Dice Game

This sequence of activities provides opportunities for pupils to explore:

- **generalising**
- **structure**
- **graphical representation.**

The activities around the dice game allow pupils to make generalisations and to explore graphical representation of results to reveal the structure of the game.

Activity overview

The game is for two players, Blue and Red, and each move consists of both players throwing a die and recording their two scores by means of a single dot on the grid. Pupils will need dice to play the game for themselves.

The game starts with a dot at (0,0) and each move results in a dot to the right and above the previous dot. In the game shown, Blue rolled 2 and Red rolled 3 on the first move. They then rolled 2 and 2, and so on. The game ends when a move results in a dot being placed in the blue or red region. In the example shown, Blue is the winner.

Language and representations

Informal representations, number lines and simple plotting should be familiar to all pupils. The contextual use of this graphical representation may be new.

Useful vocabulary would include *integer*, *horizontal*, *vertical*, and *difference*. The vocabulary that may be an output of this activity includes *co-ordinate*, *vector*, *axis*, *region*, and *gradient* (however, the contextual relationships and how they can be identified by features on the co-ordinate plane are much more important).

Rationale and transition issues

This activity allows pupils and teachers to explore a different reason for plotting points in the first quadrant. The discussion of the plotting of the game reveals the structure of the Cartesian Plane in a different way to pupils and lays the foundations of discussions of gradient which come later in KS3 and KS4. The game gives a context to discuss this by considering who is in the lead, when a player is catching up or falling behind. It gives an opportunity to meet the requirements of the KS2 curriculum in a way which supports future learning in secondary school.

Preparation and prior learning

Use of a number line to track and record the addition of integers would be useful preparation. Horizontal and vertical (like a thermometer) presentations would be beneficial. Attention should be drawn to the integer labels' association with the lines and not the spaces. This would make a suitable pre-task (Worksheet 1).

Misconceptions

Pupils may not:

- recognise that the integer values are represented by the lines/intervals and not by the spaces in between them

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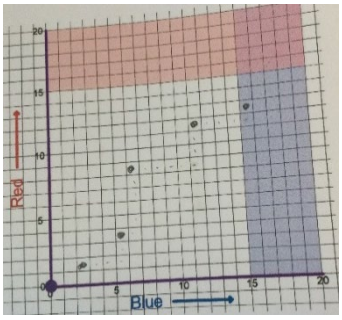
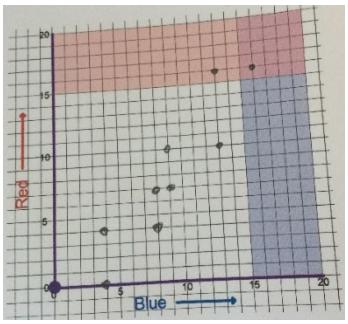


- understand that each dot represents both players' cumulative score, and instead want to make a set of dots for each player.



It is really important that pupils have the chance to play the game themselves, for long enough that the teacher can check that they fully understand the intended representation and that they start to spot patterns/features based upon their own experiences. They also enjoy it!

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Task 1: Introducing the game

Briefly explain the rules and aims of the game.

Worksheet 2 features a blank set of axes which could be used to model the game to the class with a pupil or colleague.

Online dice could be used so that all pupils can see the outcome of each roll (or large foam dice if you are lucky enough to have them).

Roll the dice one at a time (blue then red) and record the progress of the game after each pair of rolls.

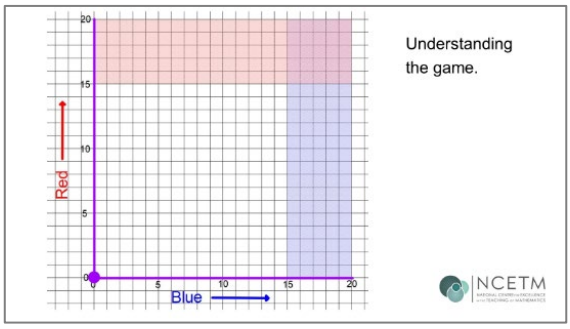
Once a couple of rolls have been modelled, question pupils to tell you where the next mark should go. Check they understand how the recording process works.

Commentary

Slide 3

Both players start with a score of zero. Where is that on the grid?

The blue player is aiming to get to the blue region, and the red player is aiming to get to the red region. The game ends when the first player gets to their target region.



Scores are recorded after **both** blue and red have rolled the dice, with one single dot/cross representing the position after both players have rolled their dice.

'Where should the next mark go?'

'What do you notice about the marks?'

'Are they on the lines or in the spaces? Why?'

Non-examples/deliberate mistakes could also be used to check understanding and prompt response/reasoning.

Who has won? Why?

The following questions will hopefully be raised by pupils (a good indicator that they are starting to think about the features of the game):

- What happens if we land in the purple region? (A draw?)
- Do we win on a total of 15, or do we need at least 16? Does the line/boundary count as a part of the winning zone/region?

(The decision you come to as a class is not that important. However, acknowledging that there is something different between the boundary of a region and being entirely within it is a worthwhile conversation. There is scope to briefly consider inequalities. Score ≥ 15 or score > 15 to win?). Once one person has won, you may wish to model another game, this time between two pupils.

Further direct questioning of the class will help you check their understanding and promote engagement.

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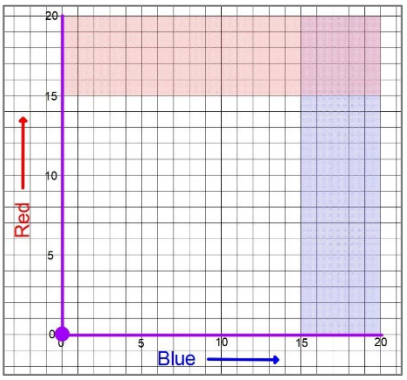
Task 2: Experiencing the game

This should not be rushed. Allow plenty of time for the pupils to play multiple games. It is vital they have plenty of experience to draw upon for the next phase of the activity and that you have the chance to check that they can reliably record the progress of their games.

Commentary



Allow the pupils to play the game in pairs, recording each game on the sheet of grids/axes (**Worksheet 2**).

Check that they understand how the recording of the game works and can do this accurately.

Whilst they play, question pupils to encourage them to start examining the features of the game and to verbally practice reasoning their responses.

This gives you a chance to set your expectations regarding the level of reasoning required. Look out for the possible misconceptions when recording games (marks that are in the spaces, rather than the intersection of lines, and marks that attempt to record blue and red scores separately). Examples of these are shown earlier in this document.

'Who has won the most?'

'Does it matter if you swap who is red and who is blue?'

'Who should win most often?'

Hopefully, pupils appreciate that chance determines the winner and may even be able to tell you that they have equal chance of winning (from the starting position). Considering who has the best chance of winning whilst part-way through a game could provoke more interesting responses:

'Does being 5 away from the winning region make you less likely to win than being 3 away from the winning region?'

The choice of who is blue/red will be irrelevant as long as scores are only recorded once **both** have rolled.

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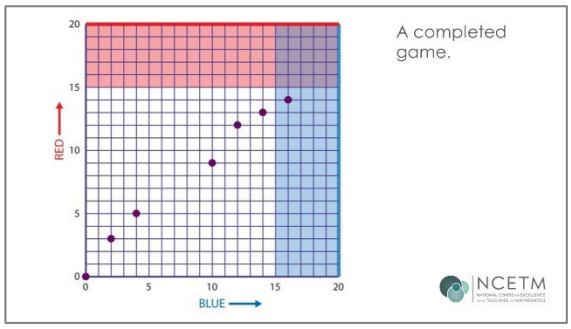
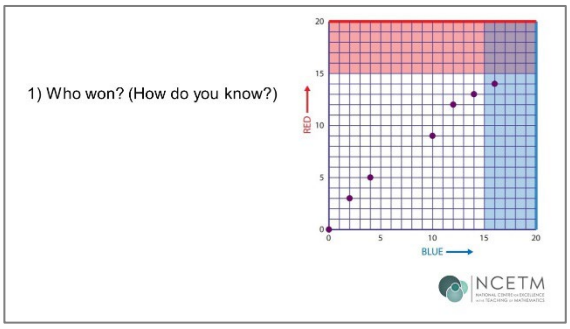
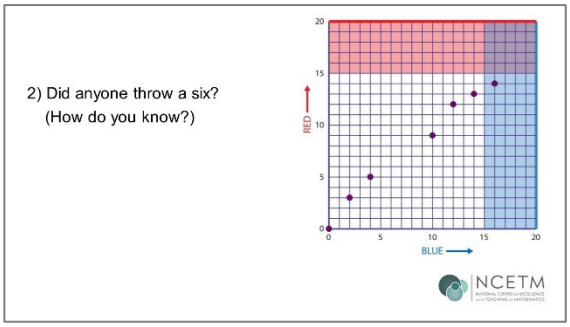
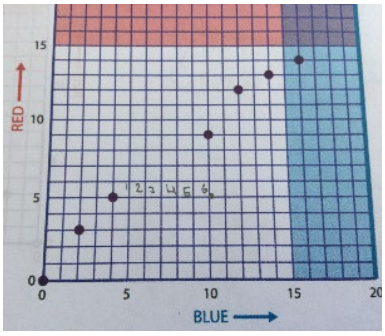
Task 3: Interrogating a game

This task involves looking at a completed game and exploring what has happened through a series of questions. These questions are on the PowerPoint slides and as a handout in Worksheet 4.

Slides 5–11	Commentary
Slide 5  	Pupils work independently (or in pairs), initially. Encourage reasoning with every response. Drawings can be a useful and productive addition to their written responses. Some potential (and actual) responses are below:
Slide 6 *'Who won? How do you know?'*  	*'Blue won because the last (most top-right) mark is in the blue region.'*
Slide 7 *'Did anyone throw a six? How do you know?'*  	*'Blue rolled a six on the third go. We know this because:* *There is a horizontal gap between the third and fourth dots on the grid.* *The mark goes from 4 across to 10 across. It has gone 6 spaces across.'* 

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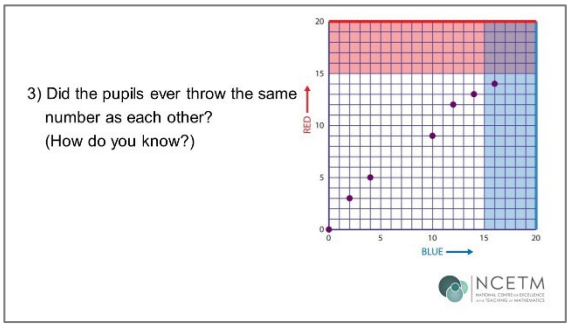
Slide 8

'Did the pupils ever throw the same number as each other? How do you know?'



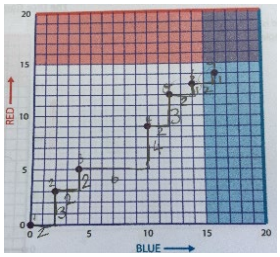
'Yes. On their second go, both blue and red scored 2. The difference in the marks is two across and two up.'

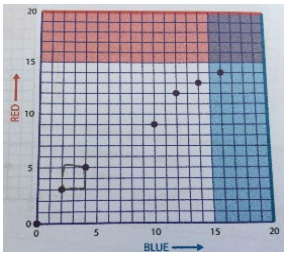
'There's a square!'

[See picture below – what potential is here to develop the understanding of gradient?]

'Squares are equal!'

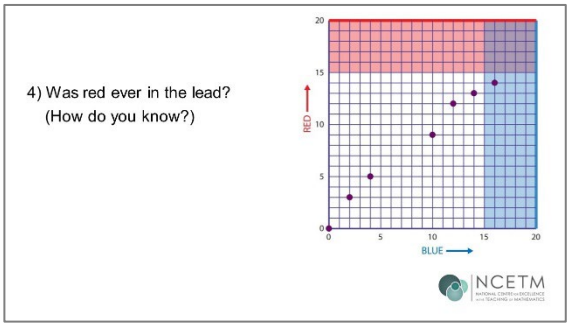
[What is equal? How could this be deepened or exploited?]





Slide 9

'Was red ever in the lead? How do you know?'



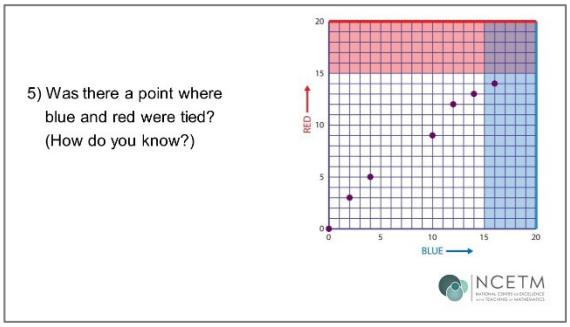
'Yes. After the first go, blue had scored 2 and red had scored 3.'

'After the first and second goes, red was in the lead because the dot was closer to the red zone than the blue zone.'

[There is a chance to develop this further after the next question.]

Slide 10

'Was there a point where blue and red were tied? How do you know?'

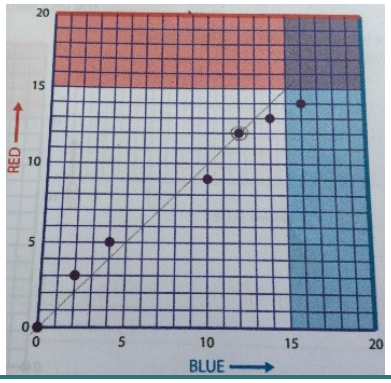


'Yes. After the fourth go both blue and red:

- *had a total of 12 points each*
- *were three (or four) away from their target zone'*

'There are the same number of jumps to both zones.'

[The line shows where blue and red would have the same scores. There is a dot on the line (an opportunity to develop Question 4).]



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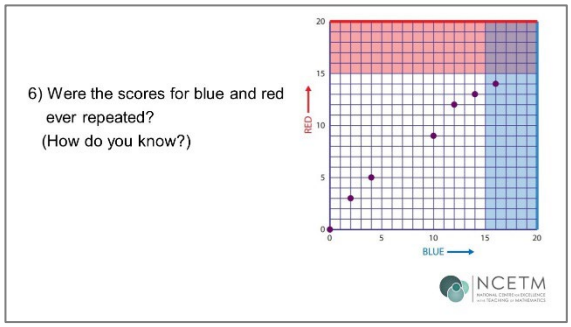
Slide 11 *'Were the scores for blue and red ever repeated? How do you know?'*  	*'The last two goes had the same scores. Blue rolled two and red rolled one. Also, the first and forth goes had the same scores. Blue rolled two and red rolled three.'*

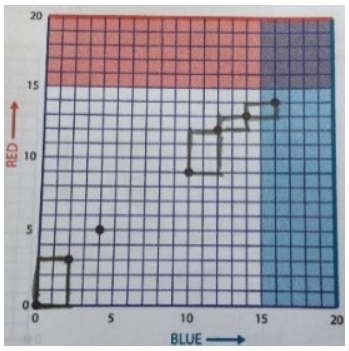
Once the pupils have had sufficient time to answer the questions on the sheet, use class discussion to guide, check, share and deepen pupil understanding.

'What conclusions can we draw?'

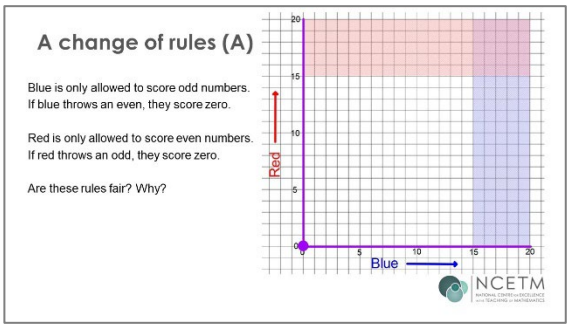
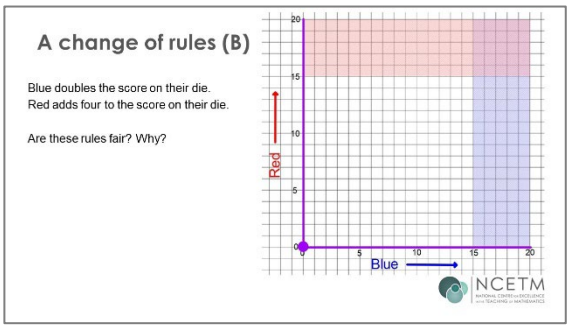
'Where have you seen elements of this before?'

'The rectangles are the same shape!' [See image, right]

'When the rectangle is tall and thin, red has scored more. When the rectangle is short and wide, blue has scored more'. [This is an opportunity to develop further understanding of gradient – are some scores equivalent in this respect?]



Task 4: Changing the rules of the game (Worksheet 4)

Slides 14 and 15	Commentary
A change of rules (A) Encourage pupils to play the game with the changes of rules (Worksheet 4). There is a blank grid on Worksheet 2, or use a new large sheet of grids/axes.  Blue is only allowed to score odd numbers. If Blue throws an even number they score zero. Red is only allowed to score even numbers. If Red throws an odd number they score zero.	A definition of 'fair' will be necessary for pupils to justify their responses. What is 'fair'? *'Blue and Red have an equal chance of winning a game'.* Reasoning could include: *'The game is fair, we both have half the numbers to choose from – we will each get to move half the time'.* Recording who wins over several games may steer pupils to explore this more deeply (especially if you can enlarge the sample size by collating results from the whole class). *'Blue can move on 1, 3, 5. Red can move on 2, 4, 6.'* So, the numbers red can move on are larger than the numbers blue can move on. *'I would expect Red to win more often'. 'Whilst it is possible for Blue to win, Red has a greater probability of winning.'* *'Red's average score is one more than Blue's average score.'* The mean score for Blue is 1. The mean score for Red is 2.
A change of rules (B)  Blue doubles the score on their die. Red adds 4 to the score on their die.	Again, encourage the pupils to **play** the game with the change of rules to develop a feel for the impact of the rules upon the numbers 1–6. Does doubling or adding 4 have a greater impact? *'It depends upon what number you start with.'* If the players roll a 4, both rules will give them a score of 8. If the players roll less than a 4, adding 4 makes a higher score than doubling. If the players roll more than a 4, doubling makes a higher score than adding 4. *'Red has an advantage when 1, 2 or 3 are rolled.'* *'Blue has an advantage when 5 or 6 are rolled.'* *'Red will score more, more of the time.'*

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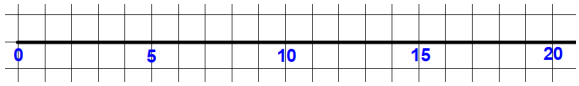
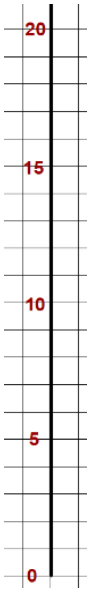
Worksheet 1

Pre-task (optional)

Hand pupils either a vertical or horizontal number line (possibly randomly).

They should roll a die and record their total (cumulative) score on the number line until they exceed 15.





- What does it look like?
- Where will the marks be?
- Where will the marks not be?

Do they understand that the integer values are represented by the lines/intervals and not by the spaces between them?

Extension

Could they represent the first three throws for **two** pupils?

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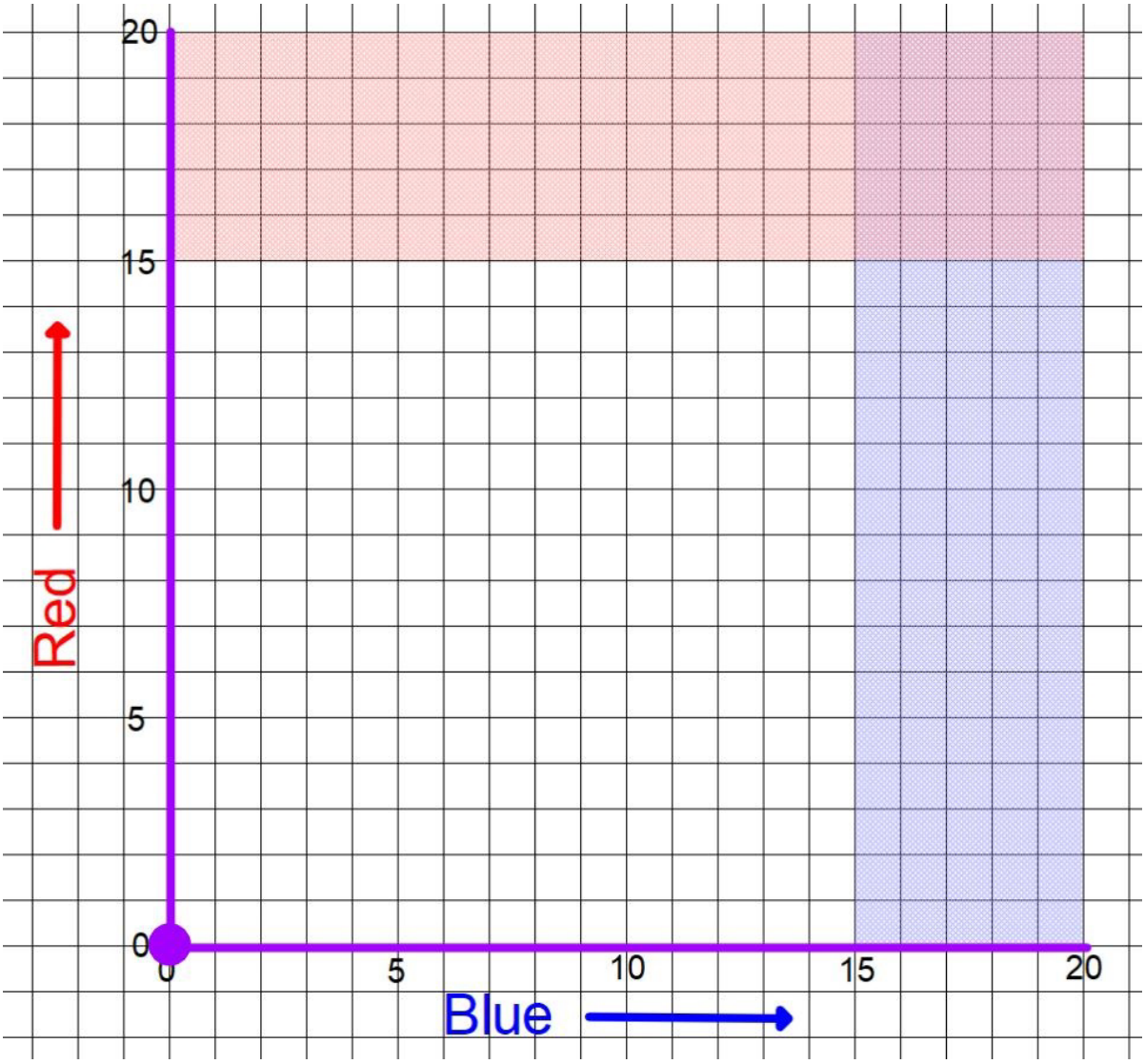
Worksheet 2

Blank axes



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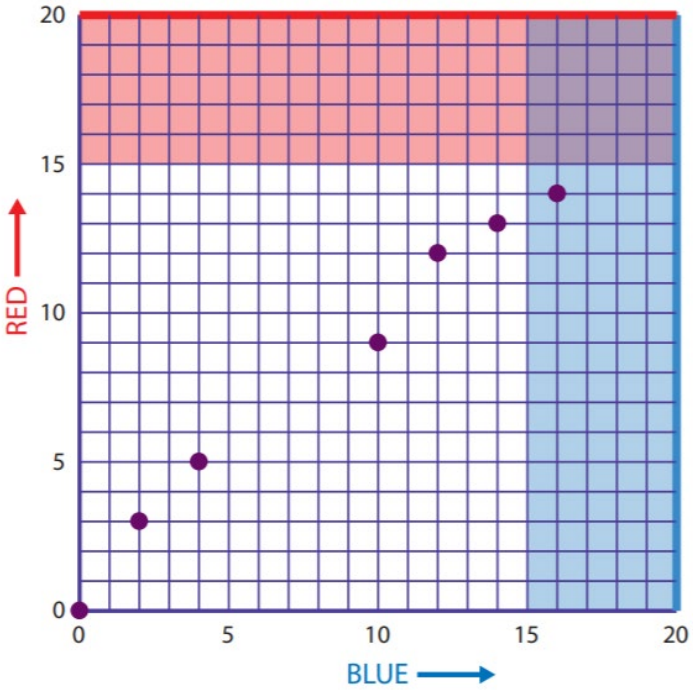
Worksheet 3

Here is a game that two pupils have already completed. Answer the questions about this game, giving **reasons** for every answer. You can draw on the grid to help your explanations.



1. Who won? (How do you know?)

2. Did anyone throw a six? (How do you know?)

3. Did the pupils ever throw the same number as each other? (How do you know?)

4. Was red ever in the lead? (How do you know?)

5. Was there a point where blue and red were tied? (How do you know?)

6. Were the scores for blue and red ever repeated? (How do you know?)

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Worksheet 4

Change of rules (A)

Blue is only allowed to score odd numbers. If blue throws and even, they score zero.

Red is only allowed to score even numbers. If red throws an odd, they score zero.

Change of rules (B)

Blue doubles the score on their die.

Red adds four to the score on their die.

Change of rules (A)

Blue is only allowed to score odd numbers. If blue throws and even, they score zero.

Red is only allowed to score even numbers. If red throws an odd, they score zero.

Change of rules (B)

Blue doubles the score on their die.

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Change of rules (A)

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Red is only allowed to score even numbers. If red throws an odd, they score zero.

Change of rules (B)

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