standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or negative integer or zero
use the number line as a model for ordering integers, decimals and fractions	use the number line as a model for ordering integers, decimals, fractions and numbers given in the form √n	use the number line as a model for ordering of the real numbers
use the symbols =, ≠, <, >, ≤, ≥ to make order statements about positive and negative integers, decimals and fractions	use the symbols =, ≠, <, >, ≤, ≥ to make order statements about integers, decimals, fractions and numbers given in the form √n	use the symbols =, ≠, <, >, ≤, ≥ to make order statements about real numbers
define percentage as 'number of parts per hundred', and know their decimal and fraction equivalents	relate percentages to decimals and fractions by showing their relative positions on a number line	relate percentages to decimals and fractions, moving efficiently between the different forms in any context
appreciate the infinite nature of the set of integers	appreciate the infinite nature of the sets of integers and rational numbers	appreciate the infinite nature of the sets of integers, real and rational numbers
use standard units of mass, length, time, money and other measures, including with decimal quantities	use standard units of mass, length, time, money and other measures, including with decimal and fractional quantities	use standard units of mass, length, time, money and other measures, including with decimal quantities and quantities given in the standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or

		negative integer or zero
round numbers and measures to different degrees of accuracy, for example to the nearest whole number or to one decimal place	round numbers and measures to different degrees of accuracy, for example, to the nearest whole number or to one or two decimal places	round numbers and measures to different degrees of accuracy, for example, to a number of decimal places or significant figures
Calculation		
Calculators should not be used as a substitute for good written and mental arithmetic. Teachers should use their judgement about when ICT tools should be used.		
use the four operations, including formal written methods, applied to integers and decimals; multiply proper and improper fractions, and mixed numbers, all both positive and negative	multiply and divide a whole number by a fraction, whether positive and negative	use the four operations applied to real numbers, whether positive or negative
use conventional notation for the priority of operations, including brackets	use conventional notation for the priority of operations, including brackets and powers,	use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
recognise and use relationships between the operations +, –, ×, ÷, including inverse operations	recognise and use relationships between the operations +, –, ×, ÷, squaring and finding the square root, including inverse operations	recognise and use relationships between any operations including inverse operations
	interpret fractions and percentages as operators	use A = $^{1}/_{n}$ of B implies B = nA, and A = n% of B implies B = $^{(100A)}/_{n}$
Structure depending on operations: integers		
use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple	use prime factorisation	use prime factorisation, including using product notation and the unique factorisation property
use square, cube, square root and cube root	use integer powers	use integer powers and associated real roots

		(square, cube and higher), recognise powers of 2, 3, 4, 5
Structure depending on operations: beyond integers		
		distinguish between exact representations of roots and their decimal approximations
work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $^{7}/_{2}$ or 0.375 and $^{3}/_{8}$)	work interchangeably with terminating decimals their corresponding fractions and percentages (such as 3.5, $^{7}/_{2}$, and 350% or 0.375, $^{3}/_{8}$, and 37.5%)	work interchangeably with terminating decimals their corresponding fractions and percentages, and know the fraction and percentage equivalents of some common recurring decimals (such as 3.5, $^{7}/_{2}$, and 350% or 0.375, $^{3}/_{8}$, and 37.5%, or 0.33333...., ⅓ and 33⅓%)
		interpret and compare numbers in standard form $A \times 10^{n}$ $1 \leq A < 10$, where n is a positive or negative integer or zero
Linking and extending percentages, decimals and fractions		
	interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%	understand why an “n% increase” is not the inverse operation of an “n% decrease”
Understanding numbers in contextual calculations		
round numbers and measures to an appropriate degree of accuracy, for example to the nearest whole number or to one decimal place	round numbers and measures to an appropriate degree of accuracy, for example to the nearest whole number or to one or two decimal places	round numbers and measures to an appropriate degree of accuracy, for example, to a number of decimal places or significant figures

use approximation, through rounding to the nearest whole number or to one decimal place, to estimate answers	use approximation, through rounding to the nearest whole number or to one or two decimal places, to estimate answers	calculate possible resulting errors expressed using inequality notation $a<x\leq b$
use a calculator and other technologies to calculate results accurately and then interpret them appropriately		