MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.18 Composition and calculation: three-digit numbers

Teacher guide | Year 3

Teaching point 1:

Three-digit numbers can be composed additively from hundreds, tens and ones; this structure can be used to support additive calculation.

Teaching point 2:

Each number on the 0 to 1,000 number line has a unique position.

Teaching point 3:

The smallest three-digit number is 100, and the largest three-digit number is 999; the relative size of two three-digit numbers can be determined by examining the hundreds digits, then the tens digits, and then the ones digits, as necessary.

Teaching point 4:

Three-digit multiples of ten can be expressed multiplicatively and additively, in terms of tens or hundreds.

Teaching point 5:

Known facts and strategies for addition and subtraction within and across ten, and within and across 100, can be used to support additive calculation within 1,000.

Teaching point 6:

Familiar counting sequences can be extended up to 1,000.

Summer 2018 pilot

Overview of learning

In this unit children will:

- extend their knowledge of the numbers 100–199 to include all three-digit numbers
- develop a sense of the size of three-digit numbers, based on both cardinal and ordinal representations, including the relative size of different three-digit numbers
- apply knowledge of place value and additive composition to calculation (e.g. *582 = 500 + 80 + 2*, *423 = 400 + 23*)
- explore the multiplicative composition of three-digit multiples of ten (for example, *320 = 32 tens*; *320 = 32 × 10*)
- apply known facts and strategies to additive calculations with three-digit numbers
- practise counting in multiples of 2, 5, 10, 20, 25, 50 and 100.

In the previous segment, children explored the numbers 100–199; this segment extends learning to all three-digit numbers. At the beginning of the segment, some time is spent counting specific quantities in order to give children a sense of the size of a three-digit number, and to give them some familiarity with three-digit numbers (200 and above) before they explore them in more detail. *Teaching point 1* presents predominantly cardinal and place-value representations, building towards additive calculations based on an understanding of place value. *Teaching points 2* and *3* present predominantly ordinal representations as children learn to place and identify three-digit numbers on number lines, and make estimates for common measures contexts (for example, estimating volumes of water relative to one litre). In the same way that children identified next and previous multiples of ten for given *two*-digit numbers (segment *1.9 Composition of numbers: 20–100*), they now identify next and previous multiples of ten and 100 for given *three*-digit numbers; here they also go a step further, identifying the *closest* multiple of ten or 100 (with the number line for support) to prepare them for formal teaching on rounding in segment *1.22 Composition and calculation: 1,000 and four-digit numbers*.

A key aim of this segment is to deepen children's understanding of number; rather than addressing all possible calculation strategies for three-digit numbers, the segment focuses on those which support this aim. *Teaching point 4* addresses the multiplicative composition of three-digit multiples of ten, linking to children's work on multiplication and division, and extending their experience of unitising (here, in tens). *Teaching point* 5 explores some key calculation structures, building on known facts and strategies. As well as solving addition and subtraction calculations, there is emphasis on the importance of children being able to partition numbers flexibly, for example:

576 = 500 + 76	576 = 306 + 270	580 = 450 + 130
576 = 400 + 176	576 = 306 + 230 + 40 etc.	580 = 450 + 120 + 10 etc.
576 = 300 + 276 etc.		

Often, it is the children who are unable to think about number flexibly who end up performing the most challenging arithmetic. It is therefore important to spend time building this fluency and flexibility.

The segment ends by extending familiar counting sequences in preparation for more detailed work on scales and graphing in segment *1.22*.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Three-digit numbers can be composed additively from hundreds, tens and ones; this structure can be used to support additive calculation.

Steps in learning

1:1

Guidance

Children have already worked with three-digit numbers below 200. Begin this segment by developing children's sense of the size of larger three-digit numbers. First, count forwards to given three-digit numbers, with you and the children tapping on the Gattegno chart (backward counting is addressed in *Teaching point 2*). For a given number (e.g. 342):

- count in hundreds, tens and ones, i.e. '*One hundred, two hundred, three hundred, three hundred and ten…three hundred and forty, three hundred and forty-one, three hundred and forty-two.*'
- count in tens and ones, i.e. '*Ten, twenty…three hundred and forty, three hundred and forty-one, three hundred and forty-two*' with children running their finger along each 'ten strip' on the hundreds pieces
- count in ones*; this will take some time, but counting in ones well into the three-digit numbers helps children to learn more deeply about these numbers.

*This may be the first time children have counted to such a high number in ones, and they may struggle at the boundaries, for example, 299 to 300, but this will be explored in more detail later (step *2:6*).

There are a variety of ways of using the Gattegno chart in this context, for example:

Representations

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

'…three hundred and forty-two.'

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- the children can chant, and you can tap each number they say (just *after* they say it)
- a child can tap while the class chants
- each child can have their own chart on which they tap while the class chants.

Whichever methods you use, it is important for children to have some experience performing the 'tapping' gestures themselves.

Counting in hundreds, tens and ones while tapping on the Gattegno chart serves as useful preparation for examining the place-value of the digits later in this teaching point.

- Count '*One hundred, two hundred, three hundred…*', tapping on the corresponding hundreds numbers.
- Count '*…three hundred and ten…*', tapping on 300 then ten, and so on, counting in tens until 340.
- Count '*…three hundred and forty-one…*', tapping on 300, then 40, then one, and then similarly for 342.

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1:2

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Now that children have done some procedural counting, explore three-digit quantities. Present, say, 342 pennies, with the majority of them arranged in groups of ten. Ask children:

- *'Do you think there are more or less than one hundred?'*
- *'How many hundreds are there?'*

Circle, or move together, each group of ten tens to show that there are three hundreds and 'some more'. Building on the different ways of counting in step *1:1*, discuss how we can work out how *many* more than 300 there are – either making further groups of ten out of the randomly arranged pennies or by counting up in ones.

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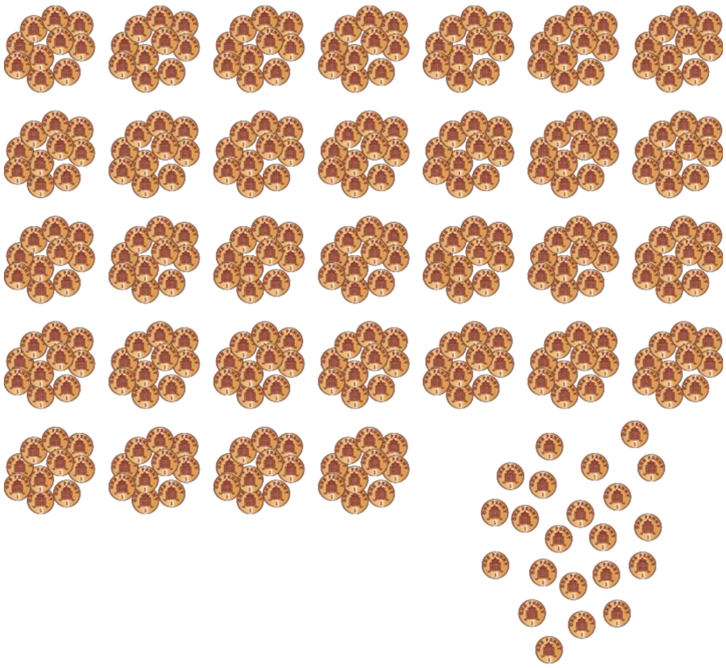
342 pennies – less structured representation:



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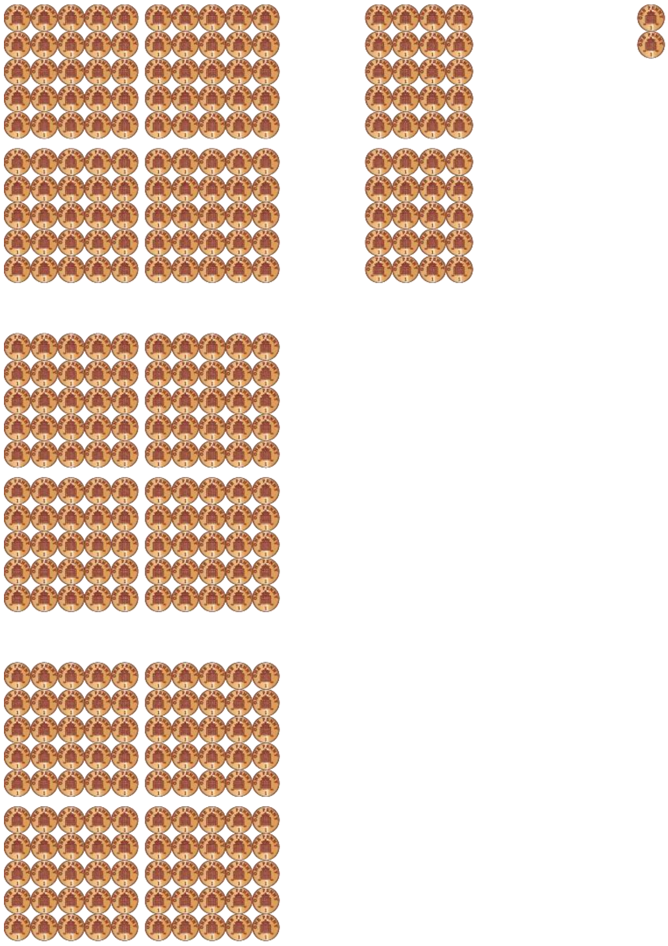
	Then move to a more structured representation of the same number of pennies, now arranged into hundreds, tens and ones, in place-value order. Record the number of hundreds, the number of additional tens and the number of additional ones, and say the number name. Reflect on how grouping items into tens or hundreds makes counting more efficient. Also discuss whether we could have found out how many pennies there were if they hadn't been grouped at all; it is important that children know we can always count in ones, but that this can be time-consuming.	342 pennies – more structured representation:  ____ hundreds ____ tens ____ ones • *'There are three hundreds and another forty-two.'* • *'There are three hundred and forty-two pennies.'*

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Now, working with the same number you explored in step 1:2, take some time to investigate further. Ask children to represent the number using Dienes, and then prompt discussions to draw out the following key concepts:

Conservation of quantity
Children may lay out the Dienes in hundreds, tens and ones 'columns'. Give value to this arrangement, but then ask children to arrange their manipulatives differently (as in the examples opposite); in each case, ask 'Does this still represent three hundred and forty-two?'
Composition
Ask children 'If you didn't have any hundreds pieces would you still be able to represent three hundred and forty-two?' Some children may be able to tell you they would need 34 tens pieces (and two additional ones), but this multiplicative reasoning will be covered in detail later. For now, ensure that children recognise that the 'missing hundreds' can be made from tens (or ones). Similarly, ask 'If you didn't have any tens pieces, would you still be able to represent three hundred and forty-two?'
Counting
Now, with the manipulatives arranged into place-value order again, count the quantity in the following ways (described in more detail in step 1:1), with children touching the relevant piece, or part of a piece, as they count in:

hundreds, tens and ones
tens, with children running their finger along each 'ten strip' on the hundreds pieces
ones, with children tapping each 'one-square' on the hundreds pieces.

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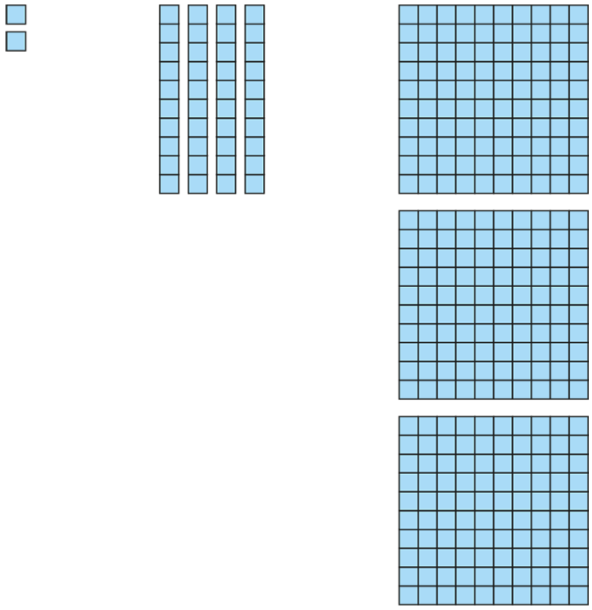
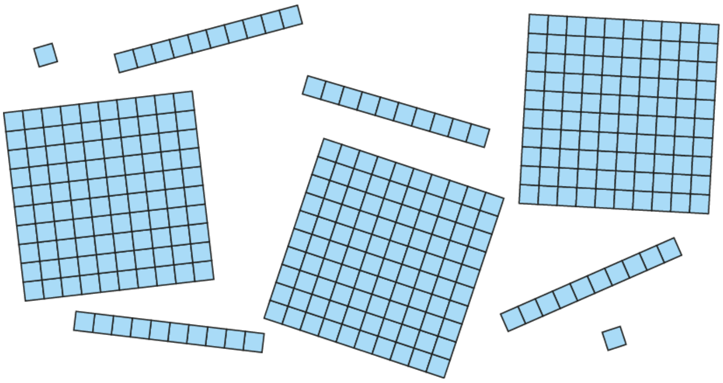
Conservation of quantity:

'Does this still represent three hundred and forty-two?'



'Does this still represent three hundred and forty-two?'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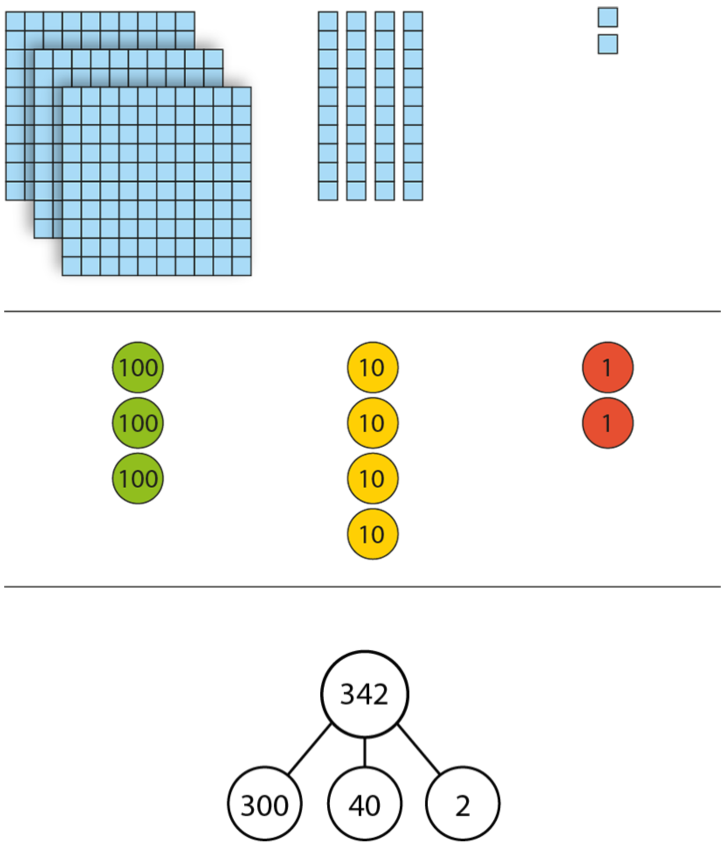
	Sometimes, moving quickly to the use of structured representations means that children are able to talk about the composition of a three-digit number in terms of hundreds, tens and ones, but struggle to see that large quantities can also be composed *just* of ones. As you go about school life, look for opportunities to highlight large quantities (for example, children in assembly, plates piled in the dinner hall, the number of millilitres of water in a bottle), and note that they are often *not* arranged into tens or hundreds. Describe numbers using the following stem sentences: • ***'___ is ___ ones.'*** • ***'___ is ___ hundreds and ___ ones.'*** • ***'___ is ___ tens and ___ ones.'*** • ***'___ is ___ hundreds, ___ tens and ___ ones.'***	
1:4	Children already have experience of writing the numbers 100 to 199 as numerals (segment *1.17 Composition and calculation: 100 and bridging 100*). Now extend this to all three-digit numbers, and explore children's understanding of place value in more detail. Continuing with the number you explored in steps *1:2* and *1:3*, use place-value counters alongside the Dienes representation. Spend some time reflecting on what is the same and what is different between the two representations: • the same: they both represent three hundreds, four tens and two ones, i.e. 342; in both cases there are the same number of individual 'pieces' • different: for Dienes, each piece size is proportional to the number it represents, whereas for place value counters it is not; for Dienes, all of the individual ones are visible,	

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whereas for place value counters they are not.

Then, show how the hundreds, tens and ones can be recorded to represent the number:
• on a part–part–part–whole diagram (300, 40 and 2)
• with numerals (342), both within and separately from a place-value chart.

Remember to take care with the place value language you use; it is easy to ask questions such as 'How many tens are there in three hundred and forty-two?' while attempting to draw attention to the four 'separate' tens; however, there are actually 34 complete tens in 342 (or 34.2 tens, though this is beyond the scope of Year 3). Step 1:6 includes some exemplar questions.
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<table>
<tr><th>100s</th><th>10s</th><th>1s</th></tr>
<tr><td>3</td><td>4</td><td>2</td></tr>
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342
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<td>1:5</td>
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The place-value composition of three-digit numbers can also be exposed using equations, as shown opposite. Children may already have some experience of combining multiplication and addition into one calculation from their work on multiplication and division. Brackets are not required here as multiplication takes place before addition; children may need reminding of this.
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Representing 342 with additive and multiplicative equations:

$342 = 100 + 100 + 100 + 10 + 10 + 10 + 10 + 1 + 1$
$342 = \underline{3 \times 100} + \underline{4 \times 10} + \underline{2 \times 1}$
$\downarrow \quad \downarrow \quad \downarrow$
$300 \quad 40 \quad 2$
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1:6

Now, for a range of three-digit numbers:

- represent a number
  - with concrete or pictorial representations (for example, Dienes, place-value counters, sticks, pennies etc.)
  - on a part–part–part–whole diagram

  or
  - by tapping it out on the Gattegno chart

  and ask children to write the number with numerals and/or say the number name out loud; use a variety of layouts for the concrete/pictorial representations and part–part–part–wholes, as shown opposite (i.e. not just in place-value order where children can 'read across')
- write numbers with numerals, and/or say them aloud, and ask children to represent them with manipulatives, pictorial representations, part–part–part–whole diagrams, or to tap them out on the Gattegno chart.

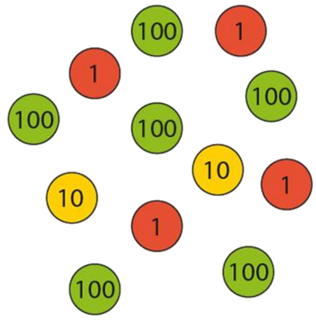
When discussing numbers written with numerals, ask questions of the form:

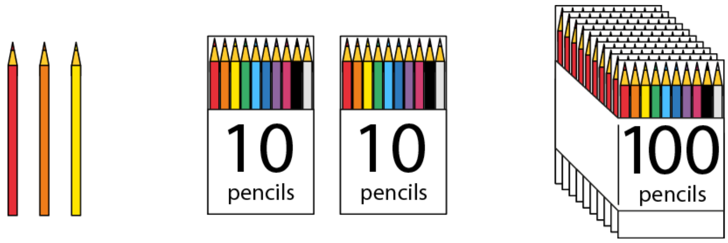
- ***'What digit is in the ___ place?'***
- ***'What is the value of the ___ digit?'***
- ***'What does the ___ represent?'***

Make sure that you include examples with a zero in either the tens or ones place (e.g. 604, 620) or in both places (i.e. a multiple of 100). The Gattegno chart is useful for highlighting the missing 'place'; for example, when tapping 604 on the chart, you can draw attention to the fact that there is no tap in the 'tens' row.

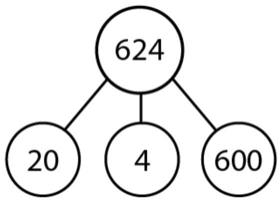
Also, ensure children practise saying numbers that are composed of a multiple of one hundred and a teen number (e.g. 312), since with the teen

Concrete/pictorial representations:





Part–part–part–whole representation:



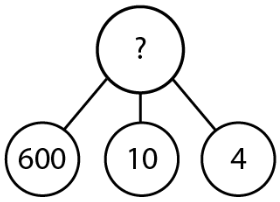
- *'What number does this represent?'*
  'This represents six-hundred and twenty-four'
  624
- *'What digit is in the tens place?'*
  'Two'
  2
- *'What is the value of the tens digit?'*
  'Twenty'
  20
- *'What does the '2' represent?'*
  'Two tens/twenty'
  2 tens/20

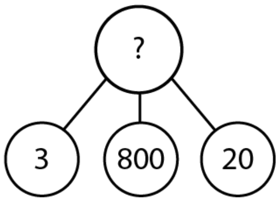
number names children can't just 'work from left to right' as they can for other numbers (for example, 742: seven-hundred and forty-two). 11 and 12 are the most irregularly named of all numbers and, although children explored these numbers in detail in segment *1.10 Composition of numbers: 11–19*, it is worth ensuring they can apply that knowledge correctly in the context of three-digit numbers.

1:7

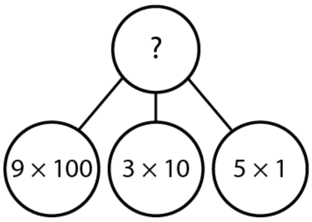
Give children independent practice moving between various representations of three-digit numbers, until you are confident that they understand the significance of each digit and the place-value composition of the numbers.

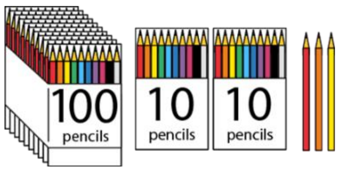
'For each representation, write down the number shown.'



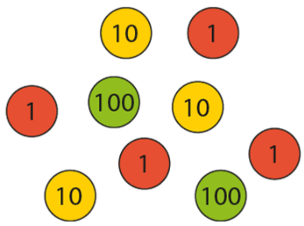
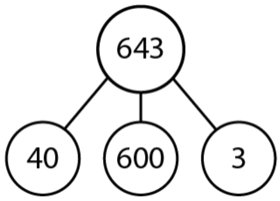
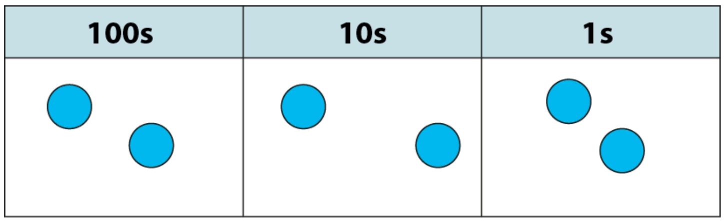
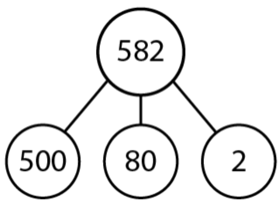


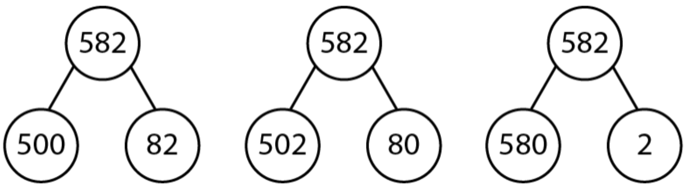
1 + 10 + 10 + 100 + 100 + 10 + 10

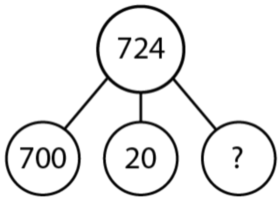
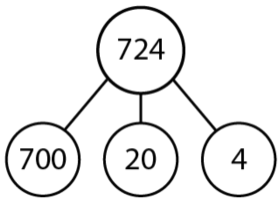


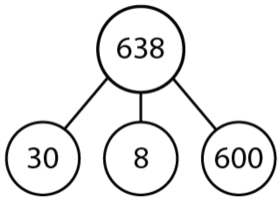
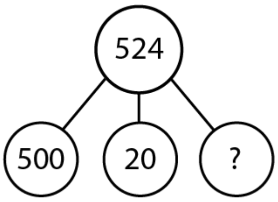
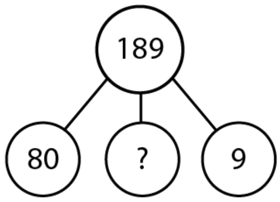


1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

		'Draw part–part–part–whole models to represent the hundreds, tens and ones parts of each these numbers.' 183 $4 \times 100 + 7 \times 10 + 3 \times 1$  *'Draw place-value counters to represent each of these numbers.'* 500 $3 \times 100 + 2 \times 10 + 9$  Dòng nǎo jīn: *'These counters have been arranged on a place value grid to show the number 222:'*  • *'Using all of the counters, how many different three-digit numbers can you make?'* • *'Have you made all the possible numbers? How do you know?'*
1:8	Now that children have a good understanding of the place-value composition of three-digit numbers, progress to application of this to addition problems (e.g. *500 + 80 + 2*). Represent a three-digit number with a part–part–part–whole diagram, as shown opposite, and ask children to	Writing addition equations: 

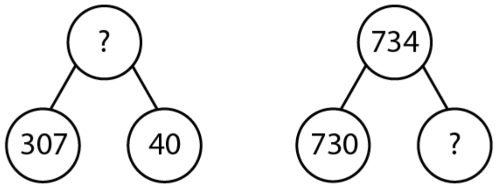
	write and describe an addition equation to go with the representations. Then ask children if they can write *another* addition equation to go with the same representations, continuing until they have written the addends in all possible orders (note that in the examples opposite, the position of the equals sign has not been varied, so as to draw attention to the changing order of addends). Children should be able to confidently add whole hundreds, tens and ones together irrespective of the order in which the addends are shown; provide some missing number problems as practice. In this and the following steps, include examples with a zero in either the tens or ones place.	• *'Five hundred and eighty-two can be composed of five hundreds and eight tens and two ones.'* • *'Five hundred and eighty-two is equal to five hundred plus eighty plus two.'* $582 = 500 + 80 + 2$ • *'We can also write this as…'* $582 = 500 + 2 + 80$ $582 = 2 + 500 + 80$ $582 = 2 + 80 + 500$ $582 = 80 + 2 + 500$ $582 = 80 + 500 + 2$ Missing number problems: *'Fill in the missing numbers.'* $70 + 3 + 100 = \square$ $\square = 900 + 40$ $300 + 1 + 70 = \square$ $\square = 400 + 9$ $30 + 100 + 7 = \square$
1:9	You should then be able, fairly quickly, to extend children's understanding to addition problems with two of the parts already 'combined' (e.g. *500 + 82*). Present these using part–part–whole diagrams and see if children can make the connection to their previous knowledge. If children don't make the connection automatically, encourage them to further partition the combined part so that they can see the hundreds, tens and ones are each still 'there' in the calculation. Then provide children with practice, as shown opposite.	Part–part–whole diagrams and equations:  $500 + 82 = 582$ $502 + 80 = 582$ $580 + 2 = 582$ Missing-number problems: *'Fill in the missing numbers.'* $400 + 20 + 3 = \square$ $400 + 23 = \square$ $420 + 3 = \square$ $403 + 20 = \square$

		• *'Six hundred and seventy-four is made of six hundreds, seven tens and four ones.* • *'Six hundred and seventy-four is also made of sixty-seven tens and four ones.* • *'Six hundred and seventy-four is made of six hundreds and seventy-four ones.'* *'Find different ways of expressing these numbers.'* • 630 • 704 • 867
1:10	In preparation for subtraction problems, progress to missing-part/addend problems, including those with: • the hundreds, tens or ones part missing (e.g. *724 = 700 + 20 + ?*) • a 'combined' part missing (e.g. *724 = 700 + ?).* Working with one three-digit number as the whole, for each calculation type, show the part–part–part–whole diagram alongside the corresponding missing-addend equation, and ask children to identify the missing part/addend. Note that the part–part–part–whole can be related to both the three-addend and the two-addend equations, as shown opposite. Again, provide children with related sets of missing-number problems as practice. Continue to include examples with a zero in either the tens or ones place.	Part–part–part–whole diagram and equations:  724 = 700 + 20 + □ 724 = 720 + □ Missing-number problems: *'Fill in the missing numbers.'* 40 + □ + 7 = 647 600 + □ = 647 600 + □ + 7 = 647 □ + 607 = 647 209 = 200 + □ 290 = 200 + □ 306 = 6 + □ 360 = 60 + □
1:11	Now move on to the use of subtraction equations, including subtraction of: • the hundreds, tens or ones (e.g. *724 – 20 = 704*) • two parts (e.g. *724 – 20 – 4*) • a 'combined' part (e.g. *724 – 24 = 700*) Represent a number on a part–part–part–whole diagram, and demonstrate writing an equation for subtraction of	Part–part–part–whole diagram and equations:  724 – 700 = 24 724 – 20 = 704 724 – 4 = 720

	one part (e.g. the hundreds); you can cover over the subtracted part to draw attention to the parts that remain. Then ask children to write equations representing subtraction of the tens part, and then the ones part. Use the same number you examined in the previous step to help children make links to the missing-addend equations and, for now, show the parts in place-value order to mirror how the numbers are written. Repeat for subtraction of two parts and of 'combined' parts. Once children are confident with calculations like these, provide sequences of related missing-number problems for practice, initially scaffolded by part–part–(part–)whole diagrams (including varying the order of the parts). Again, include examples with a zero in either the tens or ones place.	724 – 20 – 4 = 700 724 – 24 = 700 724 – 700 – 4 = 20 724 – 704 = 20 724 – 700 – 20 = 4 724 – 720 = 4 Missing-number problems: *'Fill in the missing numbers.'*  638 – 600 = □ □ = 638 – 30 – 8 638 – 30 = □ □ = 638 – 30 – 600 638 – 8 = □ □ = 638 – 608 240 – 40 = □ 204 – 4 = □ 240 – 200 = □ 204 – 200 = □
1:12	Present varied practice for addition and subtraction calculations that are based on the place-value composition of three-digit numbers, including: • part–part–(part–)whole models (bar-model or cherry diagram) with a missing 'part' or 'whole' (varying the order of the hundreds, tens and ones) • missing-number problems (varying the order of the hundreds, tens and ones, and the position of the equals symbol) • balancing equations (missing numbers or symbols) • real-life problems, including measures contexts, for example: • *'There are 200 books on the shelves and 25 books out on the tables. How*	Missing-number problems: *'Fill in the missing numbers.'*   600 + 70 + 1 = □ 3 + 500 + 40 = □ 461 = □ + 60 + 1 20 + □ + 3 = 823 953 – 50 – 3 = □ 846 – □ – 40 = 800

many books are there altogether?'
(aggregation)

- *'There are 365 days in a year. If it rains on 65 days of the year, on how many days does it not rain?'*
(partitioning)
- *'A bamboo plant was 4 m tall. Then it grew 83 cm. How tall is the bamboo plant now?'*
(augmentation)
- *'Francesco had 265 marbles. He gave 60 marbles to his friend. How many marbles does Francesco have now?'*
(reduction)
- *'The apple tree outside Cecily's house is 308 cm tall. How much further would it have to grow to reach the bottom of Cecily's bedroom window, at 3 m 68 cm?'*
(difference)



☐ = 203 + 90 ☐ = 290 + 3

628 = 20 + ☐ 628 = 8 + ☐

☐ + 40 = 240 904 = 4 + ☐

☐ = 932 – 30 ☐ = 932 – 2

700 + 30 + 9 = 700 + ☐

300 + 90 + 5 = 305 + ☐

7 + 950 = 900 + ☐ + 7

Balancing equations:

'Use symbols (< > =) to complete the equations.'

100 + 60 + 5 ◯ 105 + 60

300 + 40 + 2 ◯ 300 + 24

783 – 80 ◯ 783 – 3

839 – 9 – 30 ◯ 839 – 39

Dòng nǎo jīn:

- *'Fill in the missing digits.'*

4 __ 3 = __ 00 + 6 __

- *'Jess writes down the number '518'. She says that if she subtracts the ones she will have '508'. Is she correct? Explain why/why not.'*

Teaching point 2:

Each number on the 0 to 1,000 number line has a unique position.

Steps in learning

2:1

This teaching point looks at the ordinality of three-digit numbers:

- three-digit multiples of ten on a 0 to 1,000 number line (steps *2:1–2:5*); ensure that numbers with five tens have emphasised tick marks
- three-digit numbers with non-zero ones-digits on a section of the number line (steps *2:6–2:8*); ensure that numbers with five ones have emphasised tick marks
- three-digit numbers, in general, on the 0 to 1,000 number line (ensure that numbers with both five ones and five tens have emphasised tick marks), and in measures contexts (steps *2:7–2:8*)

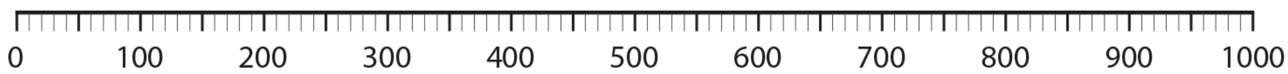
Begin by introducing children to the 0 to 1,000 number line with multiples of 100 *labelled* and multiples of ten *marked*, as shown below. Spend some time discussing the number line, including the value of the gap between each tick mark (ten) and comparison with the 0–10 and 0–100 number lines. Then practise counting, with you and/or the children pointing to the numbers as you go; count:

- forwards and backwards in hundreds
- forwards and backwards in tens, counting in two ways:
  - *'…three hundred, three hundred and ten, three hundred and twenty…'*
  - *'…thirty tens, thirty-one tens, thirty-two tens…'*

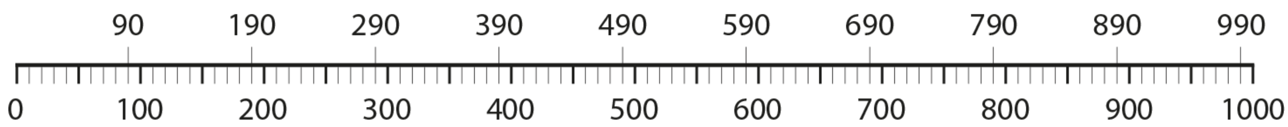
You may initially want to label the numbers with nine tens to support backwards counting over the hundreds boundaries (as shown in the second number line below).

In both cases, include starting with numbers other than zero (for forwards counting) and other than 1,000 (for backward counting).

Number line with multiples of hundred labelled and multiples of ten marked:



Labelling numbers with nine tens:



2:2 During counting, stop frequently and ask children to identify the next and previous number in the sequence. Then present children with practice in the form of:

- missing-number sequences to complete (both multiples of 100 and multiples of ten)
- identifying 100 more/less than a given multiple of 100
- identifying 10 more/less than a given multiple of ten.

Pay particular attention to moving across the hundreds boundaries in tens.

Missing-number problems:

'Fill in the missing numbers.'

900		700	600		400		200

370		390		410	420		440

	100 less		100 more			ten less		ten more	
☐	←	800	→	☐	☐	←	390	→	☐

	100 less		100 more			ten less		ten more	
☐	←	100	→	☐	☐	←	800	→	☐

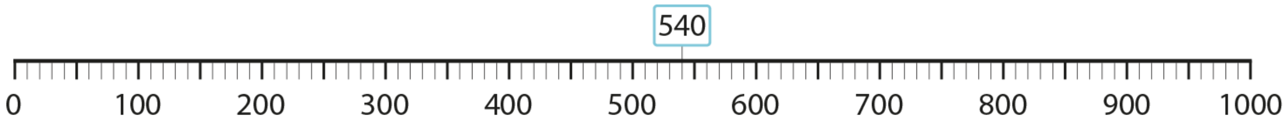
2:3 To work towards children identifying or placing numbers on the 0 to 1,000 number line, first work on identifying the two multiples of 100 that a given three-digit multiple of ten sits between. Initially, ask children questions such as *'Where are the four hundreds?'*, and have them point to or shade the appropriate section of the number line. Then, for a given multiple of ten labelled on the number line, ask children to point to or circle the previous and next multiple of 100. Be careful with your language, ensuring that you make a clear distinction between 100 less/more in the previous step, and next/previous multiple of 100 here. Use the following stem sentences:

- ***'___ is between ___ and ___.'***
- ***'___ is the previous multiple of one hundred.'***
- ***'___ is the next multiple of one hundred.'***

After practising identifying the next/previous multiple of 100 for several examples, with the scaffold of the number line, work through some examples without the number line.

Finally, returning to three-digit multiples of ten labelled on the number line, begin to ask children to also identify which is the *closest* multiple of 100. This serves as further prepartaion for identifying and placing numbers on the number line, as well as paving the way for rounding in segment *1.22 Composition and calculation: 1,000 and four-digit numbers*. For now, children do not need to identify the closest multiple of 100 without the scaffold of the number line. For numbers with five tens, children should be able to identify them as halfway between the two multiples of 100.

Identifying previous and next multiple of 100 on the number line:

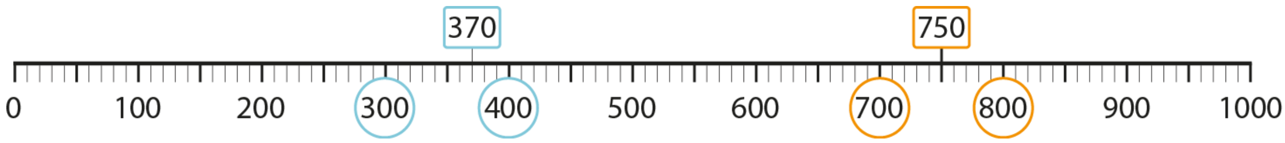


- *'Five hundred and forty is between five hundred and six hundred.'*
- *'Five hundred is the previous multiple of one hundred.'*
- *'Six hundred is the next multiple of one hundred.'*

Identifying previous and next multiple of 100 without the number line:

previous multiple of 100				next multiple of 100
☐	←	630	→	☐

Identifying the closest multiple of 100 on the number line:



- *'Three hundred and seventy is between three hundred and four hundred.'*
- *'Three hundred is the previous multiple of one hundred.'*
- *'Four hundred is the next multiple of one hundred.'*
- *'Four hundred is the* ***closest*** *multiple of one hundred.'*

- *'Seven hundred and fifty is* ***halfway*** *between seven hundred and eight hundred.'*
- *'Seven hundred is the previous multiple of one hundred.'*
- *'Eight hundred is the next multiple of one hundred.'*

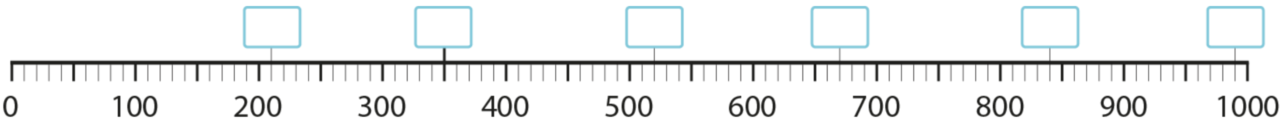
2:4 Now progress to identifying three-digit mutiples of ten indicated on the number line. The aim is for children to be able to say the number name or write the digits corresponding to an indiciated position on the number line. You can scaffold this as follows:

- Show a number line with a given number 'pointed at' but not identified; choose a number with the tens digit below five (e.g. 230). Initially show the children how to look at the previous multiple of hundred (200), modelling language such as *'I know this is going to be two hundred and "something".'* Then count up from the previous multiple of hundred, in tens, or apply knowledge of the zero to ten number line or subitise, to identify the number indicated.
- Similarly, indicate a number with a tens digit above five (e.g. 280), and demonstrate identifying and working back from the next multiple of hundred (300).
- Return to noting that the emphasised tick mark halfway between two multiples of 100 always has a tens value of five, and demonstrate how this knowledge can be used to identify numbers such as 350 without counting forward or back from a multiple of 100.

- Then look at numbers that are one ten more or less than a number with a tens value of five (e.g. 340 and 360), demonstrating that it is more efficient to work from the 'halfway' point than from a multiple of 100. Similarly you can look at numbers that are two tens more/less than the halfway point (e.g. 330 and 370).

Practise these strategies until children can confidently provide spoken number names (e.g. *'three hundred and forty'*) or written numbers (e.g. 340). Also have children practise labelling marked points on the number line. To promote and assess depth of understanding, present dòng nǎo jīn problems such as those shown below.

'Identify the missing numbers.'



Dòng nǎo jīn:

'Three friends described the number '680' in different ways:'

- Annie says it is 600 plus 80.
- Bashir says it is 68 tens.
- Carys says it is 20 less then 700.

'Find different ways to describe these three-digit numbers:'

560 910 390

2:5 Now move on to positioning a given number on the number line, drawing on learning from the previous step. For example, ask children to indicate the correct position when you:

- say a number name (e.g. '*six hundred and thirty*')
- write a number as digits (e.g. 630)
- describe a number in terms of multiples of ten (e.g. '*sixty-three tens*')
- write a number in terms of multiples of ten (e.g. 63 tens).

Discuss the strategies that children use to place the numbers, giving value to all strategies that show good reasoning (analaogus to those in the previous step), but also discuss which strategies are more efficient in particular cases.

Provide children with independent written practice positioning given numbers on the number line, for example:

- *'Mark these numbers on the number line:'*
  140, 290, 320, 430, 720, 860, 940

2:6 Now focus in on different sections of the number line, with the multiples of ten *labelled* and the ones *marked*, as shown in the examples below. Follow a similar progression to that described in steps *2:1–2:5*, with children:

- counting forwards and backwards in ones; with their prior experience in counting, children often find it easier to cross the boundaries in ones than in tens (step *2:1*), although it is worth paying attention to going backwards across the hundreds boundaries
- completing missing-number sequences, and identifying one more/less for given numbers

- identifying the next and previous multiple of ten, as well as the closest multiple of ten, for given numbers
- identifying three-digit numbers (with non-zero ones digits) indicated on the number line; children should explain their reasoning (for example, *'I know this is five hundred and eighteen because it is two less than five hundred and twenty'*, *'…because it is eight more than five hundred and ten'* or *'…because it is three more than five hundred and fifteen.'*)
- positioning a given number on the number line.

480–530 number line:

Missing-number problems:

'Fill in the missing numbers.'

	480		482	483		485	

502		500		498	497		595

one less   one more

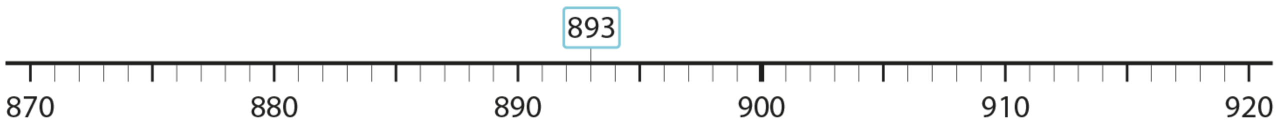
☐ ← 490 → ☐

one less   one more

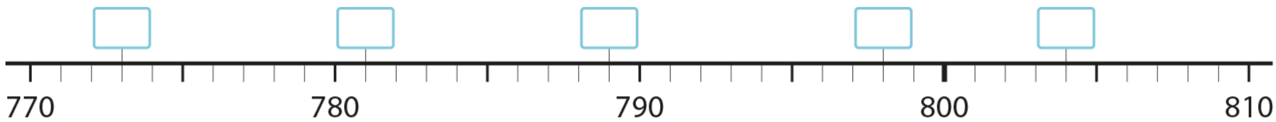
☐ ← 500 → ☐

Identifying next and previous multiple of ten, and closest multiple of ten:

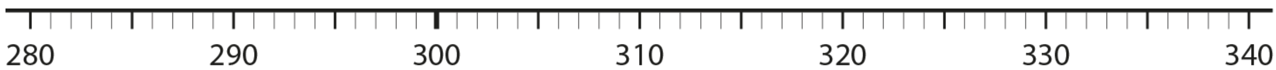
- *'Circle the closest multiple of ten.'*



- *'Identify the missing numbers.'*



- *'Place these numbers on the number line below: 285, 292, 299, 304, 314, 339.'*



2:7

Although logistically challenging, it is important to look at general three-digit numbers (i.e. those that are not multiples of ten) in the context of the *entire* 0 to 1,000 number line. To do this, you can draw out a 10 m (1,000 cm) number line in the playground with the hundreds labelled and the tens marked. A 1 m ruler marked with millimetre tick-marks could also be used (1,000 mm = 1 m), but ensure that the intervals are labelled in millimetres (often called a 'millimetre ruler') rather than in centimetres.

For given three-digit numbers, repeat work on identifying the next and previous multiple of ten, as well as the closest multiple of ten, but also extend to include identifying the next, previous and closest multiple of 100. Now that you are working in the context of the full number line, make sure you include examples where the next/previous/closest multiple of ten is also a multiple of 100.

Then look at identifying three-digit numbers (with non-zero ones digits) indicated on the number line, and placing such numbers on the number line. In both cases children should continue to explain their reasoning, for example:

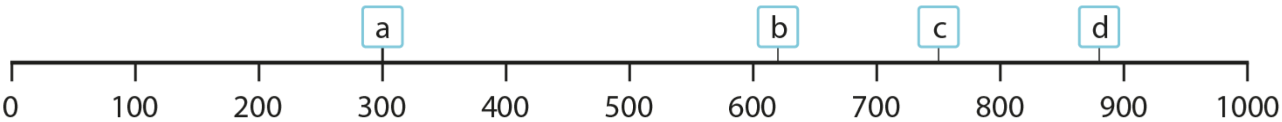
- *'To place eight hundred and twenty-one, I find eight hundred…plus two tens… eight hundred and twenty-one is just after that.'*
- *'To place four hundred and ninety-eight, I find five hundred (half the full length)… four hundred and ninety-eight will be just before that.'*

To promote and assess depth of understanding use dòng nǎo jīn problems such as those shown here and below:

- *'Sasha is thinking of a number. She says: "The number is between five hundred and six hundred. The closest multiple of ten is also a multiple of hundred." What could her number be? Are there any other possibilities?'*

Dòng nǎo jīn:

- *'Which of these numbers has a hundreds digit that is larger than the tens digit?'*
- *'Which of these numbers has a zero in the tens place?'*
- *'Which of these numbers could have all the digits the same?'*
- *'Which of these numbers has a five in the tens place?'*



2:8

Finally, move on to working with a blank 0 to 1,000 number line (no intervals labelled or marked), challenging children to identify/place numbers without any marked intervals as a scaffold.

Start with identification/placement of multiples of 100. While it useful to teach children to mark the halfway point (500) to use as a reference point, discourage them from trying to mark on all of the multiples of 100 and from counting up in hundreds from zero. Aim, instead, to encourage *proportional* thinking, for example, positioning 600 relative to *both* zero and 1,000 (or to 500 and 1,000).

Then move on to asking children to approximately identify/place three-digit multiples of ten (e.g. 430) and three-digit numbers with non-zero ones digits (e.g. 605, 625). Encourage children to use appropriate benchmarks, explaining their chain of reasoning, for example:

- *'Five hundred must be here…'* (indicating the midpoint) *'…so four hundred and eighty is about here.'* (indicating a position 'a bit before' 500).
- *'Seven hundred and fifty would be about here…'* (halfway between 700 and 800) *'…so seven hundred and sixty-two would be just after that.'*

You can use estimation of measures as a more hands-on approach; this will also serve to give children further experience working with various measures, for example:

- draw out lines of various lengths in the playground, each alongside a pre-drawn 1,000-centimetre/ten-metre reference line, as shown below; ask children to estimate the length of the lines, before measuring each as a class to check
- pour some water into a one-litre container (with no intervals marked, or at least not visible to the children), and ask children to estimate the volume in mililitres; similarly tell the children you want to pour in, for example, 230 ml, and ask children to shout 'stop!' when they estimate that point has been reached.

When reviewing the lengths/volumes, make sure children in no way see the exact value as the 'only right answer'; reasonable estimates should be celebrated for showing good proportional understanding. In each case, note that children's ability to convert between, say 100 cm and 1 m, or 1 l and 1,000 ml is implied; you may need to remind children of the conversions or add the equivalent measure to the diagram (e.g. 1,000 g alongside 1 kg).

Also present dòng nǎo jīn problems such as those shown below to further deepen children's proportional-thinking skills.

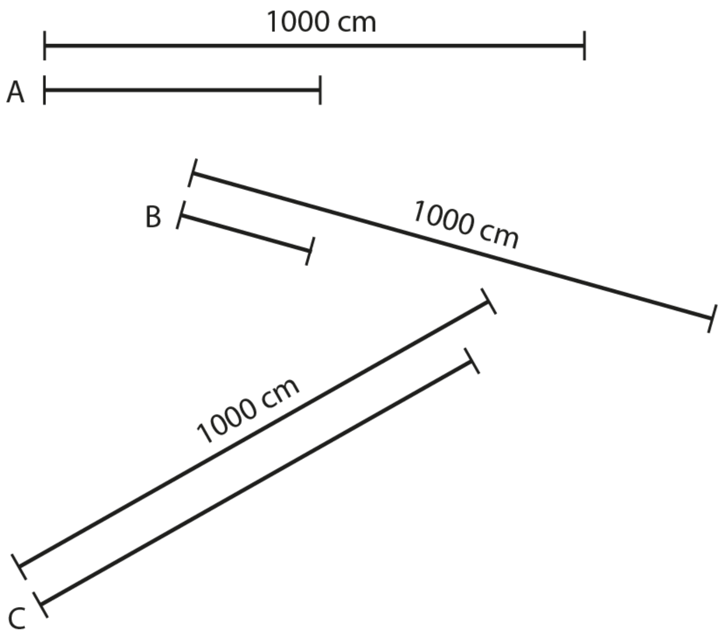
Placing numbers on a 'blank' 0 to 1,000 number line:

'Place these numbers on the number line:'

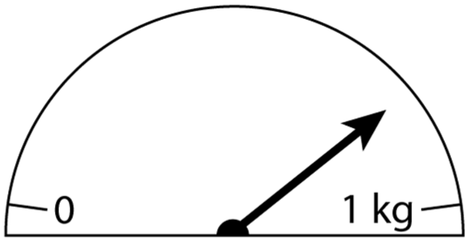
600, 200, 480, 840, 762, 195

Estimating measures:

- *'Look at lines A, B and C. Can you guess how long they are by comparing them to the 1,000 cm lines?'*

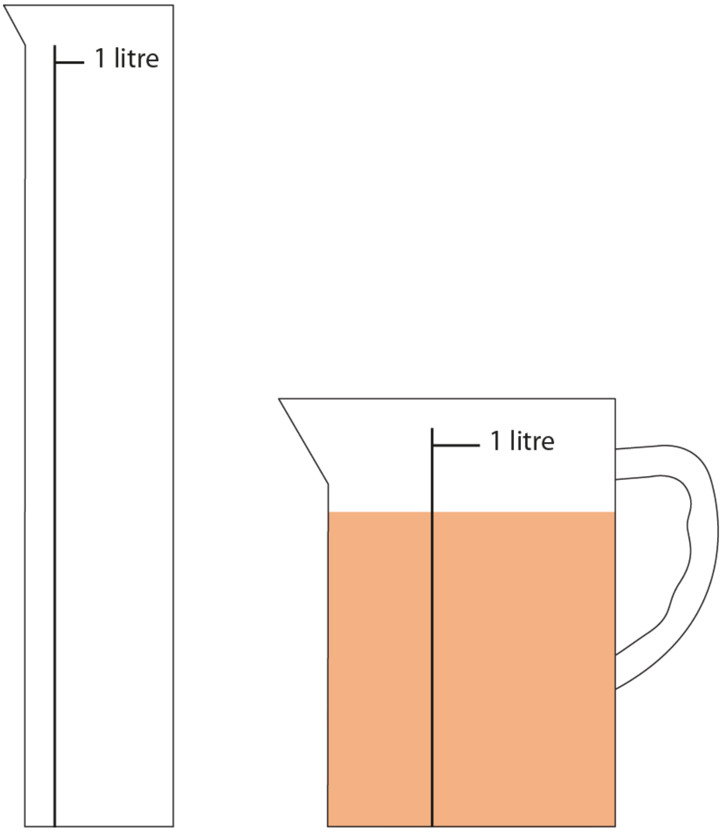


- *'What mass does this show?'*

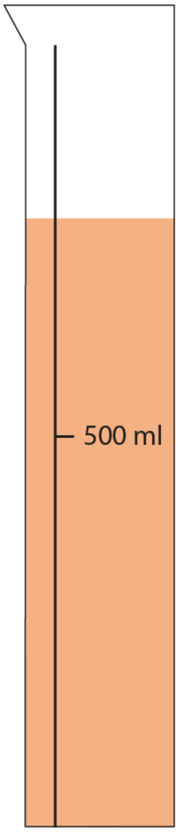


Dòng nǎo jīn:

- *'Here is a 1 litre measuring cylinder and a 1 litre jug. The jug has some juice in it. Shade the same volume of juice in the measuring cylinder.'*



- *'Estimate how much liquid is in this measuring cylinder.'*



Working without a visual scaffold:

'Join each number to the set that it belongs to.'

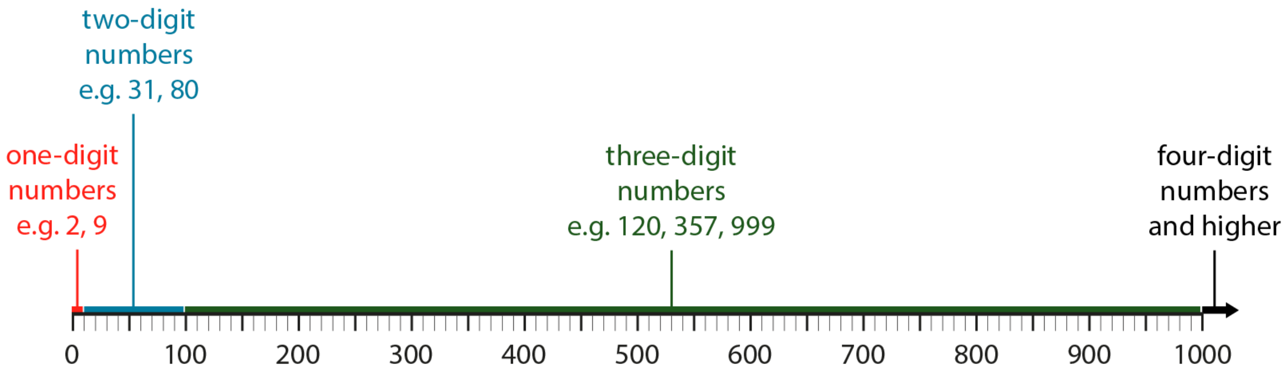
449	1 to 100
178	101 to 200
999	201 to 300
99	301 to 400
327	401 to 500
	greater than 500

Teaching point 3:

The smallest three-digit number is 100, and the largest three-digit number is 999; the relative size of two three-digit numbers can be determined by examining the hundreds digits, then the tens digits and then the ones digits, as necessary.

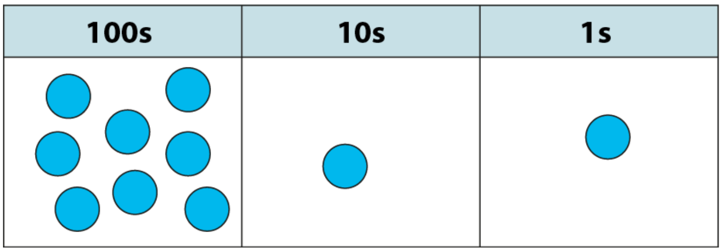
Steps in learning

3:1 Now move on to comparison of three-digit numbers. Start by looking at the 0 to 1,000 number line and identifying where the one-, two- and three-digit numbers lie. Ask children if they can then identify the smallest and largest three-digit numbers (100 and 999 respectively) and locate these on the 0 to 1,000 number line. Using the number line, draw attention to the fact that three-digit numbers are always larger than one- and two-digit numbers: the smallest three-digit number (100) is still greater than the largest two digit number (99).



Dòng nǎo jīn:

'Carla says "This represents the largest three-digit number you can make using all ten counters." Is she correct? Explain your reasoning.'



3:2 In segment *1.9 Composition of numbers: 20–100*, children learnt to compare pairs of two-digit numbers by first examining the tens digits and then, if the tens digits are the same, going on to examine the ones digits. Now extend this idea to the comparison of three-digit numbers, using the following progression supported by the number line:

- Begin by comparing two numbers with different hundreds digits (e.g. 347 and 526); then demonstrate that we can change the tens or ones digits without affecting the comparison (slide the numbers along on the number line).
- Then compare two numbers that have the same hundreds digits but different tens digits (e.g. 347 and 362) demonstrating that we can no longer determine the relative size of the numbers by looking only at the hundreds digits – you can initially hide the tens and ones digits so children are unable to say which number is greater, then reveal the tens digits to

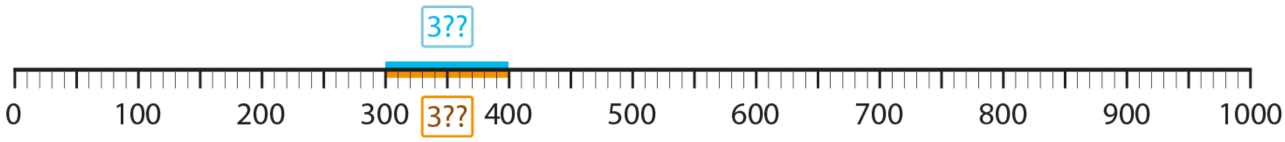
enable comparison. Demonstrate that we can change the ones digit without affecting the comparison.

- Finally, compare two numbers that have the same hundreds and tens digits (e.g. 347 and 349), demonstrating that we now need to look at the ones digits; you can initially hide the ones digits, before revealing them to enable comparison.

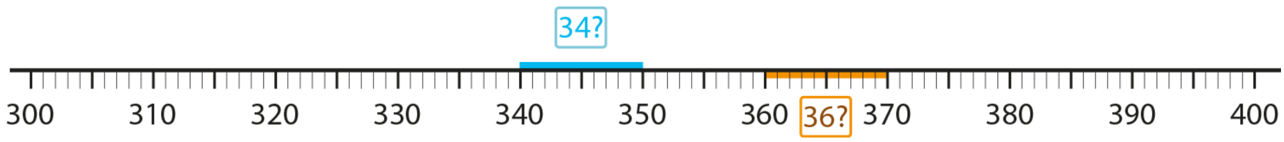
For more detail on this progression, see segment *1.9, Teaching point 4*.

Numbers with equal hundreds digits:

- *'Which number is greater?'*



- *'The hundreds digits are the same, so we can't tell which number is greater by looking at the hundreds digit.'*



- *'We need to look at the tens digits. The tens digit are different.'*

3:3	Once children can confidently compare three-digit numbers supported by the number line, move on to comparing numbers looking only at the digits. Complete missing symbol problems using the > and < symbols. After making each comparison, confirm the answer by looking at the number line. Work towards the generalised statement: • ***'To compare three-digit numbers, we need to compare the hundreds digits; if the hundreds digits are the same, we need to compare the tens digits; if both the hundreds and the tens digits are the same, we need to compare the ones digits.'*** Provide children with varied practice. Initially they can use the number line as a scaffold, but should progress to working only with the numbers written as numerals. Include comparison of three-digit numbers with two-digit	'Fill in the missing symbols.' (< > or =) 542 ○ 342 624 ○ 644 442 ○ 342 634 ○ 644 342 ○ 342 644 ○ 644 242 ○ 342 654 ○ 644 321 ○ 322 322 ○ 322 323 ○ 322 324 ○ 322

	numbers. Also present dòng nǎo jīn problems such as those shown opposite.	109 ◯ 99 573 ◯ 753 223 ◯ 43 428 ◯ 482 602 ◯ 62 361 ◯ 316 410 ◯ 41 423 ◯ 324 Dòng nǎo jīn: • *'What is the biggest multiple of ten you can use to correctly complete each inequality?'* 200 + ☐ + 5 < 200 + 40 + 24 200 + ☐ + 5 < 200 + 40 + 25 200 + ☐ + 5 < 200 + 40 + 26 • *'How many different three-digit numbers are there? How do you know?'*
3:4	Finally, move on from comparing pairs of numbers to ordering sets of numbers, providing practice as shown opposite.	*'For each set, put the numbers in order from smallest to largest.'* 352 746 549 810 401 515 115 55 155 511 708 680 68 678 80 867 Dòng nǎo jīn: *'Fill in the missing digits so that the numbers are in order from smallest to largest.'* ___ 4 6 | ___ 3 2 | 3 ___ 1 | ___ 6 6 | ___ 35

Teaching point 4:

Three-digit multiples of ten can be expressed multiplicatively and additively, in terms of tens or hundreds.

Steps in learning

	Guidance	Representations
4:1	By now children have a good understanding of unitising and addition facts. It should be a very simple task for children to apply this to the addition and subtraction of whole multiples of 100, so briefly cover this now. You can use part–part–whole models, as shown opposite, to link number facts within ten to calculations with hundreds. You may also wish to use place-value counters or Dienes hundred squares as concrete representations. Note that, at this stage, children only need to add/subtract multiples of 100 within 1,000; bridging 1,000 is covered in segment *1.22 Composition and calculation: 1,000 and four-digit numbers*. Provide children with some missing-number problems, as shown opposite.	Part–part–whole diagrams: (see below)

Part–part–whole diagrams:

7	
4	3

7 hundreds	
4 hundreds	3 hundreds

700	
400	300

Missing-number problems:

'Fill in the missing numbers.'

2 + 4 = 6   6 + 3 = ☐

200 + 400 = ☐   600 + 300 = ☐

7 – 5 = 2   8 – 4 = ☐

700 – 500 = ☐   800 – 400 = ☐

☐ = 400 + 500   ☐ = 900 – 700

700 = ☐ + 400   400 = 600 – ☐

4:2

In segment *1.17 Composition and calculation: 100 and bridging 100*, children learnt about the *multiplicative* composition of multiples of ten up to 190 (e.g. 160 = 16 tens). Now extend this to larger three-digit multiples of ten.

Return to the use of a concrete or pictorial representation that clearly shows the ten tens within each 100, such as tens frames with ten-value place-value counters or Dienes hundred squares. Begin by representing 100 (one full tens frame or one Dienes hundred square), and then add additional whole hundreds, one at a time, to look at the composition of all of the multiples of 100 up to 1,000. To draw attention to the emerging pattern, write equations to show the relationship between the multiple of 100 and the number of tens:

- *without* the multiplication symbol (e.g. 300 = 30 tens)
- *with* the multiplication symbol (e.g. *300 = 30 × 10*)

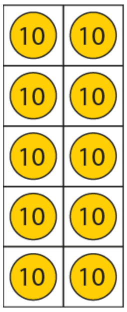
Then, represent a three-digit multiple of ten (e.g. 320), now partially filling the final tens frame, or adding tens Dienes, depending on your chosen representation. Write expressions as before, and use the following stem sentences to describe the resulting number:

- ***'This is ___ hundred and ___.'***
- ***'This is ___ tens.'***

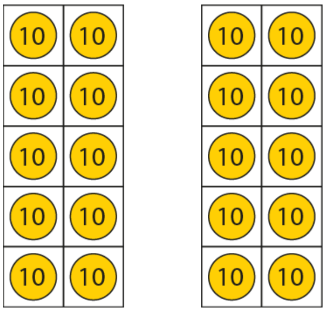
Present a variety of three-digit multiples of ten in this way, asking children to identify the numbers made. Ask questions such as:

- *'How many tens are there in ___?'*
- *'What three-digit number is composed of ___ tens?'*

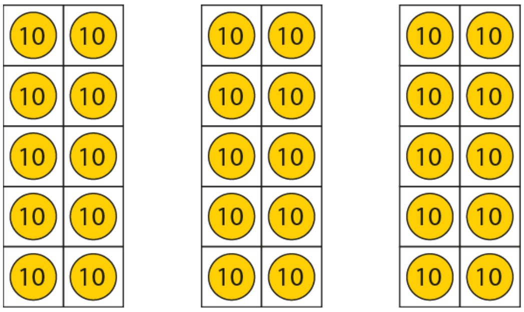
Multiples of 100:



100 = 10 tens 100 = 10 × 10



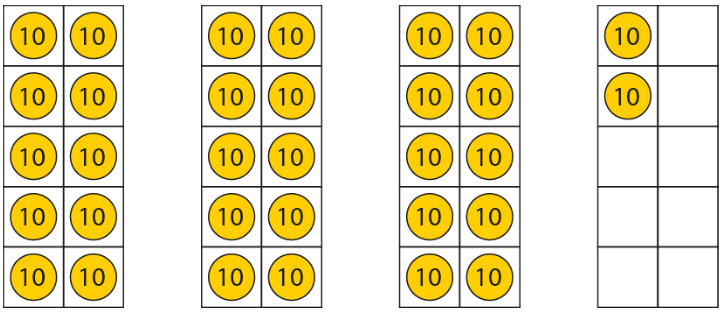
200 = 20 tens 200 = 20 × 10



300 = 30 tens 300 = ☐ × 10

etc.

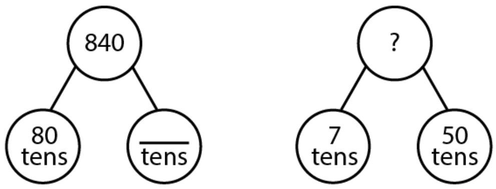
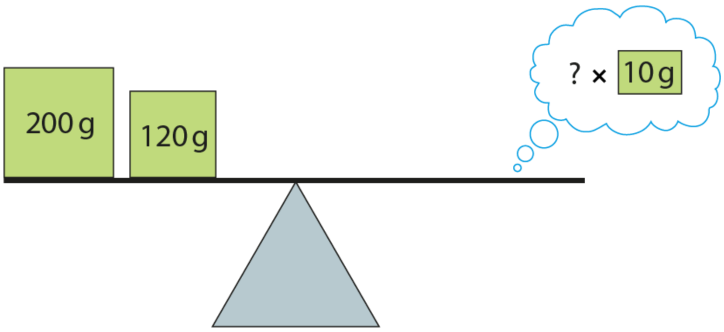
Three-digit multiple of ten:



320 = 32 tens 320 = 32 × 10

- *'This is three hundred and twenty.'*
- *'This is thirty-two tens.'*

	This step further highlights the need to frame questions carefully; earlier in the segment we emphasised the importance of not asking, for example, '*How many tens are there in four hundred and ten?*' when trying to elicit the number of *additional* tens (four hundred and *one additional ten*). Here the question is used correctly, to elicit the full number of tens from which the three-digit number is composed.	

4:3	To complete this short teaching point, present children with practice applying the learning from step *4:2*, including:
	• missing-number problems (equations and part–part–whole diagrams)
	• real-life problems, including measures contexts, such as those shown opposite and below:
	  ◦ *'Jed has 3.5 m of ribbon. How many 10 cm strips can he cut from that?'*
	  ◦ *'A shop has fifty-one £10 notes in the till at the end of the day. How much money is this?'*
	Also present dòng nǎo jīn problems that require children to link together their understanding of additive and multiplicative composition (as opposite), and link to more complex measures conversions, for example:
	• *'Emma is training for a marathon; it takes her 10 minutes to run 1 mile. 'Emma runs for 2.5 hours; how many miles does she run in this time? 'Show that it will take Emma 4 hours and 20 minutes to run the 26 miles of the marathon.'*
		Missing- number problems.
		'Fill in the missing numbers.'
		50 tens = □ 400 = □ tens
		27 tens = □ 360 = □ tens
		□ × 10 = 200 89 × 10 = □
		
		Dòng nǎo jīn:
		'How many ten-gram weights are needed to balance the scales?'
		
		300 + 60 = □ × 10 □ × 10 = 40 + 700
		29 × 10 = □ + 90 41 × 10 = 10 + □

Teaching point 5:

Known facts and strategies for addition and subtraction within and across ten, and within and across 100, can be used to support additive calculation within 1,000.

Steps in learning

5:1

Guidance

By this stage, children should be able to confidently add hundreds, tens and ones parts to form three-digit sums, and partition three-digit numbers into these constituent parts. In this teaching point different calculation types are considered; rather than attempting to address all possibilities, key calculation groups which deepen children's understanding of three-digit numbers are covered. These include:

- adding and subtracting hundreds, tens, or ones to/from a three-digit number, without bridging a multiple of 100 for the latter two, e.g.:
  - *576 + 200*
  - *586 – 10*
  - *576 + 3*

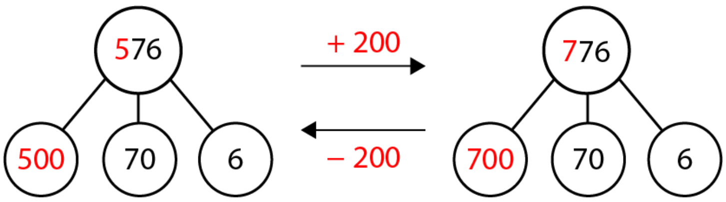
  (steps *5:1* and *5:3*)
- partitioning three-digit numbers in different ways, e.g.:
  - *576 = 400 + 176*
  - *580 = 450 + 120 + 10*
  - *576 = 306 + 230 + 40*

  (steps *5:2* and *5:4*)
- calculating to/from multiples of 100 in tens or ones, e.g.:
  - *500 – 10 and 500 – 30*
  - *499 + 1 and 497 + 3*

  (steps *5:5–5:7*)
- bridging a multiple of one hundred with addition/subtraction of tens or ones, e.g.:
  - *470 + 50 and 520 – 50*
  - *498 + 30 and 528 – 30*
  - *498 + 7 and 505 – 7*

Representations

Addition and subtraction of 100s:



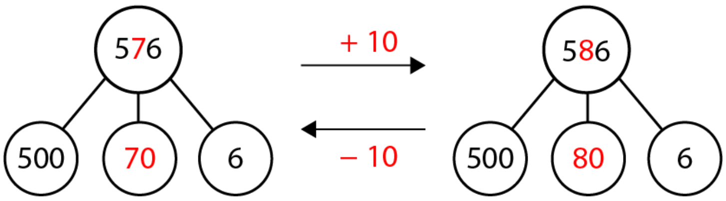
576 + **2**00 = **7**76

776 – **2**00 = **5**76

Example description – addition:

- *'We had* ***five*** *hundreds, then we added* ***two*** *hundreds, so now we have* ***seven*** *hundreds.'*
- ***'Five*** *hundred and seventy-six plus* ***two*** *hundred is equal to* ***seven*** *hundred and seventy-six.'*

Addition and subtraction of tens:

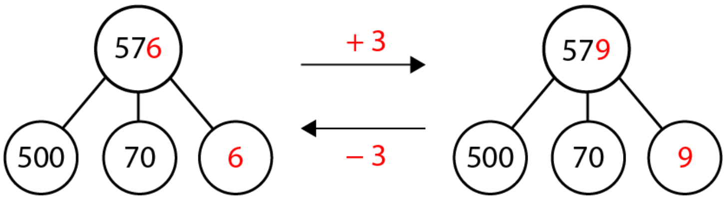
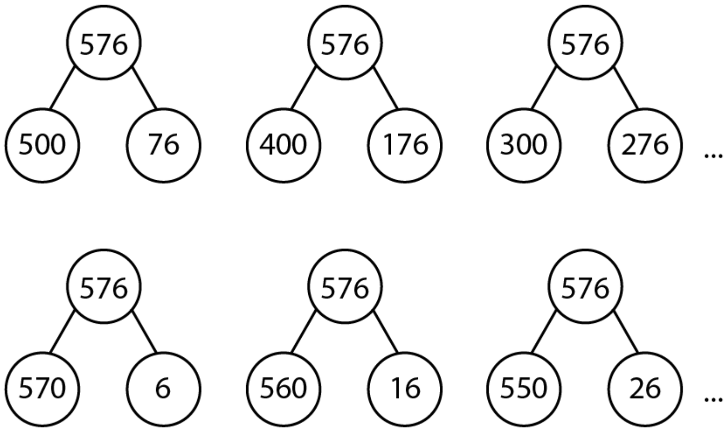


5**7**6 + **1**0 = 5**8**6

5**8**6 – **1**0 = 5**7**6

Example description – subtraction:

- *'We had* ***eight*** *tens, then we subtracted* ***one*** *ten, so now we have* ***seven*** *tens.'*
- *'Five hundred and* ***eighty****-six minus* ***ten*** *is equal to five hundred and* ***seventy****-six.'*

	(steps *5:8–5:13*) Throughout, ensure that children are using known facts and strategies, rather than 'counting on/back'. After each calculation type, intelligent practice is suggested; examples of contextual practice are given at the end of this teaching point. Begin with the addition of ones, tens or hundreds parts to a three-digit number, avoiding addition of tens or ones that involve bridging a multiple of 100. Initially use a part–part–part–whole model as a scaffold, but use the language shown opposite to support children so that they can then work with the equations only. Use the same quantities and part-part-part–whole diagrams to explore similar subtraction calculations, to help support the link between addition and subtraction. You can include examples for which addition of the ones involves bridging a tens boundary (e.g. *576 + 7 = 583*).	Addition and subtraction of ones:  57**6** + **3** = 57**9** 57**9** – **3** = 57**6** Example description – addition: • *'We had **six** ones, then we added **three** ones, so now we have **nine** ones.'* • *'Five hundred and seventy-**six** plus **three** is equal to five hundred and seventy-**nine**.'*
5:2	It is also important for children to be able to partition numbers flexibly. This supports a deeper understanding of number and underpins some later mental strategies. Spend some time exploring different ways of partitioning a given three-digit number, based on children's knowledge of adding/subtracting from the hundreds or tens, as shown opposite. If extra support is needed, you can initially work with Dienes on the part–part–part–whole diagrams, but this scaffold should then be removed.	

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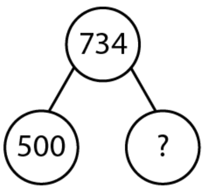
<td>**5:3**</td>
<td>Provide children with practice completing missing-number problems, including both sequences that highlight the patterns, and standalone calculations.</td>
<td>
Missing-number problems:
'Fill in the missing numbers.'

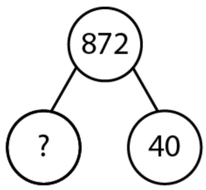
125 + 100 = □   125 + 10 = □   125 + 1 = □
125 + 200 = □   125 + 20 = □   125 + 3 = □
125 + 300 = □   125 + 30 = □   125 + 5 = □
125 + 400 = □   125 + 40 = □   125 + 7 = □

543 – 100 = □   183 – 10 = □   273 – 1 = □
543 – 200 = □   183 – 20 = □   273 – 2 = □
543 – 300 = □   183 – 40 = □   273 – 3 = □
543 – 400 = □   183 – 60 = □   273 – 4 = □

623 + 30 = □   287 + 400 = □   593 + 4 = □
831 – 500 = □   766 – 5 = □   258 – 30 = □

630 = 600 + 30   457 = 400 + 57
630 = 500 + □   457 = 410 + □
630 = 400 + □   457 = 420 + □
630 = □ + 330   457 = □ + 27
630 = □ + 430   457 = □ + 17
630 = □ + 530   457 = □ + 7





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<td>
Dòng nǎo jīn:

- *'What is the largest multiple of ten which makes this inequality true?'*

582 – □ > 500

- *What is the smallest multiple of ten which makes this inequality true?*

582 – □ < 500
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<tr>
<td>5:4</td>
<td>
In Teaching point 1, children partitioned three-digit numbers into the usual hundreds, tens and ones; in step 5:2 they explored 'redistributing' either the hundreds or the tens. Now encourage further flexibility in partitioning, redistributing combinations of hundreds, tens and ones, e.g.:

- *580 = 450 + 130*
- *580 = 450 + 120 + 10*
- *576 = 306 + 270*
- *576 = 306 + 230 + 40*
- *576 = 300 + 200 + 46*

etc.

First, ask children to make a three-digit multiple of ten using place-value counters (for example, 580 from five hundred-value counters and eight ten-value counters), then challenge them to partition the counters, in any way, into two or three parts. You can provide them with printed part–part–part–whole diagrams to work on and, to avoid any 'bridging', restrict children to using the original counters rather than allowing exchange (for example, ten ten-value counters for one hundred-value counter). Record children's suggestions with numerals on part–part–part–whole diagrams. Then repeat the process for a three-digit multiple of ten with a non-zero ones digit (e.g. 576). For each partitioning, ask children to confirm
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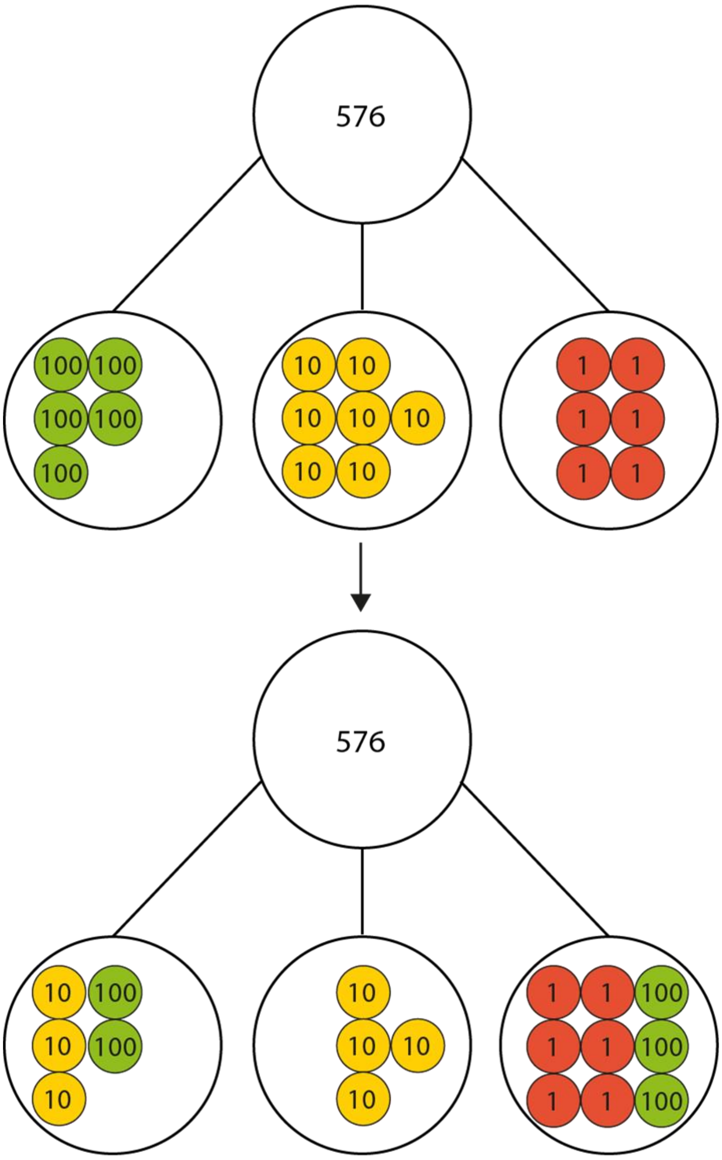
<td>
Flexible partitioning of three-digit multiples of ten:

576 = 500 + 70 + 6



576 = 230 + 40 + 306
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that the parts still combine to make the same whole, emphasising that the original number is conserved however we partition it.

Then work through some examples with missing parts. For example, show a part–part–part–whole diagram with the whole-value (e.g. 475) in digits; show one part containing 200 in place-value counters, another part containing 170 in place-value counters, and a blank part. Then model how to deduce the missing part: '*I already have seven tens; I have three hundreds in total, but I need four hundreds – so that's one hundred; I also need five ones – so the missing part must be one hundred and five.*' Work through several examples, encouraging children to explain their reasoning.

Then remove the place-value counters scaffold, working only with numerals on the part–part–part–whole diagrams, and finally work just with equations. Finish by providing children with some practice problems based on flexible partitioning.

This type of additive decomposition will be useful later when children are working on division. For example, it is not obvious that 450 is a multiple of six because 45 isn't a multiple of six, but by partitioning 450 into three numbers which are more obviously multiples of six (e.g., *450 = 300 + 120 + 30*), the connection becomes clear.

Missing-number problems:

- *'Fill in the missing numbers.'*

340 = 200 + 140	200 + 100 + 70 = 370
340 = 210 + ☐	200 + 110 + ☐ = 370
340 = 220 + ☐	200 + 120 + ☐ = 370
340 = 230 + ☐	200 + ☐ + 40 = 370
340 = 240 + ☐	200 + ☐ + 30 = 370

'Three children write the following expressions.'

Jo: 120 + 30 + 5

Ella: 110 + 20 + 25

Oscar: 100 + 40 + 15

'Are their numbers the same in total? Explain how you know.'

Dòng nǎo jīn:

- *'I have some £1 coins and 10p coins. How can I use twelve coins to make £3?'*
- *'I've got 12 tens and 12 ones. What have I got altogether?'*
- *'I've got 245 altogether, made up of 56 place-value counters. Each of my place-value counters is a ten or a one. How many of each counter have I got?'*

	10	1
How many?		

5:5 Now, to extend children's understanding of the composition of multiples of 100, and to prepare them for calculations across hundreds boundaries, move on to calculating to/from multiples of 100 in tens or ones, for example:

- *470 + 30* *500 – 30*
- *497 + 3* *500 – 3*

Children are likely to find the addition calculations fairly straightforward, so focus most of your time on the subtraction calculations; children tend to find subtraction of a multiple of ten from a multiple of 100 (e.g. 500 – 30) challenging.

Begin by revisiting sums to, and subtraction from, 100 in tens (covered in detail in segment *1.17 Composition and calculation: 100 and bridging 100*), as shown below; children should now be fluent with these facts. Then make the link to sums to, and subtraction from, *multiples* of 100, by partitioning as follows:

- For addition, partition the three-digit number into hundreds and tens, to reveal the multiples of ten that bond to 100.
- For subtraction, partition the multiple of 100 into 100 and another part, as shown below.

It may be sufficient, at this stage, to just use partitioning 'jottings' as used here. However, if you feel children need further support, particularly for the subtraction calculations, you can return to the use of pictorial representations or manipulatives. In this case, Dienes or hundred squares are useful representations, since the tens to be subtracted can be 'seen' in the context of the existing 100 (if you were to work with place-value counters, you would first need to exchange one of the 100 counters for a tens counter).

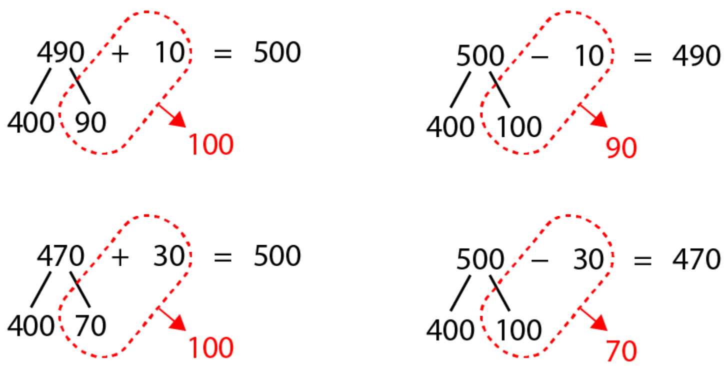
You can also show the calculations on a number line, but discourage children from counting on/back to find the answers, ensuring they instead use known facts.

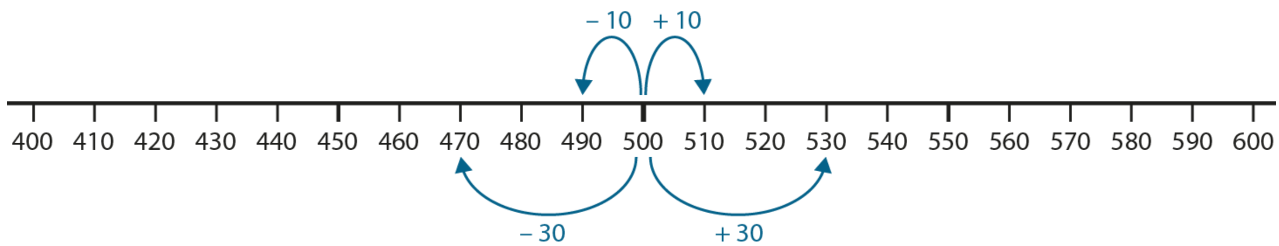
Some children may suggest other strategies; give value to reasonable approaches and, where appropriate, discuss efficiency.

Calculating to/from 100 in tens – known facts:

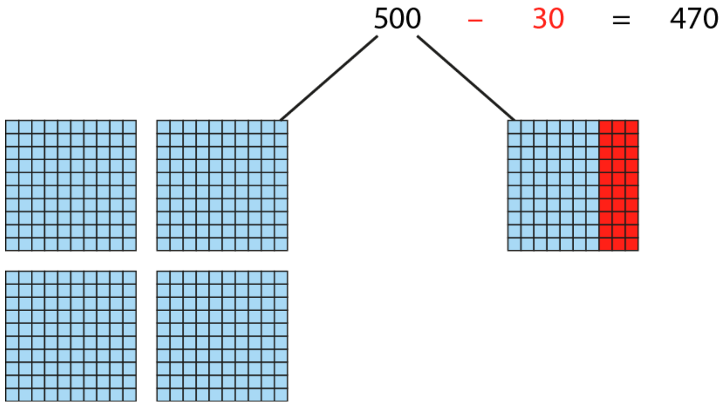
90 + 10 = 100	100 – 10 = 90
70 + 30 = 100	100 – 30 = 70

Calculating to/from *multiples* of 100 in tens – jottings and number line:





Calculating to/from *multiples* of 100 in tens – with Dienes:



5:6

Repeat the sequence, this time for addition/subtraction of ones instead of tens. Build from sums to, and subtraction from 100 in ones (again, this was secured in segment *1.17 Composition and calculation: 100 and bridging 100*).

Children might suggest strategies other than those suggested here, for example:

- 297 + 3 = 290 + 7 + 3
  = 290 + 10

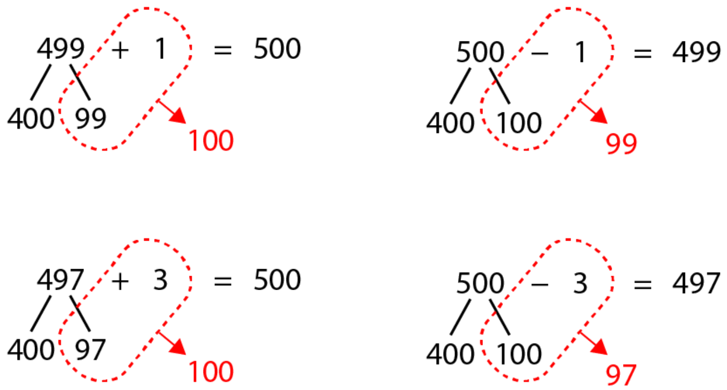
Discuss, and give value to, any reasonable strategies proposed.

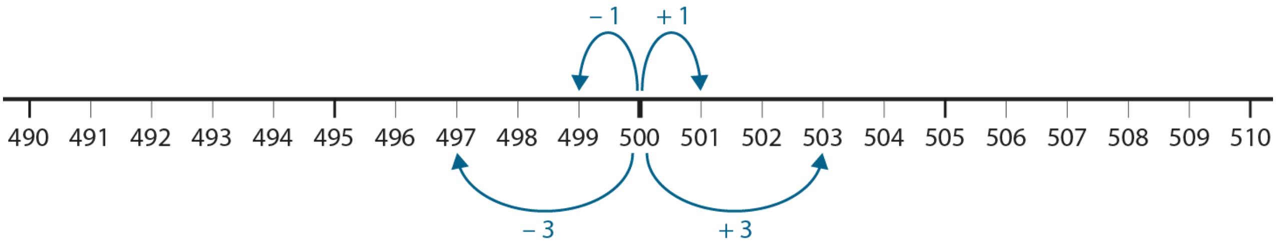
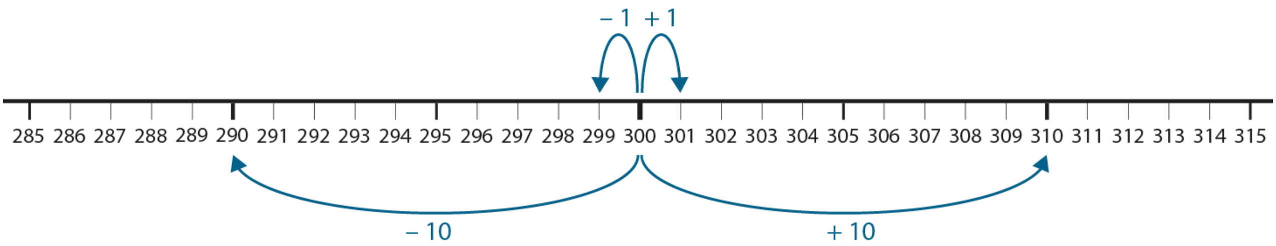
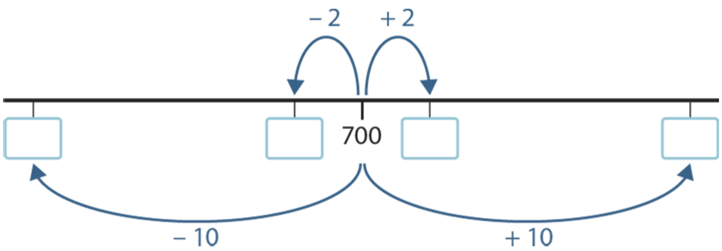
After working through addition/subtraction of ones, show both addition/subtraction of tens and ones together on the same number line to draw attention to the difference between the two types of calculation. Remind children that addition/subtraction of a multiple of ten from a multiple of 100 will always give a multiple of ten.

Calculating to/from 100 in ones – known facts:

99 + 1 = 100	100 – 1 = 99
97 + 3 = 100	100 – 3 = 97

Calculating to/from *multiples* of 100 in ones – jottings and number line:



	 Comparing addition/subtraction of tens and ones: 	
5:7	Provide children with missing-number problems for practice, including both sequences of calculations and isolated problems, as shown opposite.	Missing-number problems: *'Fill in the missing numbers.'*  800 + 100 = ☐ 900 – 100 = ☐ 890 + 10 = ☐ 900 – 10 = ☐ 899 + 1 = ☐ 900 – 1 = ☐ 400 + ☐ = 800 800 – 400 = ☐ 760 + ☐ = 800 800 – 40 = ☐ 796 + ☐ = 800 800 – 4 = ☐

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<td>700 – 200 = ☐   400 – ☐ = 300
700 – 50 = ☐   400 – ☐ = 370
700 – 8 = ☐   400 – ☐ = 397</td>
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<tr>
<td>5:8</td>
<td>Children can already count forward and back in ones and tens over the 100 boundary (segment 1.17 Composition and calculation: 100 and bridging 100). Now extend to counting over other hundreds boundaries, initially using the number line and Gattegno chart for support, and then progressing to counting without a scaffold, for example:
• 'Two hundred and eighty, two hundred and eighty-one… three hundred and nineteen, three hundred and twenty.'
and
• 'Five hundred and twenty, five hundred and ten… four hundred and eighty, four hundred and seventy.'
• 'Fifty-two tens, fifty-one tens… forty-eight tens, forty-seven tens.'
(For more guidance, and representations, see segment 1.17, Teaching point 3.)
Check children are secure with this step by providing missing-number sequences.</td>
<td>'Fill in the missing numbers.'
| 405 | 404 | ☐ | 402 | ☐ | 400 | ☐ | 398 |

| 670 | ☐ | 690 | ☐ | 710 | ☐ | 730 |</td>
</tr>
<tr>
<td>5:9</td>
<td>Now begin to look at calculations that bridge multiples of 100. First explore addition/subtraction to/from a multiple of ten, in multiples of ten (e.g. 480 + 50 and 530 – 50). Using number lines, as shown on the next page, begin by looking at an addition calculation that bridges 100, and then link it to a related calculation that bridges a multiple of 100. Then repeat for subtraction.</td>
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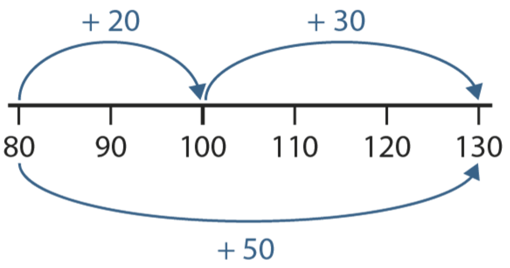
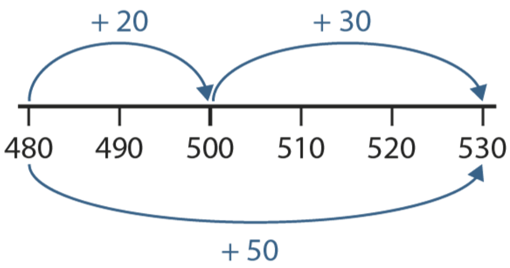
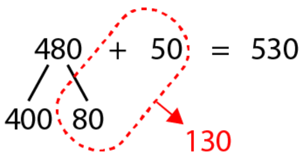
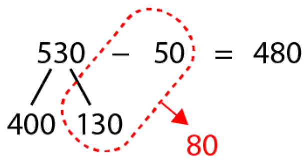
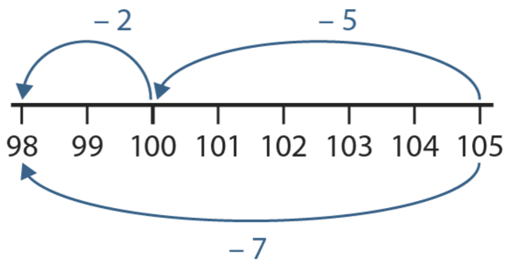
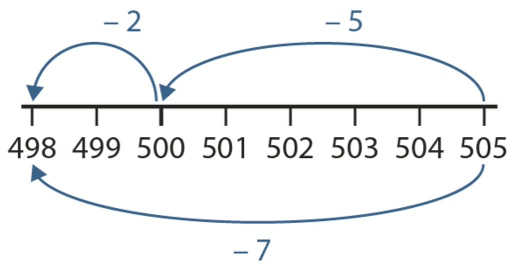
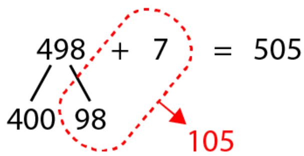
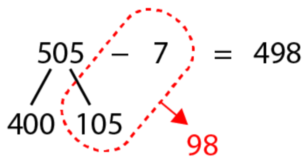
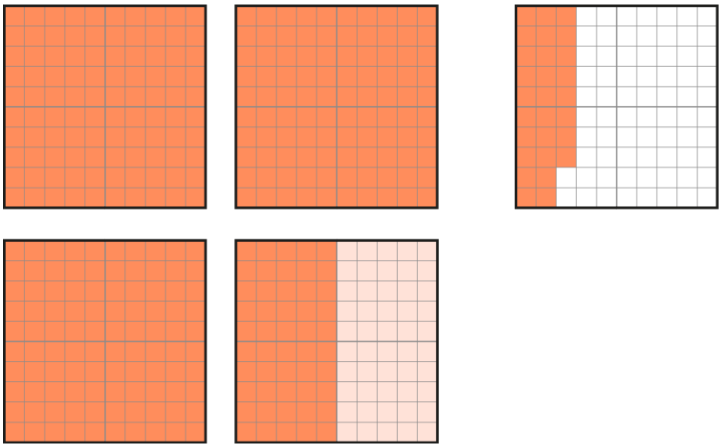
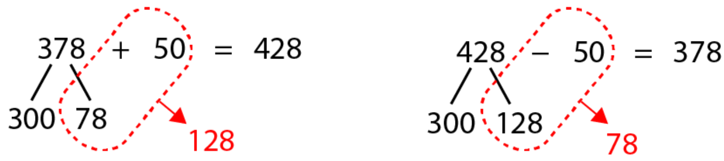
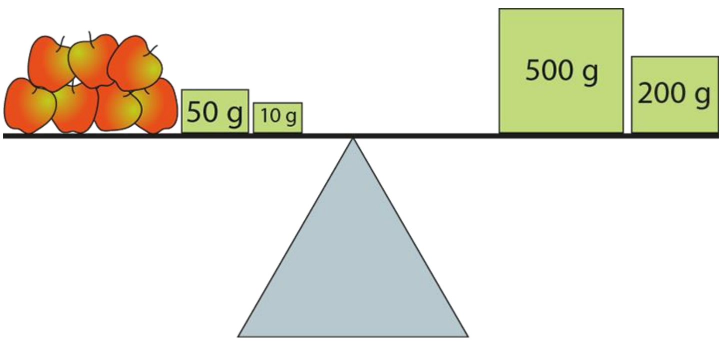
	This strategy emphasises using knowledge of bridging 100, but children may well propose other appropriate strategies, such as directly bridging the multiple of 100 by partitioning the addend (i.e. *480 + 20 + 30*) or unitising in tens (i.e. *48 tens + 5 tens = 52 tens*).	Bridging in multiples of ten – number lines:  80 + 50 = 130  480 + 50 = 530 Bridging in multiples of ten – jottings:  
5:10	Provide children with missing-number problems for practice, including both sequences of calculations and isolated problems, as shown opposite.	Missing-number problems: *'Fill in the missing numbers'* ☐ = 90 + 30 150 – 70 = ☐ ☐ = 190 + 30 250 – 70 = ☐ ☐ = 290 + 30 350 – 70 = ☐ ☐ = 590 + 30 650 – 70 = ☐ ☐ = 890 + 30 950 – 70 = ☐ 70 + ☐ = 110 180 – ☐ = 90 170 + ☐ = 210 380 – ☐ = 290 140 + ☐ = 210 ☐ – 90 = 490

		450 + 70 = ☐ 920 – 30 = ☐ 690 + 40 = ☐ 740 – 80 = ☐ 380 + 40 = ☐ 210 – 20 = ☐
5:11	Now, following the same progression as in step *5:9*, explore addition/subtraction to/from a three-digit number in ones (e.g. *498 + 7* and *505 – 7*).	Bridging in ones – number lines:  105 – 7 = 98  505 – 7 = 498 Bridging in ones – jottings:  
5:12	As before, provide children with missing-number problems for practice.	Missing-number problems: *'Fill in the missing numbers'.* ☐ = 96 + 8 103 – 6 = ☐ ☐ = 196 + 8 203 – 6 = ☐ ☐ = 296 + 8 303 – 6 = ☐ ☐ = 596 + 8 603 – 6 = ☐ ☐ = 896 + 8 903 – 6 = ☐

		95 + ☐ = 104 106 – ☐ = 99 195 + ☐ = 204 206 – ☐ = 199 ☐ + 9 = 304 = 210 ☐ – 7 = 299 493 + 8 = ☐ 902 – 5 = ☐ 695 + 8 = ☐ 703 – 8 = ☐ 397 + 6 = ☐ 204 – 7 = ☐
5:13	Children can already count forward and back across the hundred boundary in multiples of ten from/to a two-digit number (e.g. *76, 86, 96, 106, 116*). Now extend to counting over other hundreds boundaries, for example: • *'Three hundred and seventy-six, three hundred and eighty-six… four hundred and sixteen.'* • *'Five hundred and twenty-two, five hundred and twelve… four hundred and eighty-two.'* Initially use a section of the 10 × 100 grid (as shown opposite), a number line and a Gattegno chart. Then progress to counting without a scaffold. (For more guidance, and representations, see segment *1.17 Composition and calculation: 100 and bridging 100, Teaching point 4*.) Check children are secure with this step by providing missing-number sequences. Then give them practice identifying ten more/less over hundreds boundaries, before moving on to formal addition and subtraction of ten.	Counting in tens across the boundary: 371 372 373 374 375 **376** 377 378 379 380 381 382 383 384 385 **386** 387 388 389 390 391 392 393 394 395 **396** 397 398 399 400 401 402 403 404 405 **406** 407 408 409 410 411 412 413 414 415 **416** 417 418 419 420 Missing-number problems: *'Fill in the missing numbers.'* 483 | 493 | ☐ | 513 | ☐ | 533 629 | ☐ | 609 | ☐ | 589 | ☐ ten less ten more ☐ ← 802 → ☐ ten less ten more ☐ ← 207 → ☐

	A typical error that children make is: 703 – 10 = 697 ✘ Here they realise that 'ten less' will take them to the 690s, but then they apply their number bonds to 10 (*10 – 3*) to give an incorrect ones digit. Remind children that when we add/subtract ten, the ones digit remains unchanged, and provide sufficient practice for children to secure this point before moving on.	96 + 10 = □ □ = 106 – 10 196 + 10 = □ □ = 206 – 10 296 + 10 = □ □ = 306 – 10 596 + 10 = □ □ = 406 – 10 896 + 10 = □ □ = 506 – 10 105 = 95 + □ 101 – □ = 91 205 = 195 + □ 201 – □ = 191 305 = □ + 10 □ – 10 = 291 405 = □ + 10 □ – 10 = 391 593 + 10 = □ 908 – 10 = □ 492 + 10 = □ 702 – 10 = □
5:14	Now consider the final type of calculation in this segment, addition or subtraction of a multiple of ten to/from a three-digit number, e.g.: • *378 + 50* • *428 – 50* There are a variety of strategies for these calculations so, at this stage, it is useful to give the children the opportunity to explore these. Beginning with addition, present a calculation (e.g. *378 + 50*) and challenge children to offer ways of calculating the answer. You may find it useful to provide children with Dienes, number lines, and/or printed hundred squares. Discuss the different strategies	Hundred squares – subtracting *from* a multiple of 100: 428 – 50 = 400 – 50 + 28 

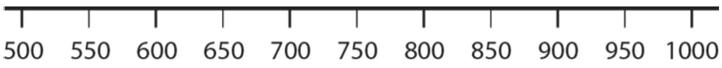
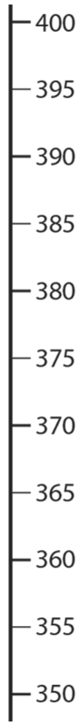
	suggested, considering efficiency and simplicity. Then repeat for subtraction. As mentioned earlier, children who do not have a deep understanding of number often end up performing the most challenging arithmetic. For example, a child with a deep awareness of composition of number may suggest the following strategy: $428 - 50 = 400 - 50 + 28$ $= 350 + 28$ while a child with a lesser understanding may draw on earlier work bridging through ten, suggesting the following: $428 - 50 = 428 - 28 - 22$ $= 400 - 22$ A few strategies and representations are exemplified opposite. Ensure that, by the end of this step, all children are able to confidently, correctly and consistently apply at least one strategy. Provide missing-number problems for practice and to assess children's ability to apply a reasonable strategy.	Jottings – applying knowledge of bridging 100:  Equations – adding the multiples of ten first: $378 + 50 = 370 + 50 + 8$ $= 420 + 8$ $= 428$
5:15	Complete the teaching point by providing contextual practice, including measures, for example: • *'Miss Evans has 142 pencils in the store cupboard and 3 packs of 10 pencils on her desk. How many pencils does she have altogether?'* (aggregation) • *'There are 623 books in the library. If 400 of them are fiction, how many are non-fiction?'* (partitioning) • *'George and his family travelled 7 km by bus to get to Cardiff, and then another 497 km by train to Edinburgh. How far did they travel in total?'* (augmentation)	*'How much do the apples weigh?'* 

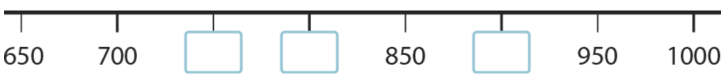
	• *'Stephanie had 400 pennies in her piggy bank. Then she spent 30 p. How many pennies does she have left?'* (reduction)
	• *'Jake swam 275 metres. Felicity swam 50 metres further than Jake. How far did Felicity swim?'* (difference)
	For challenge and depth, present a dòng nǎo jīn problem, for example:
	• *'There are 372 children in a school. At the end of term, they will each be given a bookmark. The bookmarks are sold in packs of 100 and packs of 10. The head teacher already has two packs of 100.*
	  ◦ *'What is the smallest number of packs the head teacher needs to buy?'*
	  ◦ *'How many packs should the head teacher buy so she has as few bookmarks as possible left over?'*	

Teaching point 6:

Familiar counting sequences can be extended up to 1,000.

Steps in learning

	Guidance	Representations
6:1	Earlier in this segment, children practised counting in ones, tens and hundreds within 1,000. To complete this segment, extend other familiar counting patterns, including counting in multiples of: • two (segments *1.4 Composition of numbers: 6–10* and *1.7 Addition and subtraction: strategies within 10*) • five (*Spine 2: Multiplication and Division*) • 20, 25 and 50 (segment *1.17 Composition and calculation: 100 and bridging 100*). A lot of graphing and measures work that children will encounter will depend on them being able to confidently read scales in these intervals. This is addressed explicitly in segment *1.22 Composition and calculation: 1,000 and four-digit numbers*; for now, keep the focus on counting-fluency. Taking each sequence in turn, practise counting forwards and backwards in the chosen multiple, initially supported by number lines, as exemplified opposite. Include vertical number lines to prepare children for reading vertical scales. For counting in multiples of 50, you can count all the way from 0 to 1,000 and back. For the smaller multiples (e.g. two and five), choose a certain section of the 0 to 1,000 number set to count within (for example, count in multiples of five from three hundred and fifty to four hundred and back again).	Example number line – multiples of 50:  Example number line – multiples of five: 

	Children may find it challenging to count back across a multiple of 100 in these less familiar multiples, e.g.: • 440, 420, 400, <u>380</u> • 750, 725, 700, <u>675</u> Pay particular attention to these in your teaching. You can use the Gattegno chart to support counting in multiples of two, then 20, then 200; the similarity of gestures (and numerals) draws attention to the link between these counting patterns. Similarly, count in multiples of five, then 50, then 500. Once children's confidence increases, remove the scaffold of the number line and Gattegno chart. Throughout, make sure you and the children describe the sequences using the word 'multiple', for example: • *'We are counting in multiples of twenty-five.'* not • *'We are counting in twenty-fives.'* This will help children to become more familiar with the word, and it reinforces the fact that the numbers being spoken (e.g. 750, 725…) are multiples of the counting-interval number (e.g. 25). Counting the following sequence: • 305, 330, 355, 380… is also '*counting in twenty-fives*', but the focus in this segment is *multiples* of 25 (and the other multiples), due to their significance in graphing and measures.	
6:2	Once the children have mastered the counting patterns, present some missing-number sequences using these multiples (with and without a number-line scaffold).	Missing-number sequences: *'Fill in the missing numbers.'* 

425			500		550		

385	390			405

806		802			798

Dòng nǎo jīn:

'What could the missing numbers be?

Find several different possibilities.'