MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.23 Composition and calculation: tenths

Teacher guide | Year 4

Teaching point 1:

When one is divided into ten equal parts, each part is one tenth of the whole.

Teaching point 2:

Tenths can be expressed as decimal fractions; the number written '0.1' is one tenth; one is ten times the size of 0.1.

Teaching point 3:

We can count in tenths up to and beyond one.

Teaching point 4:

Numbers with tenths can be composed additively and multiplicatively.

Teaching point 5:

Known facts and strategies, including column algorithms, can be applied to calculations for numbers with tenths.

Teaching point 6:

Numbers with tenths can be rounded to the nearest whole number by examining the value of the tenths digit.

Overview of learning

In this segment children will:

- extend their understanding of place value to tenths
- express numbers with tenths using decimal fractions (e.g. 0.1, 5.2)
- explore the additive composition of numbers with tenths (e.g. $2.4 = 2 + 0.4$)
- explore the multiplicative composition of numbers with tenths (e.g. $2.4 = 24 \times 0.1$)
- extend additive calculation strategies, including column addition and subtraction, to numbers with tenths
- learn how to round numbers with tenths to the nearest whole number.

This segment will deepen children's understanding of the base-ten structure of our number system as they explore how this extends to tenths.

Children begin by identifying one tenth in different contexts, then multiples of one tenth. By exploring the relationship between the hundreds, tens and ones places, children learn about the tenths place and the purpose of the decimal point. Children practise counting in tenths, using both number names ('*zero-point-one, zero-point-two…*') and unitising language ('*one tenth, two tenths…*'), and begin unitising in tenths by considering the multiplicative composition of numbers with tenths. With this foundation, children can build on existing knowledge and strategies, including:

- comparing the size of different numbers with tenths (note that, now we are working with decimal fractions, the number with the most digits is not necessarily the larger number, e.g. $1.7 < 2$)
- applying facts for additive calculations within ten to additive calculations within one (e.g. $0.5 + 0.2 = 0.7$ and $1 - 0.3 = 0.7$)
- applying strategies and facts for bridging ten to calculations that bridge one (e.g. $0.6 + 0.7 = 1.3$ and $1.6 - 0.8 = 0.8$)
- using column addition and column subtraction to calculate for numbers with tenths
- applying their knowledge of rounding in order to round numbers with tenths to the nearest whole number.

Also included is a brief discussion of how any facts and strategies for two-digit whole numbers can be applied to numbers with tenths (step *5:4*).

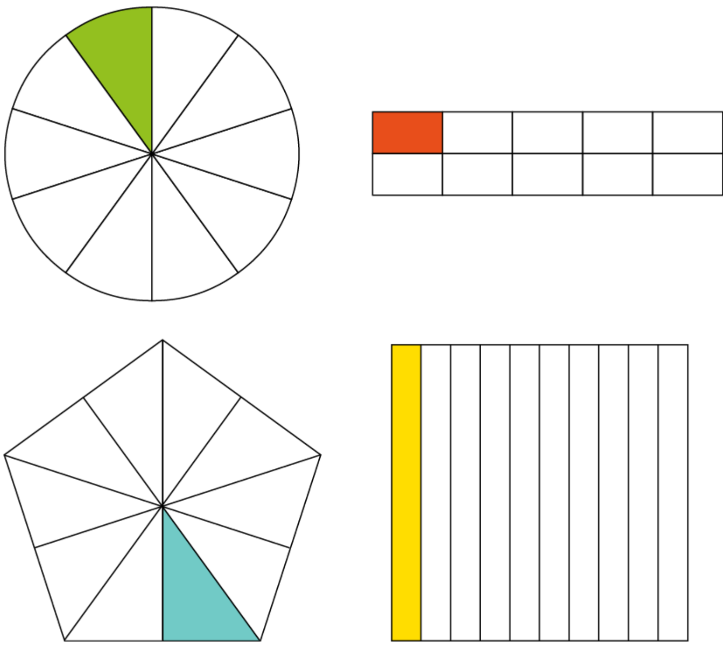
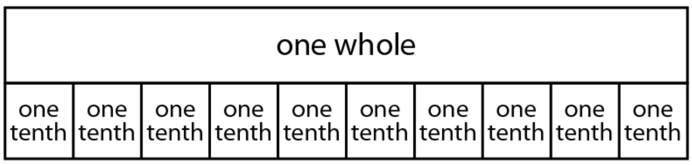
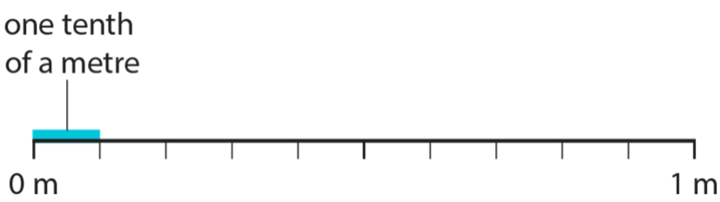
Throughout this segment, one Dienes hundred square is used to represent one, and therefore one Dienes tens rod represents one tenth; this representation can then easily be extended to hundredths in the next segment (where a hundred square is still equal to one, so a Dienes one cube is equal to one hundredth). This is likely to be the first time that children use Dienes in such a way – with the individual Dienes ones pieces no longer representing one, so teachers may need to spend some time ensuring that children understand what is being represented before using Dienes to support understanding of structures and calculation strategies. Teachers could, instead, choose to use a Dienes thousand cube to represent one, thereby allowing for extension to thousandths. In that case, one hundred square should be used to represent one tenth, one tens rod to represent one hundredth and a single Dienes one cube to represent one thousandth.

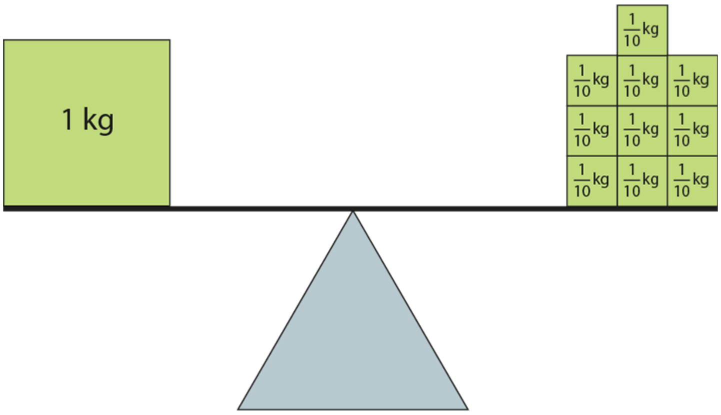
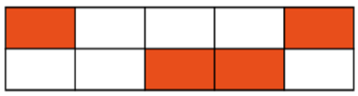
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

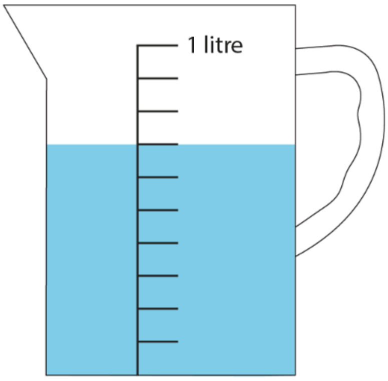
Teaching point 1:

When one is divided into ten equal parts, each part is one tenth of the whole.

Steps in learning

	Guidance	Representations
1:1	Children should already be familiar with the term 'tenth' from their work on fractions. Begin this short teaching point by exploring one tenth of different 'wholes', using the term 'tenth' but not representing it with numerals. Use both practical work and pictorial representations of wholes with different shapes and arrangements. The square representation shown opposite is important, since it links to Dienes and Cuisenaire® rods, which children will use later in this segment when counting and calculating with tenths. Use the following generalised sentence to describe pictorial representations such as those shown opposite: ***'The whole is divided into ten equal parts and one of them is shaded; this is one tenth of the whole.'*** Use the bar model to link to children's understanding of 'parts' and 'wholes'.	Shapes:  Bar model: 
1:2	Also include examples of measures, as shown opposite. Practical work could include, for example, dividing one kilogram of rice into ten equal parts.	Measures – length: 

		Measures – mass: 
1:3	Then extend to examples with more than one tenth identified. Make sure you include cases where the different highlighted tenths are not adjacent, as with the rectangle opposite. Extend the generalised sentence used in step *1:1*: ***'The whole is divided into ten equal parts and ___ of them is/are shaded; this is ___ tenth(s) of the whole.'*** Use similar language to describe measures contexts, as exemplified opposite.	More than one tenth:  *'The whole is divided into ten equal parts and three of them are shaded; this is three tenths of the whole.'*  *'The whole is divided into ten equal parts and four of them are shaded; this is four tenths of the whole.'*

		More than one tenth - measures:  *'The litre jug is divided into ten equal parts and there is water up to the seventh mark; this is seven tenths of a litre.'*

Teaching point 2:

Tenths can be expressed as decimal fractions; the number written '0.1' is one tenth; one is ten times the size of 0.1.

Steps in learning

	Guidance	Representations

2:1

Now work towards expressing one tenth as 0.1. Children should already be able to confidently represent numbers on a place-value chart, up to the thousands column. For whole numbers, explore what happens when a counter or digit is moved between columns on a place-value chart, as shown opposite. Ask children to describe, in words, what happens to the value of the number represented, working towards use of the following generalised statements:

- ***'If a digit is moved one column to the left, the number represented becomes ten times bigger/ten times the size.'***
- ***'If a digit is moved one column to the right, the number represented becomes ten times smaller; we can also say it becomes one tenth the size.'***

Before introducing tenths to the place-value chart, practise describing the relationship between the different place-value columns:

- *'1,000 is divided into ten equal parts; each part is equal to 100; 100 is one tenth of 1,000.'*
- *'100 is divided into ten equal parts; each part is equal to 10; 10 is one tenth of 100.'*
- *'10 is divided into ten equal parts; each part is equal to 1; 1 is one tenth of 10.'*

Representations:

1,000s	100s	10s	1s
●			
	●		
		●	
			●

ten times smaller → one tenth the size	ten times smaller → one tenth the size	ten times smaller → one tenth the size

1,000s	100s	10s	1s
1			
	1		
		1	
			1

ten times smaller → one tenth the size	ten times smaller → one tenth the size	ten times smaller → one tenth the size

2:2

Now move on to explore what happens when 'we go one tenth smaller' from the ones column; explain that another column is needed in the place-value chart: the tenths column.

Ask children to describe what the digit in each column represents, for example:

- *'The "1" in the hundreds column represents one hundred.'*
- *'The "1" in the tenths column represents one tenth.'*

Extend the descriptive sentences about the relationship between the different columns: *'One is divided into ten equal parts; each part is equal to one tenth.'*

Then ask children what will happen if we remove the place-value headings.

Make a second chart without the headings; then add zeros as placeholders in the empty places to the right of the digits on both charts to see what happens. Encourage children to notice that when the place-value headings are removed, there are too many zeros; the numbers are now ten times larger than they should be. Working from top to bottom in the charts, ask, for example:

- *'What number does the second row of the place-value chart represent?'* (There is a one in the hundreds column and zeros in the other columns, so the number represented is one hundred.)
- *'What number does the second row of the other chart represent?'* (There is a one followed by three zeros, so the number represented is one thousand.)

You can then introduce the decimal point, demonstrating that it is needed to separate the whole number from the part that is less than one (children

Adding a column to the place-value chart:

1,000s	100s	10s	1s	tenths
1				
	1			
		1		
			1	
				1

Including the zeros with place-value headings:

1,000s	100s	10s	1s	tenths
1	0	0	0	0
	1	0	0	0
		1	0	0
			1	0
				1

Including the zeros without place-value headings:

1	0	0	0	0
	1	0	0	0
		1	0	0
			1	0
				1

Introducing the decimal point:

1	0	0	0 •	0
	1	0	0 •	0
		1	0 •	0
			1 •	0
			0 •	1

should be familiar with the term 'whole number' from their work on fractions).

Explain:

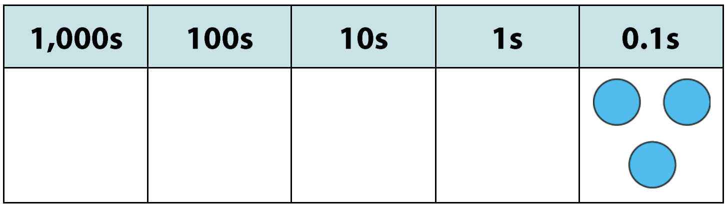
- that the decimal point marks the separation of the ones and the tenths, so that we can write these numbers without needing a place-value chart
- that when we represent a number smaller than one, we write a zero in the ones place, so one tenth is written as '0.1'; the '0' makes it clear that we have a decimal point.

2:3

Extend by including different numbers of counters in the tenths column and using decimal notation to write the numbers (for numbers up to and including 0.9).

For each number ask children to describe how the number is written, using the stem sentence: ***'One tenth can be written as "0.1", so ___ tenths can be written as "0.___".'***

You could revisit the examples you used in step *1:3*, asking children to now write the numbers with numerals.



1,000s	100s	10s	1s	0.1s
				3

			•	3

			0 •	3

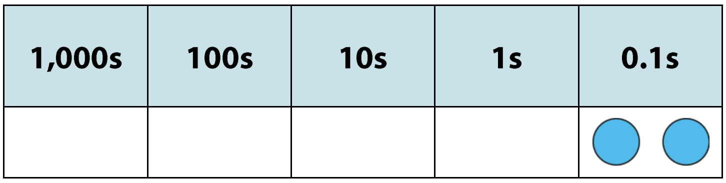
'One tenth can be written as "0.1", so three tenths can be written as "0.3".'

2:4

Provide children with varied practice writing numbers less than one using the decimal point or interpreting numbers written in this way, including:

- presenting numbers represented on place-value charts and asking children to write the numbers (as numerals), and vice versa
- presenting shapes divided into tenths with some parts shaded and asking children to write the numbers (as numerals), and vice versa

Place-value chart with counters:



'Fill in the blanks.'

- *'This represents ___ tenths.'*
- *'We can write this as ___.'*

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- asking children to label or interpret measures contexts (as shown opposite)
- presenting word problems such as *'A pizza is cut into ten equal parts; Joe eats four pieces. Write down how much of the pizza Joe eats.'*

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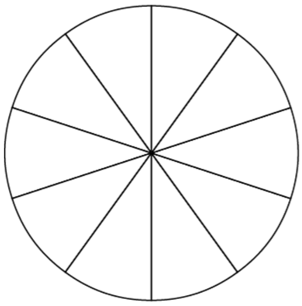
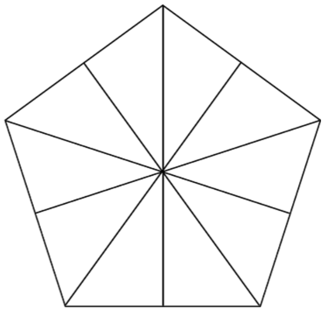
Shapes:

'Colour in the shapes to represent the numbers shown.'

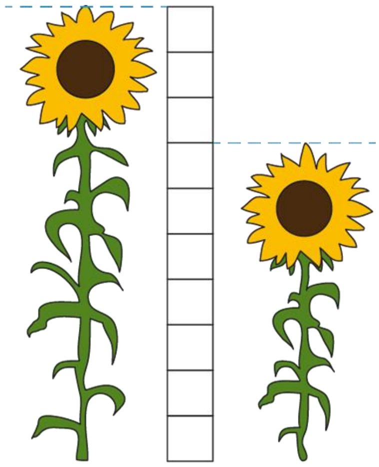


0.4



0.7

Measures context:



Alicia's sunflower

Brian's sunflower

'Fill in the blanks.'

- *'Brian's sunflower is ___ tenths the size of Alicia's sunflower.'*
- *'We can write this as ___.'*

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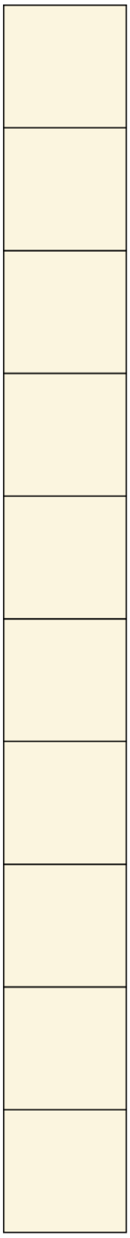
Teaching point 3:

We can count in tenths up to and beyond one.

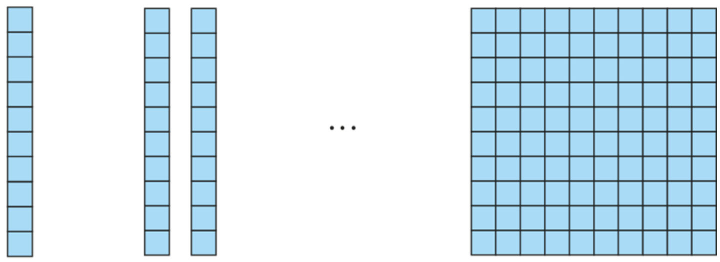
Steps in learning

	Guidance	Representations
3:1	Now explore counting in tenths, initially only up to ten tenths/one. You could use Cuisenaire® rods as a concrete representation: beginning with the white rods, children count as they add more white rods (one tenth, two tenths, etc.) until they have ten white rods, which match the length of one orange rod (ten tenths or one whole). Use dual counting as shown opposite to emphasise unitising in tenths and to reinforce that, for example, 0.3 represents three tenths. Unitising in tenths will be important when children come to apply known additive strategies to numbers with tenths.	
3:2	Practise counting using a variety of cardinal and ordinal representations, including Cuisenaire® rods, Dienes, number lines and the Gattegno chart. Note that each Dienes tens rod is used to represent one tenth, with the	

ten tenths	one	1.0
nine tenths	zero-point-nine	0.9
eight tenths	zero-point-eight	0.8
seven tenths	zero-point-seven	0.7
six tenths	zero-point-six	0.6
five tenths	zero-point-five	0.5
four tenths	zero-point-four	0.4
three tenths	zero-point-three	0.3
two tenths	zero-point-two	0.2
one tenth	zero-point-one	0.1

hundred square being used to represent one; this will facilitate extension to hundredths in the next segment. However, since this is the first time that children will be using Dienes other than with one small cube representing one, make sure you are clear about the use of one rod to represent one tenth.

Dienes:



0.1 0.2 1

Number line:

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0.0 1.0

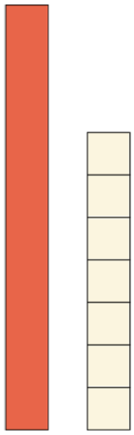
Gattegno chart:

100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

3:3 Now, using Cuisenaire® rods or Dienes, start to explore counting beyond one in tenths. Count to one using the unitising language (*'one tenth, two tenths…'*) until one is reached (*'…ten tenths, which is equal to one'*), then regroup the ten tenths manipulatives, replacing them with one one-value piece, before beginning to count a new set of tenths. For now, count *'… one and one tenth, one and two tenths…'*, as this will prepare children for representing the numbers on a place-value chart in the next step. Using a cardinal representation (Cuisenaire® rods or Dienes) here helps children to see the 'one' and some more tenths. Once children are confident with this structure, count further, going beyond two, then three.

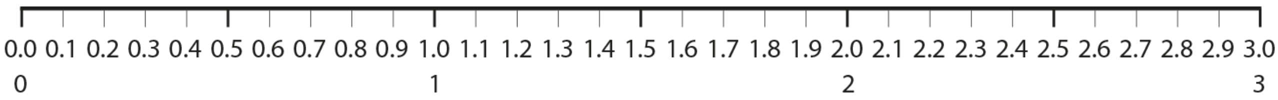
Then count in the same way using a number line and/or Gattegno chart. With the Gattegno chart you will need to tap twice for each number – once on the whole number, and one on the 'part'.

Cuisenaire® rods:



'One tenth, two tenths… one and seven tenths.'

Number line:

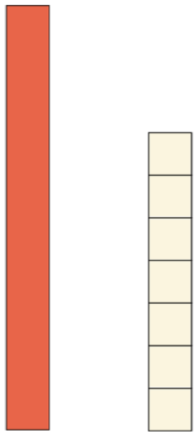


3:4 Now take time to discuss how these numbers are written and named. Present a given number (e.g. 1.7) using one of the representations you used in step *3:3* and ask children to represent the number on a place-value chart. Then ask them to write the number without the place-value chart (that is, with the decimal point, e.g. 1.7), and say the number name (e.g. *'one-point-seven'*).

Repeat for several numbers, including:

- examples where the ones digit is greater than one (e.g. 3.2)
- examples where the tenths digit is zero (e.g. 2.0).

Ensure that children can record whole numbers at both levels of precision, for example, as both '2' and '2.0', and that children understand these are equivalent/have the same value in this context.



1,000s	100s	10	1s	0.1s
			1	7

1 . 7

<table>
<tr><td>3:5</td><td>Then repeat the counting exercise from step 3:3, with supporting representations, but now using the number names: 'Zero-point-one, zero-point-two… one-point-seven.'</td><td></td></tr>
<tr><td>3:6</td><td>Now use dual-counting with unitising in tenths beyond one, so that children recognise the equivalence of, for example, 1.2 and 12 tenths:
• '…nine tenths, ten tenths, eleven tenths, twelve tenths…'
• '…zero-point-nine, one-point-zero, one-point-one, one-point-two…'
Once children are confident counting in this way, extend to counting from starting points other than zero and to counting backwards.</td><td></td></tr>
<tr><td>3:7</td><td>Now, briefly spend some time illustrating that we can also include non-zero digits in the tens, hundreds and thousands places, and that the numbers to the right of the decimal point continue to 'work' in the same way.</td><td><table><tr><th>1,000s</th><th>100s</th><th>10s</th><th>1s</th><th>0.1s</th></tr><tr><td></td><td>2</td><td>0</td><td>3</td><td>7</td></tr></table>203 . 7
'This is two hundred and three and seven tenths. We can also say two hundred and three-point-seven.'
<table><tr><th>1,000s</th><th>100s</th><th>10s</th><th>1s</th><th>0.1s</th></tr><tr><td>4</td><td>2</td><td>5</td><td>1</td><td>7</td></tr></table>4,251 . 7
'This is four thousand two hundred and fifty-one and seven tenths. We can also say four thousand two hundred and fifty-one-point-seven.'</td></tr>
<tr><td>3:8</td><td>You can link to work on fractions by reminding children that they now have two different ways to write numbers with tenths:</td><td>42.3 $\left(42\frac{3}{10}\right)$
'I say forty-two-point-three, but I think forty-two and three tenths.'</td></tr>
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- as common fractions $\left(\frac{7}{10}\right)$ or mixed numbers $\left(42\frac{3}{10}\right)$
- as decimal fractions (e.g. 0.7, 42.3).

To reinforce the link between the two forms, encourage children to use the stem sentence: ***'I say ___-point-___ but I think ___ and ___ tenth(s).'***

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3:9

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Provide children with varied practice writing numbers greater than one using the decimal point or interpreting numbers written in this way, including:

- presenting them with numbers represented on place-value charts, and asking children to write the numbers (as numerals), and vice versa
- presenting pictorial/concrete representations of a value and asking children to write the numbers (as numerals), and vice versa
- asking children to label or interpret measures contexts (as shown opposite)
- presenting word problems, such as: *'After a party there are three whole pizzas and four tenths of a pizza left over. Write this amount in numerals.'*
- asking children to convert from common fractions/mixed numbers to decimal fractions and vice versa.

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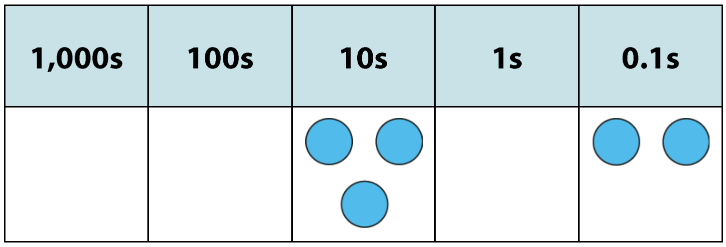
Place-value chart with counters:



'Write the number represented as a decimal fraction.'

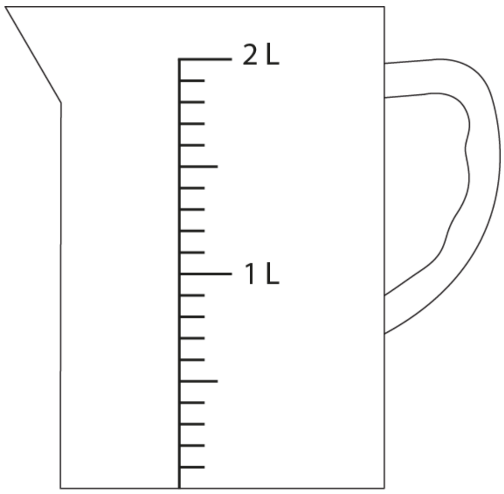
Concrete/pictorial representations:

'Represent the following numbers using Cuisenaire® rods. Sketch your answers.'

1.4   2.0   3.7   4.2

Measures context:

'Colour in 1.6 litres of juice.'



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		Converting between common fractions/mixed numbers and decimal fractions: *'Complete the table.'* $\frac{3}{10}$ → 0.3 $\frac{7}{10}$ → → 1.0 $302\frac{4}{10}$ → → 1,056.9
3:10	Counting is a useful step in helping children to understand the relative value of different numbers. In a similar way to segment *1.9 Composition of numbers: 20–100, Teaching point 4*, present children with two numbers and ask them to identify which is larger/smaller using a cardinal representation (for example, Dienes) or ordinal representation (for example, number line) for support. They may reason, for example: • *'Four-point-three is greater than three-point-seven because four-point-three has one more large Dienes square.'* (where one 'large'/hundred square represents one). • *'Four-point-three is greater than three-point-seven because it is further along the number line.'*	
3:11	Then, beginning with numbers with the same number of significant digits (e.g. 3.7 and 4.3), explore how the numbers can be compared just by examining the digits. Discuss how, if the ones digits are different, we can determine the relative size of the numbers just by looking at the ones	

Table in the first row:

$\frac{3}{10}$	0.3
$\frac{7}{10}$	
	1.0
$302\frac{4}{10}$	
	1,056.9

digits; but if the ones digits are the same, we must then examine the tenths digits. Initially use Dienes for support, but then work with only the digits.

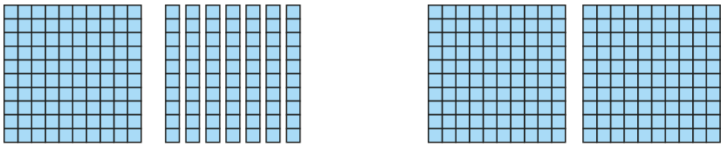
3:12

Now ask children to compare a number pair such as 1.7 and 2 (i.e. where the larger number has fewer digits), looking only at the digits. Children may say that the number with more digits is larger, since this would be true for whole numbers. If children do give the correct answer, challenge them to explain why. In either case, after examining only the digits, investigate together in detail, asking children to represent each number with Dienes and on a place-value chart, to confirm that 2 is larger than 1.7 despite having fewer digits.

Then repeat with several similar number pairs. In each case, before children decide which number is larger/smaller, work with them to identify the place-value of each digit and emphasise that we need to compare 'like digits' starting with the largest place-value digit. Use language such as that shown opposite to draw attention to the importance of place value.

Gradually move away from use of the place-value chart until children are able to identify which digits to compare without this support. Use the generalised statement: ***'To compare two numbers, we compare digits with the same place value, starting with the largest place-value digit.'***

Comparing numbers using Dienes:



1.7 < 2

Comparing numbers on a place-value chart:

100s	10s	1s	0.1s
		1	7
		2	

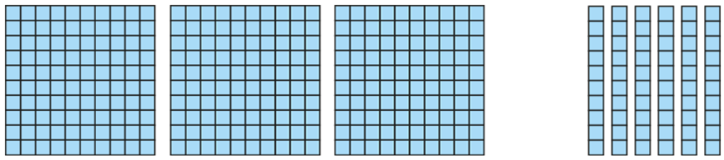
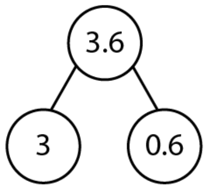
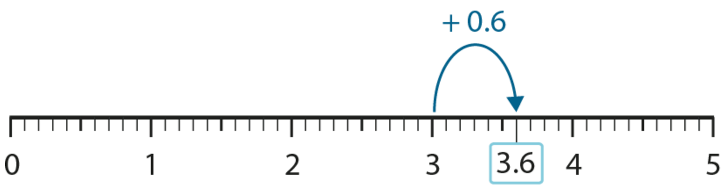
'I know that one-point-seven is smaller than two because the largest place value is the ones and there are more ones in two than in one-point-seven.'

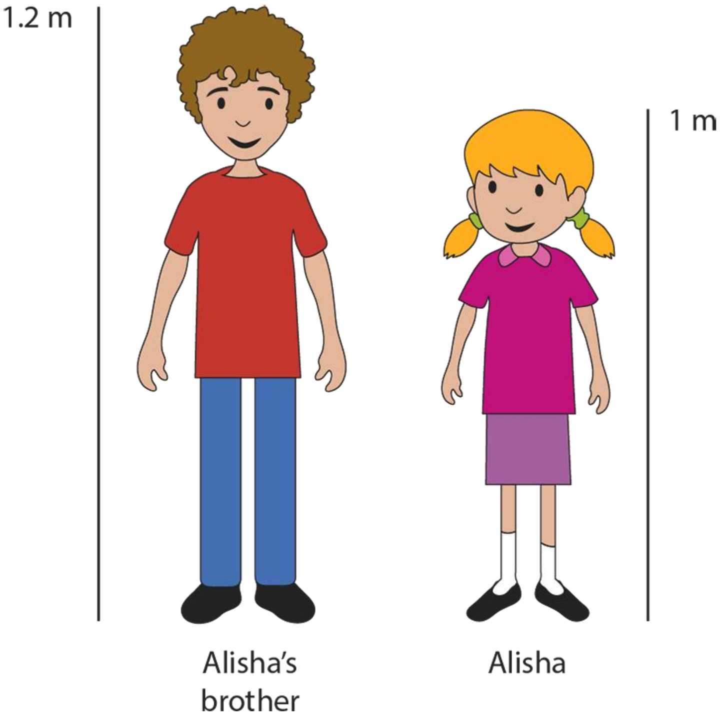
3:13	Provide children with practice comparing numbers using symbols or putting sequences of numbers in ascending order, as shown opposite. Also present dòng nǎo jīn problems, such as the following example, to promote and assess depth of understanding: *'Sally says she can think of four numbers, between one and five, that each have two digits which sum to eight. What numbers is Sally thinking of?'*	• *'Fill in the missing symbols (< > or =).'* 0.7 ○ 0.4 1.0 ○ 1 1.7 ○ 1.4 0.6 ○ 1.6 0.2 ○ 2.0 1.6 ○ 0.9 0.4 ○ 4.0 0.2 ○ 2 9 ○ 9.0 14 ○ 1.4 • *'Put these numbers in order from the smallest to the largest.'* 131 0 1.3 3 13 0.3 13.1 smallest □ □ □ □ □ □ □ largest

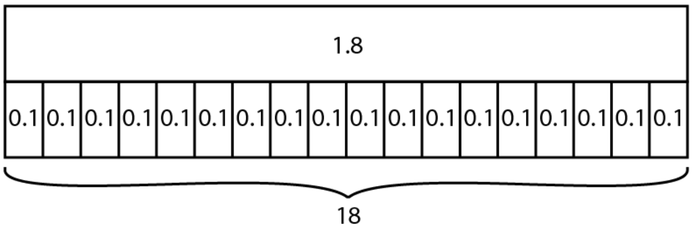
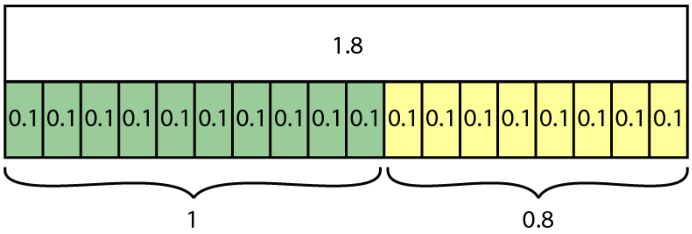
Teaching point 4:

Numbers with tenths can be composed additively and multiplicatively.

Steps in learning

	Guidance	Representations
4:1	Now provide children with experience in decomposing numbers with one decimal place. Begin with additive decomposition, i.e. partitioning into ones (whole numbers) and tenths. Use Dienes to represent a given number and ask children to identify the whole ones and the additional tenths; ask them to represent this on a part–part–whole model and to write the corresponding addition and subtraction equations. Work through a range of examples, gradually removing the scaffold of the Dienes and then the part–part–whole models, until children can write the additive equations without support. Children should practise expressing additive decomposition using both decimal fractions (e.g. 0.6) and common fractions (e.g. $\frac{6}{10}$): $3.6 = 3 + 0.6 = 3 + \frac{6}{10}$ Encourage children to use sentences of the form *'I say three-point-six, but I think three and six tenths'* to support their understanding of additive composition, as well as the link between decimal fractions and mixed numbers.	Dienes:  Part–part–whole model:  Additive equations: $3.6 = 3 + 0.6$ $\quad$ $3.6 - 3 = 0.6$ $3.6 = 0.6 + 3$ $\quad$ $3.6 - 0.6 = 3$
4:2	Also use the number line to represent the decomposition into ones and tenths, and to support children's understanding of magnitude of numbers. For example, encourage children to recognise and reason that 3.6 is between three and four because	Number line: 

	it is *'three-and-some tenths'*. Children will build on this in *Teaching point 6* when they learn to round to the nearest whole number. Link to real-world contexts that involve scales and measures.	Measures context: *'How much shorter is Alisha than her brother?'*  1.2 – 1 = 0.2 *'Alisha is zero-point-two metres shorter than her brother.'*
4:3	Present missing-number problems such as those shown opposite; the equations are shown in pairs to help children focus on each part of the number (whole numbers and tenths).	*'Fill in the missing numbers.'* 1.8 = 1 + ☐ 3.6 = ☐ + 0.6 1.8 = ☐ + 0.8 3.6 = 3 + ☐
4:4	Also explore multiplicative decomposition. Begin with multiple addition of numbers smaller than one, for example: 0.3 = 0.1 + 0.1 + 0.1 0.3 = 3 × 0.1 0.1 + 0.1 + 0.1 = 3 × 0.1 Then move on to numbers greater than one.	

	Use the bar-model representation to link multiplicative and additive reasoning, for example: $1.8 = 1 + 8 \times 0.1$ $1.8 = 10 \times 0.1 + 0.8$ Once children have grasped the multiplicative decomposition, promote further depth of understanding by encouraging them to use the fact that *0.1 = 1 ÷ 10* to rewrite the expressions, for example: $1.8 = 18 \times 1 \div 10 = 18 \div 10$ Throughout this teaching point, make sure you vary placement of the equals symbol to represent both 'decomposition' and 'composition', for example: $1.8 = 18 \times 0.1$ $18 \times 0.1 = 1.8$	 
4:5	Provide children with a range of carefully chosen practice problems to increase their understanding and fluency of the multiplicative composition of tenths. Encourage them to use the language of unitising to secure understanding that three 0.1s is equal to 0.3. Children will have used this language when learning about the composition of multiples of ten in segment *1.8 Composition of numbers: multiples of 10 up to 100.*	*'Fill in the missing numbers and words.'* $3 \times 0.1 =$ 0.3 three 0.1s $4 \times 0.1 =$ 0.4 ________ 0.1s $5 \times 0.1 =$ ☐ five 0.1s $10 \times 0.1 =$ ☐ ten 0.1s $11 \times 0.1 =$ ☐ ________ 0.1s $12 \times 0.1 =$ ☐ twelve 0.1s $22 \times 0.1 =$ ☐ ________ 0.1s $23 \times 0.1 =$ ☐ twenty-three 0.1s

Teaching point 5:

Known facts and strategies, including column algorithms, can be applied to calculations for numbers with tenths.

Steps in learning

5:1

Guidance

The purpose of this teaching point is for children to apply existing number facts, strategies and methods to numbers with tenths, including:

- applying facts for additive calculations within ten to additive calculations within one (steps *5:1* and *5:2*)
- applying strategies and facts for bridging ten to calculations that bridge one (step *5:3*)
- applying two-digit number facts and strategies to numbers with tenths (step *5:4*)
- applying column addition and subtraction algorithms to numbers with tenths (steps *5.5* and *5.6*).

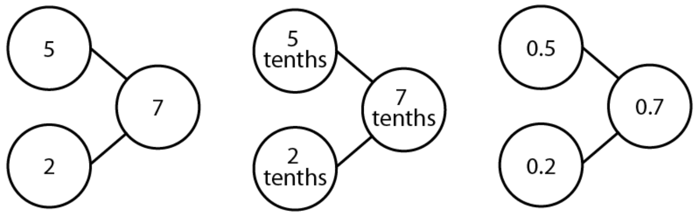
Necessarily, not all strategies are included, but those that are exemplify understanding that can be extended to other calculations and strategies.

Begin with addition and subtraction calculations within one, using part–part–whole models and either Dienes or tenth-value place-value counters on tens frames to link calculations with tenths (e.g. *0.5 + 0.2 = 0.7*) to related single-digit facts (e.g. *5 + 2 = 7*). Write the calculations alongside the representations and use the following stem sentence with the language of unitising: ***'___ tenths plus/minus ___ tenths is equal to ___ tenths.'***

Work through examples until children no longer require support from concrete/pictorial representations and then give children some practice

Representations

Part–part–whole models:

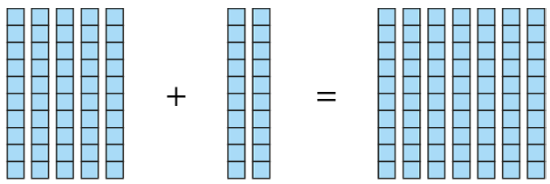


- *'Five tenths plus two tenths is equal to seven tenths.'*
- *'Seven tenths minus two tenths is equal to five tenths.'*

0.5 + 0.2 = 0.7

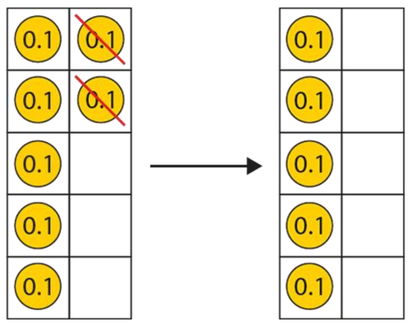
0.7 – 0.2 = 0.5

Dienes – example addition calculation:

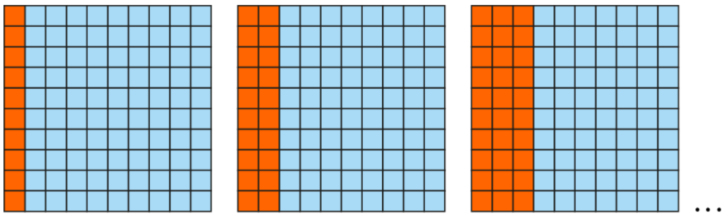
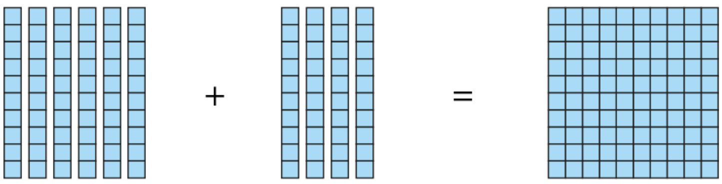


0.5 + 0.2 = 0.7

Tens frames and place-value counters – example subtraction calculation:



0.7 – 0.2 = 0.5

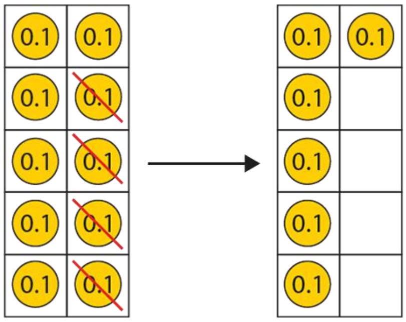
	completing missing-number problems, as shown opposite.	Missing-number problems: *'Fill in the missing numbers.'* 3 tenths + 2 tenths = □ tenths 0.3 + 0.2 = □ 9 tenths – 3 tenths = □ tenths 0.9 – 0.3 = □ 3 tenths + 2 tenths = □ tenths 0.3 + 0.2 = □ □ = 0.3 + 0.4 0.7 – 0.3 = □ 0.8 = □ + 0.2 0.8 – □ = 0.2
5:2	Now extend to complements to one (using known bonds to ten). Begin with addition and use hundred squares, Dienes or tens frames with counters to emphasise the equivalence between ten tenths and one, for example: • use pictorial Dienes with two colours, linking to work on complements to 100 (see segment *1.17 Composition and calculation: 100 and bridging 100*) • use Dienes tens rods, replacing the ten tens rods with one hundred square after addition • use tenth-value place-value counters, drawing attention to the full tens frame after addition. Then work through subtraction calculations in a similar way.	Pictorial Dienes representation:  Dienes – example addition calculation:  0.6 + 0.4 = 1 *'Six tenths plus four tenths is equal to ten tenths, which is equal to one.'*

Continue to use the language of unitising:

- ***'___ tenths plus ___ tenths is equal to ten tenths, which is equal to one.'***
- ***'One is equal to ten tenths; ten tenths minus ___ tenths is equal to ___ tenths.'***

Then present practice problems as shown opposite, using intelligent practice to extend to complements to other whole numbers.

Tens frames and place-value counters – example subtraction calculation:



1 – 0.4 = 0.6

'One is equal to ten tenths; ten tenths minus four tenths is equal to six tenths.'

Missing-number problems:

'Fill in the missing numbers.'

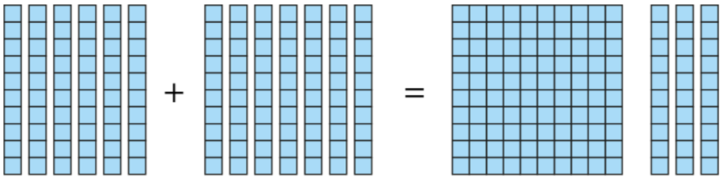
1 – 0.1 = ☐	2 – ☐ = 1.9	0.9 = 1 – ☐
1 – 0.2 = ☐	2 – ☐ = 1.8	0.8 = 1 – ☐
1 – 0.3 = ☐	2 – ☐ = 1.7	0.7 = 1 – ☐
1 – 0.4 = ☐	2 – ☐ = 1.6	0.6 = 1 – ☐
1 – 0.5 = ☐	2 – ☐ = 1.5	0.5 = 1 – ☐
1 – 0.6 = ☐	2 – ☐ = 1.4	0.4 = 1 – ☐
1 – 0.7 = ☐	2 – ☐ = 1.3	0.3 = 1 – ☐
1 – 0.8 = ☐	2 – ☐ = 1.2	0.2 = 1 – ☐
1 – 0.9 = ☐	2 – ☐ = 1.1	0.1 = 1 – ☐

5:3

Now build on the previous step, exploring addition and subtraction across one. Continue to use Dienes and/or tens frames with counters and unitising language. You can also use number lines to link to known 'bridging strategies'.

As before, work through examples until children no longer require concrete/pictorial support, and then provide missing-number problems for practice.

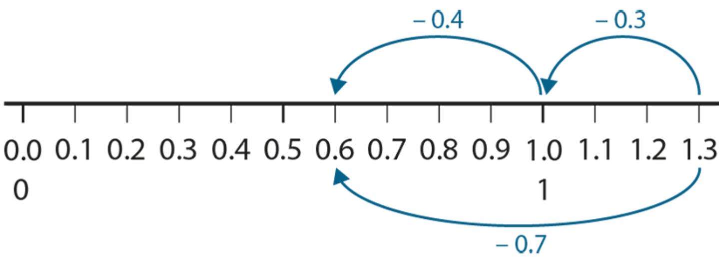
Dienes – example addition calculation:



0.6 0.7 1.3

'Six tenths plus seven tenths is equal to thirteen tenths, which is equal to one-point-three.'

Number line – example subtraction calculation:



1.3 – 0.7 = 0.6

Missing-number problems:

'Fill in the missing numbers.'

8 tenths + 3 tenths = ☐ tenths

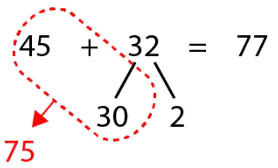
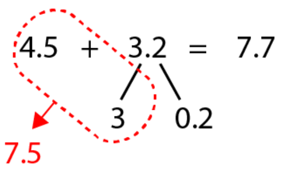
0.8 + 0.3 = ☐

16 tenths – 8 tenths = ☐ tenths

1.6 – 0.8 = ☐

☐ = 0.9 + 0.4   1.3 – 0.4 = ☐

1.5 = ☐ + 0.8   1.5 – ☐ = 0.7

5:4	So far in this teaching point, children have been combining known facts with unitising in tenths to mentally calculate for two-digit numbers with tenths (i.e. one digit on either side of the decimal point). Other known strategies for calculation with two-digit whole numbers can be extended to calculation with tenths – using either unitising or knowledge of place value. For example, children already have strategies for adding two two-digit numbers (e.g. *45 + 32*; partitioning one addend); this can be applied to the corresponding calculation with tenths (i.e. *4.5 + 3.2*), as shown opposite. You may wish to include such strategies for efficient mental calculation with tenths at a later stage.	Combining known strategies with unitising in tenths:  $45 + 32 = 77$ $4.5 + 3.2 = 45$ tenths + 32 tenths = 77 tenths = 7.7 Applying known strategies – partitioning numbers with tenths: 
5:5	Extend children's knowledge of column addition and subtraction (see segments *1.20 Algorithms: column addition* and *1.21 Algorithms: column subtraction*) by applying it to adding numbers with tenths. Work through the following key steps; for example, for column addition: • Start with two-digit numbers (ones and tenths) with no regrouping required. • Align the digits correctly, paying attention to the decimal point. • Begin with the least significant digit (now the tenths rather than the units). • Emphasise the alignment of the decimal points in the addends with the decimal point in the sum. • Move to calculations requiring regrouping in the tenths, reminding children that ten tenths is equal to one. • If we have a total of ten or more in any column we need to regroup.	*'Complete the calculations.'* $\begin{array}{r} 3\,.\,2 \\ +\ 5\,.\,7 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 4\,.\,5 \\ +\ 3\,.\,9 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 6\,.\,5 \\ -\ 2\,.\,3 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 4\,.\,3 \\ -\ 1\,.\,7 \\ \hline \\ \hline \end{array}$

<table>
<tr><td></td><td>

- Add the numbers in each column in the most efficient order.

Then repeat for subtraction, beginning with cases where no exchange is required, then extending to include exchange across the decimal point.

Provide children with practice until they are confident with such calculations, before progressing to the next step.</td><td></td></tr>
<tr><td>**5:6**</td><td>Now extend column addition to cases where the addends have different numbers of digits. Then repeat for subtraction calculations where the minuend has more digits than the subtrahend. Provide children with practice, both laying out the column calculations correctly and completing column calculations.</td><td>

- *'Write these as column calculations.'*

6.3 + 1.4
16.3 + 1.4
16.3 + 21.4
16.3 + 1.4 + 12.8

5.6 – 2.1
15.6 – 2.1
15.6 + 12.1

- *'Complete the calculations.'*

$$\begin{array}{r} 13.2 \\ +\ 5.7 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 14.5 \\ +\ 23.9 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 36.5 \\ -\ 2.3 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 34.3 \\ -\ 2.1 \\ \hline \\ \hline \end{array}$$
</td></tr>
<tr><td>**5:7**</td><td>To complete this teaching point, present varied practice, including opportunities for children to apply their understanding of calculating for numbers with tenths in real-world contexts, including measures and statistics, for example:

- *'Dave and Amira are picking cherries.'*
  - *Dave picks 0.5 kg of cherries and Amira picks 0.3 kg of cherries. How much have they picked altogether?* (aggregation)</td><td>Dòng nǎo jīn:

- *'What could the missing numbers be?'*
- *'What could they not be?'*
- *'How do you know?'*

$$\begin{array}{r} 5\,.\,\square \\ +\ 1\,.\,3 \\ \hline 7\,.\,2 \\ \hline \end{array} \qquad \begin{array}{r} 3\,.\,2 \\ +\ \square\,.\,4 \\ \hline 7\,.\,6 \\ \hline \end{array}$$
</td></tr>
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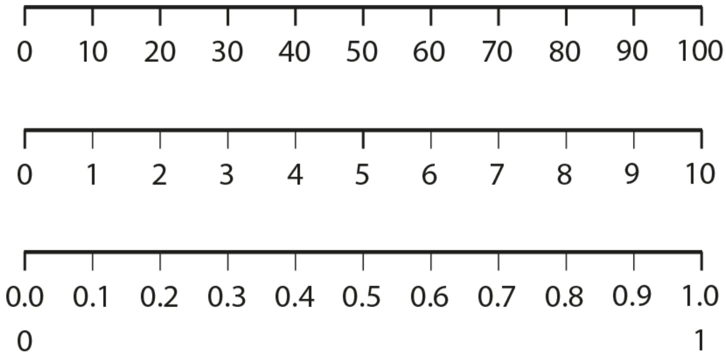
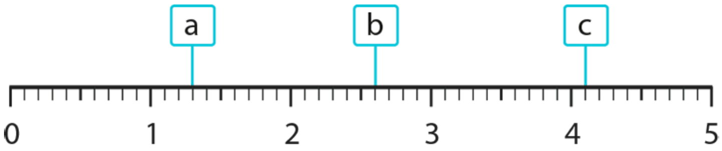
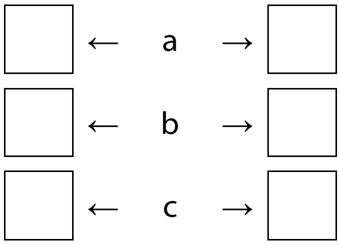
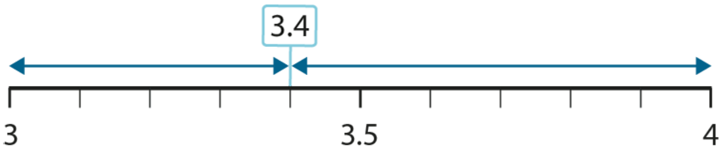
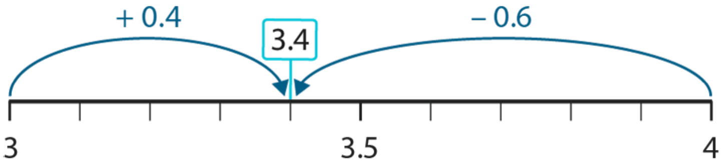
	• *Later, Amira picks another 0.4 kg of cherries. How much do they have now?* (augmentation) • *Felicity had a one litre bottle of water; then she drank 0.2 litres of it. How much does water does Felicity have left?* (reduction) • *Adrian had a 5 litre bottle of water; then he drank 0.2 litres of it. How much water does Adrian have left?* (reduction) • *Nina's journey to school is 5.5 miles. She uses her scooter for 3 miles of the journey. For what distance does she not use her scooter?* (partitioning) • *A dressmaker has 5.2 m of red ribbon and 2.6 m of blue ribbon. How much more red ribbon than blue ribbon does she have?* (difference) • *Sidney the snake was 3.6 m. After one year he was 1.1 m longer. After one more year he was another 0.3 m longer. How long is he now?* (augmentation)	

Teaching point 6:

Numbers with tenths can be rounded to the nearest whole number by examining the value of the tenths digit.

Steps in learning

	Guidance	Representations
6:1	Children learnt the principle of rounding and how to round whole numbers to the nearest 10, 100 and 1,000 in segment 1.22 *Composition and calculation: 1,000 and four-digit numbers.* Now children will need to apply their understanding that tenths are numbers between whole numbers on a number line in order to round to the nearest whole number. To begin, you can gradually 'zoom in' on a number line, showing multiples of ten, then ones, then tenths – this helps children to orient the new piece of learning within their understanding of place value.	
6:2	As a first step in rounding, ask children to identify which whole numbers a given number lies between on the number line; for example, 3.4 lies between three and four. Use the following stem sentences to link to earlier work: • ***'___ is between ___ and ___.'*** • ***'___ is the previous whole number.'*** • ***'___ is the next whole number.'***	Identifying previous and next whole numbers: *'Which whole numbers does 3.4 lie between?'* 3 ← 3.4 → 4 • *'Three-point-four is between three and four'* • *'Three is the previous whole number.'* • *'Four is the next whole number.'*

		Practice problems: *'For each letter on the number line, say which two whole numbers the value is between.'*  
6:3	Once children can locate a number with tenths between the relevant whole numbers, move on to identifying which of these whole numbers the given number is closest to. Initially, make sure that children can identify the half-way point between two whole numbers – without being able to do this they will be unable to easily find which whole number a given number is closest to. Extend the stem sentences from the previous step to include the closest whole number: ***'___ is the closest whole number.'***	 • *'Three-point-four is between three and four'* • *'Three is the previous whole number.'* • *'Four is the next whole number.'* • *'Three is the **closest** whole number.'*
6:4	As a step towards working without the number line, build on children's understanding of complements to a whole number to identify how far a given number is from each whole number. Then, for a range of examples, and still using the number line for support, ask children to identify the closest number.	 3 + 0.4 = 3.4 4 – 0.6 = 3.4

6:5	Now work without the number line, drawing attention to the following points: • When we round to the nearest whole number, we are particularly interested in the tenths digit (the bit that is not a whole number). • What number is half way between consecutive whole numbers? • How far is a particular number from the previous/next whole number? Looking at complements, use variation exercises with missing-number problems, supported by a number line, to help draw out these points. Build on children's previous knowledge of rounding to the nearest ten (see segment *1.22 Composition and calculation: 1,000 and four-digit numbers*) to cover cases with five tenths. Use the generalised statement: ***'If there are five tenths or more round up to the next whole number; if there are fewer than five tenths round down to the previous whole number.'*** Children should memorise this rule.	*'Fill in the missing numbers, and for each bold number, circle the closest whole number.'* **1.1** + [0.9] = 2 **1.6** + [ ] = 2 **1.1** – [0.1] = (1) **1.6** – [ ] = 1 **1.2** + [ ] = 2 **1.7** + [ ] = 2 **1.2** – [ ] = 1 **1.7** – [ ] = 1 **1.3** + [ ] = 2 **1.8** + [ ] = 2 **1.3** – [ ] = 1 **1.8** – [ ] = 1 **1.4** + [ ] = 2 **1.9** + [ ] = 2 **1.4** – [ ] = 1 **1.9** – [ ] = 1 **1.5** + [ ] = 2 **1.5** – [ ] = 1
6:6	Complete this teaching point by providing rounding practice, including practice without the support of a number line (as opposite) and contextual problems, for example: • *'Pandora the python is 4.2 m long. How long is this to the nearest whole metre?'* • *'Gemma the gerbil has mass 34.5 g. How much is this to the nearest gram?'* • *'Samuel the snail moved 0.5 m one morning and another 0.6 m that afternoon. To the nearest whole metre, how far did Samuel go?'*	*'Round each of these numbers to the nearest whole number.'* 2.1 nearest whole number → [ ] 3.5 nearest whole number → [ ] 4.3 nearest whole number → [ ] 2.9 nearest whole number → [ ] 16.2 nearest whole number → [ ] 5.5 nearest whole number → [ ]

		8.7 nearest whole number → ☐ 9.9 nearest whole number → ☐ 12.3 nearest whole number → ☐