Secondary Magazine
Issue 66

National Centre
for Excellence in the
Teaching of Mathematics

Contents

Welcome to Issue 66 of the Secondary Magazine. This is a special edition published during the summer break. First we have an interview with Jon Keating, Professor of Mathematical Physics at the University of Bristol. Then we introduce a collection of five short films about the nature of mathematics. These *Mathematical Ethnographies* resulted from an invitation from Professor Keating to other distinguished professional mathematicians to talk about what they believe characterises real mathematics and mathematical activity. We hope you will find their thoughts inspirational as you seek ways of enabling and encouraging your own students to act mathematically. The next issue will include all the usual features.

Contributors to this issue include: Jon Keating and Mary Pardoe.

From the editor

Welcome to this special *Mathematical Ethnographies* issue of the Secondary Magazine.

In our teaching we aim to find ways of enabling and encouraging our students to 'act mathematically', as mathematicians, rather than doing what can be summarised as 'merely being told facts and passively copying methods that they are shown'. We probably all have various ideas about what it means to 'act mathematically' – but how do actual professional mathematicians themselves describe characteristics of really doing mathematics?

Jon Keating FRS, Professor of Mathematical Physics and Dean of the Faculty of Science at the University of Bristol, whose 'interview' is the first article in this issue, recently conceived the idea of inviting professional mathematicians to share publicly, and informally on film, their thoughts about what they do – about how they 'see' mathematical activity and mathematics itself.

Professor Keating, Chrystal Cherniwchan and Azita Ghassemi, who all work in Bristol University's Department of Mathematics, set about producing a collection of short films that they have called Mathematical Ethnographies. The films focus on personal reflections by professional mathematicians on the processes of exploring and creating mathematics. The mathematicians who we see describing their experiences and communicating their thoughts are currently researching and developing mathematics. They are also teaching the professional mathematicians of the future.

Chrystal Cherniwchan, who is Heilbronn Administration Manager of the department of mathematics and who also has a background in photographical art and design, explains: "The interviews begin from the first spark of childhood interest in maths and cover topics such as the research process, ideas, and that elusive moment when it all comes together. Some people call it the 'Aha! moment'. I wish I could know what it feels like to work on something for years and then finally solve it. If more people thought about that, maybe they would be more interested in mathematics and the sciences."

Azita Ghassemi, research coordinator of SuSTaIn (Statistics underpinning Science, Technology and Industry), adds: "These films have been a year-long labour of love, and have put me in touch with the hidden aspects of what goes on emotionally and psychologically with mathematicians when they seem to be 'preoccupied' in their world. For me, this project brought into the light the humanity of what it means to be a mathematician: their agonies and ecstasies, joys and sorrows, triumphs and disappointments."

In our lessons we want our students to feel what it is like to think and act mathematically. We devise tasks and activities, and reflect on our roles, and our students' roles, and on the learning atmosphere in our classroom. While we are preparing ourselves in these ways it is inspirational to bear in mind some thoughts of mathematicians themselves about mathematics and some of the complex human processes involved in doing it.

The Interview

Name: Jon Keating

Photograph by Chrystal Cherniwchan

About you: I am Professor of Mathematical Physics at the University of Bristol, where I am also currently the Dean of Science. I went to school (the local comprehensive, followed by the local sixth-form college) in Manchester. I read Physics at Oxford University (1982-85), and did a PhD in Theoretical Physics at Bristol University (1985-1989). After a period of postdoctoral research I became a lecturer in Applied Mathematics at Manchester University. Later I returned to Bristol. I have been a professor here since 1997. From 2001-2004 I was Head of the Mathematics Department, and from 2004-2009 I was an EPSRC Senior Research Fellow. My main mathematical interests are in Quantum Chaos (the quantum mechanical properties of systems whose classical dynamics is chaotic), Random Matrix Theory (matrices whose entries are random), and Number Theory (principally, the theory of the Riemann zeta function and its close relatives, the L-functions). What really excites me is exploring the connections and similarities between these subjects.

The most recent use of mathematics in your job was…
I do mathematics every day. When I wake up I usually have mathematics on my mind – some problem I am stuck on, or something someone has told me. Right now I am trying to use Random Matrices to describe the properties of elliptic curves. I am also working on a theory of anyons on quantum networks. In three dimensions and higher, quantum particles are either Fermions or Bosons - this is one of the most important facts about the world. In two dimensions one can have more unusual particle-like objects ('anyons'). These explain some very remarkable phenomena (eg the Fractional Quantum Hall effect). Jonathan Harrison, Jonathan Robbins and I have shown recently that in more exotic (discrete) spaces, one can have much richer anyonic behaviour determined by the topology.

Some mathematics that amazed you is…
the Dirac equation. Dirac wanted to unify quantum mechanics and special relativity. There are many possible equations that do this. Dirac wrote down one on the basis that it seemed mathematically the most natural and beautiful. It appears to be the right one, and led him to understand how

electrons spin and to predict the existence of antimatter. This is just one example of beautiful mathematics turning out to be the best way, as far as we know, of describing nature.

Why mathematics?
Because it is beautiful, and so often surprising.

A significant mathematics-related incident in your life was…
The most significant was finding a general formula for the moments of the Riemann zeta function (numbers that describe the size of the zeta function on the line where the Riemann Hypothesis says its zeros lie) by calculating the corresponding moments of the characteristic polynomials of random matrices. I did this with my then Ph.D. student Nina Snaith. People had been searching for such a formula since Hardy and Littlewood computed the first number in the sequence in 1918. Before our work, people had calculated the first two numbers in the sequence, and had conjectured the third and fourth. We found the general formula using, surprisingly, techniques invented to describe the quantum properties of complex systems. We knew our formula gave the first four numbers that were already known/guessed. Nina and I went to a conference in Vienna where I was to speak about our formula. Just a few minutes before my talk, Brian Conrey, a well-known number theorist, told me that he and Steve Gonek had guessed the fifth number. It took a nervous ten minutes of feverish calculation, with lots of mistakes and corrections along the way, to check that their number (24024/9!) also came out of our formula. When that worked, we knew we were on the right track. That was the most exciting ten minutes of my mathematical life.

A mathematics joke that makes you laugh is…
"You love mathematics more than you love me, don't you?"
"No, of course not – I love you much more."
"Prove it to me!"
"OK… Let *R* be the set of all lovable objects…"

The best book you have ever read is…
Subtle is the Lord - Abraham Pais' biography of Einstein.

Who inspired you?
My PhD advisor (Sir) Michael Berry. He showed me how to think, how to struggle with a problem, how to have fun, and when to move on to the next challenge. He was and remains a great teacher.

If you weren't doing this job you would…
wish that I was.

Mathematical Ethnographies - introduction

In *Mathematical Ethnographies*, five films from the University of Bristol, mathematicians reflect on their personal experiences of engaging in mathematics.

Image from the *Mathematical Ethnographies* Project

In each film the mathematicians explore questions about a particular aspect of mathematical enquiry:

- are mathematicians creative?
- are mathematical proofs beautiful?
- how do mathematicians deal with disappointments and frustrations?
- how does it feel to see the 'light at the end of the tunnel'?
- what is mathematics like? … what is it about? … where is certainty? … is mathematics created or discovered? … what kinds of questions do mathematicians ask? … how and where do they do their thinking? … is the 'path' to a mathematical result as important as the result itself?

The mathematicians talk about what they believe are general characteristics of mathematical research. Although they have a wide variety of different interests, these professional mathematicians describe some kinds of struggles and joys that they have found to be typical of all their mathematical endeavours.

The academically distinguished mathematicians who express in these films their inferences from experience about the nature of mathematics, between them have a great variety and number of research interests.

The mathematicians taking part:
Prof Christoph Andrieu, Prof Sir Michael Berry, Dr Andrew Booker, Dr Li Chen, Dr Isaac Chenchiah, Dr Sean Collins, Prof Brian Conrey, Dr Carl Dettmann, Dr Vanessa Didelez, Prof Jens Eggers, Prof Gilles Francfort, Dr Ayalvadi Ganesh, Dr Alex Gorodnik, Prof Peter Green, Dr Aram Harrow, Dr Oliver Johnson, Dr Thomas Jordan, Prof Jon Keating FRS, Prof Sir John Kingman FRS, Dr Arne Kovac, Dr Cornelia Leitgeb, Prof Hannes Leitgeb, Dr David Leslie, Prof Noah Linden, Dr Tanniemola Liverpool,

Prof Jens Marklof, Prof John McNamara, Dr Francesco Mezzada, Dr Sebastian Muller, Prof Guy Nason, Dr Yuri Netrusov, Dr Jonathan Pila, Prof Elmer Rees, Prof Jeremy Rickard, Dr Tim Riley, Dr Jonathan Robbins, Dr Jonty Rougier, Dr Misha Rudnev, Prof Aidan Schofield, Dr Roman Schubert, Emeritus Prof John Shepherdson (Fellow of the British Academy), Dr Martin Sieber, Dr Valeriy Slastikov, Dr Nina Snaith, Dr Vladislav Tadic, Dr Yves Tourigny, Dr Corina Ulcigrai, Dr Stanislav Volkov, Prof Van H. Vu (winner of the George Polya Prize 2008), Dr Lynne Walling, Prof Philip Welch, Dr Karoline Wiesner, Prof Andreas Winter, Prof Trevor Wooley, Dr Feng Yu, Dr Maria Zaturska.

Are Mathematicians Creative?

Jens Eggers, Professor in Applied Mathematics, fellow of the American Physical Society
Image from the *Mathematical Ethnographies* Project

On the website of the Mathematics Department at the University of Bristol the first film of the *Mathematical Ethnographies* Project is [Are Mathematicians Creative?]

The mathematicians' insights that are shared with us in this film are good starting points for further thought, discussion and debate. For example, which aspects of what your students' do in mathematics lessons are illuminated by the mathematicians' observations about creativity in mathematics?

The following quotations from the film will give you some idea of the variety in the mathematicians' ideas. For the full impact you need to watch the film.

- *A good question I think is worth a hundred good answers. So I would say the deepest creative idea lies in asking the right or the deep questions.*
  Dr Isaac Chenchiah
- *To do good mathematics you have to develop a new insight – calculation everybody can learn, you can even teach a machine to do it very well – in mathematics you have to know what to calculate and what to predict.*
  Prof Van H. Vu
- *When I am working on a piece of physics or mathematics related to it, and I see something appearing and gradually building up, and you put a piece here and a piece there, and it doesn't work, and it does work, and so on, I think more of carpentry, of constructing something, cutting it into shape and polishing it, and so on – I suppose it is creative, of course it is!*
  Prof Sir Michael Berry
- *I always think it's nearer to architecture, perhaps, than to other arts – one is trying to sort of build this formal structure up – and there are supports and girders, and there are connections – so it's the same kind of aesthetic appeal as that of a piece of good architecture.*
  Prof Philip Welch
- *There are mathematicians who maybe build a house which is similar to all the others, but then there are also visionary ones who create a totally new building which has nothing to do with the (existing) buildings – and then they become our model for new buildings in the future – so somehow there are different levels of creativity.*
  Dr Corinna Ulcigrai

- *How can you be a researcher and not be creative? – it's impossible! So it's two things at the same time – it's science, it's rigorous, it's about truth or falsity AND it's artistic, it's creative, it's innovative, it's something new – and that's the great fun of it!*
  Prof Hannes Leitgeb
- *It's hard to see how asking a question which no-one had thought of before is a logical process.*
  Prof Noah Linden
- *You have to conform to precise rules, but having that structure there to constrain you somehow enhances the creativity.*
  Dr Tim Riley
- *A lot of the things that I find I'm doing in statistics and in research is inventing new algorithms, new procedures for new problems, and so therefore I think you need some…some creativity!*
  Dr Arne Kovac
- *I always think of artistic creative talent – you've got some medium to work with, you can do anything with it – you can really conjure up something which doesn't exist, doesn't have to be true – there's something different there for me – it's a much scarier place because there's so much flexibility!*
  Prof Trevor Wooley
- *Part of it is trying to do something that nobody's done before. In that sense you're an adventurer, an explorer, and you get to go somewhere maybe that nobody's been.*
  Prof Brian Conrey
- *We don't reach people immediately, but what we do as mathematicians still has an effect, a very long delayed effect, on the rest of humanity – and we develop things that are then used – it's a creativity that is somehow hidden from view, but it's certainly there!*
  Prof Andreas Winter

You can watch this film on YouTube, or via the University of Bristol's Mathematical Ethnographies page.

Some Mathemapedia entries explore strategies to encourage creativity in mathematics lessons:

- Creativity in Mathematics
- Doing and Undoing, and
- Affordances and Constraints.

Is a Mathematical Proof Beautiful?

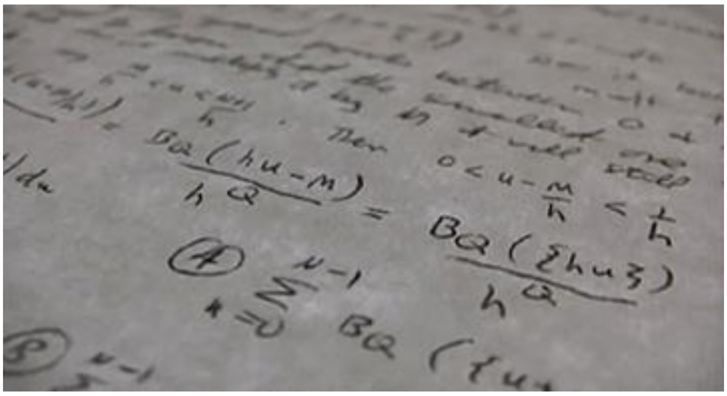

Image from the *Mathematical Ethnographies* Project

Is a Mathematical Proof Beautiful? is the second film of the *Mathematical Ethnographies* Project on the website of the Mathematics Department at the University of Bristol.

Before thinking about the mathematicians' ideas shared in this film, about beauty in mathematical proof, it may be stimulating first to watch this video of an NCETM online debate about teaching and learning proof.

You can download and read the report Looking for Structure, produced by the Proof Materials Project, and mentioned towards the end of the debate.

You can also watch the film, referred to during the debate, of Professor Andrew Wiles proving Fermat's last theorem. It is in five parts on YouTube:

- part 1
- part 2
- part 3
- part 4
- part 5.

Having first thought about the role of proof in school mathematics, and about one specific proof, you will be better prepared to appreciate the mathematicians' views expressed in *Is a Mathematical Proof Beautiful?* The mathematicians' responses to this question include:

- *One of the points about a beautiful proof is that it should be minimalistic – it should be concise – but in many cases the proof is also unexpected.*
  Dr Alex Gorodnik
- *There is beauty particularly if it's a simple proof where you can immediately follow it the first time round – that is obviously a very beautiful proof. If a proof runs to a whole book – well OK it's a proof, but are we going to read a whole book? So that's not beautiful!*
  Dr Maria Zaturska

- *Yes - if it's simple – if it's not very complicated – if one can explain the idea of the proof in a simple way – then it can be beautiful…*
  Dr Martin Sieber
- *…Yes, somehow if you feel why it's true, not only that you have the proof – that it's a logically correct proof – but that you understand why the result is true.*
  Dr Sebastian Muller
- *A proof is a process – you move from one step to another – and the more you can make that process as straight and as sleek as possible with no little deviations the more beautiful it becomes.*
  Dr Nina Snaith
- *The main appeal of mathematics is its beauty, but again, as I hinted at before, I think this beauty derives from the clarity of mathematics. Absolutely! Mathematics to some extent is like an art.*
  Prof Hannes Leitgeb
- *I remember I had once a conversation with some artists about beauty. They asked me what beauty is in mathematics – when do we call a formula beautiful, for instance?* $E = mc^2$ *is a simple formula that everybody knows, and which most people find beautiful who understand it. If you think why you find it beautiful – it contains a lot of information in a very condensed way. And it connects things that were previously unconnected, like energy and mass and the speed of light. It gives a totally new perspective on something.*
  Dr Roman Schubert
- *Still I think that the beauty is more beauty of the idea – so it's the idea or the reasoning, or the way the idea is deduced, or some of the connections which are made and that have their own beauty. So it's not visual beauty of the imagination – it's beautiful thinking, or it's beautiful reasoning.*
  Dr Corinna Ulcigrai
- *I went into a classroom recently and explained the proof of Pythagoras' Theorem to a group of nine year-olds – and I really did the proof. And it's so exceptionally beautiful! It's an idea that when you see it you just think Oh Wow!*
  Prof Jon Keating
- *In mathematics there is a certain inevitability about a good proof. There's a certain feeling that this must be the right way to do it. The great Hungarian mathematician Paul Erdős, whom I knew quite well, used to say that when he went to heaven God would show him the Golden Book, which had the best proofs of all the theorems. And his great ambition in life was that when he went to heaven and saw the Golden Book he'd find one of his own proofs in the book.*
  Prof Sir John Kingman.

You can also reach this second *Mathematical Ethnographies* film via the Mathematics Department of the University of Bristol.

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False Trails

We have a habit in writing articles published in scientific journals to make the work as finished as possible, to cover up all the tracks, to not worry about the blind alleys or describe how you had the wrong idea first, and so on. So there isn't any place to publish, in a dignified manner, what you actually did in order to get to do the work.
Richard Feynman, Nobel Lecture, 1966.

Image from the *Mathematical Ethnographies* Project

The third film of the *Mathematical Ethnographies* Project is False Trails. It focuses on the mathematicians' frustrations and disappointments that are an unavoidable aspect of creative mathematical research.

These are typical observations of the mathematicians who talk in the film. To appreciate the full impact of what they say you must watch the film.

- *Research can be really soul-destroying because you can work for hours or weeks and at the end of the game maybe you've made zero progress. Sometimes you even make negative progress because you discover errors in what's supposedly been proved before.*
  Prof Trevor Wooley
- *…inspiration comes often but there's a lot of work to do to check if it's right. Very often it's wrong – you've spent most of your time wasting your time on going the wrong way!*
  Emeritus Prof John Shepherdson
- *I think what happens is that, as you get older, as you develop as a mathematician, you tend to attack steadily harder problems. So when I was a graduate student I was proving things every other week, and I wanted to solve harder problems. And it went into this routine where I'd be thinking for…the first time I attacked something really, really, quite tough it was a couple of years before I finally solved it – and those two years were fairly nasty – and then it was three years before I finally solved the next one I tried to do, and so on. It got gradually longer until most recently I worked on something for years and years and years – six or seven years – and I haven't solved it!*
  Prof Aidan Schofield

- *I think the worst thing that can happen to you is maybe that you think you've proved something – maybe you've even published it – and then one year later you find a mistake, or even worse somebody else tells you the mistake!*
  Prof Andreas Winter
- *I kind of like the romance of not having it solved. I was rather disappointed when Fermat's last theorem was actually finally proved – because it's really romantic, I think, to have an easily stated problem that has led to all kinds of complicated mathematics being developed – and yet we still can't solve the easily stated problem.*
  Dr Lynne Walling
- *So you're also taking significant risks. You really spend huge chunks of your life on something that might never result in anything.*
  Prof Jens Eggers.

In this film the mathematicians are talking about their feelings in two different kinds of situation – they are reflecting on their experiences of frustration on 'being stuck', and on their experiences of disappointment when they find that they have 'gone up a blind alley'. If you are enabling your students to think mathematically they will experience both of these kinds of situation.

An effective way of rediscovering situations of, and beginning to think more deeply about, 'being stuck', 'going up blind alleys' and other features of the process of thinking mathematically, is to engage in some mathematical activity and reflect on your experiences. You will find excellent practical help and guidance with this in Thinking Mathematically, by John Mason, Leone Burton and Kaye Stacey, Addison-Wesley, 1982, which is an extremely illuminating book to *use* rather than just read.

Hopefully the research mathematicians who contributed to the *False Trails* film would agree with the opening statements of the *Responses to being Stuck* section of *Thinking Mathematically*:

> *Everyone gets stuck. It cannot be avoided, and it should not be hidden. It is an honourable and positive state, from which much can be learned.*

If your students are conjecturing, trying to get a sense of underlying reasons for their conjectures, and then convincing themselves and others that they can justify their arguments, they are bound to 'go up some blind alleys'. The Bristol mathematicians would possibly concur with the following statement in the *Attack: Justifying and Convincing* section of *Thinking Mathematically*:

> *On extremely recalcitrant problems mathematicians take the age-old advice:*
>> *Believe it true on Monday, Wednesday and Friday.*
>> *Believe it false on Tuesday, Thursday and Saturday.*
>> *On Sunday take a neutral stance and look for another approach!*

The Moment of Truth

Image from the *Mathematical Ethnographies* Project

The Moment of Truth is the fourth *Mathematical Ethnographies* film from the Mathematics Department at the University of Bristol.

The mathematicians' descriptions of their experiences when things fall into place include some memorable images. Watch the film!

- *When you start a new problem it's like finding your way around a strange landscape in the dark. At first you're stumbling around in circles getting nowhere, getting back to the same place, slowly perhaps getting some idea of what the country's like. And then suddenly there's a flash of light, and you see there's a very beautiful way, which is the obvious way to go. And if you're lucky, at the end – you've had enough of these flashes of light – the whole thing's absolutely daylight, and you see the clear simple beautiful way right from the beginning to the end. And that's very satisfying.*
  Emeritus Prof John Shepherdson
- *What is the elementary particle of sudden understanding? I call it the clariton. We all recognise it – suddenly 'aha!' you understand something, great. The problem is that there are also anti-claritons that come the next day, and annihilate with the one from yesterday, leaving you nowhere – very frustrated.*
  Prof Sir Michael Berry
- *It all comes together in a very short space of time, and you have a day where you're completely over the moon! Actually by now I've learnt when I've got those days I don't actually look very carefully to make sure that I'm right at that point - because I might be wrong and then it's back to the '… … uuuuuugh!'*
  Prof Aidan Schofield
- *We'd been thinking a lot about a problem, and then one night we were together in an Institute in the centre of Paris, and we figured out how to derive some formula that described the patterns amongst quantum energy levels. And we were so excited by this!*
  Prof Jon Keating
- *An exciting period is when you think you know how you're going to get there. And it's going to take you a while, but you're pretty sure that you've got a grip on this, and you're not going to let go – you're just executing the plan, and, yes, as you're executing it, you know, it's yes, yes,*

yes, you were right - this is the way! And then you say 'Aaaah!, this is wonderful, now I've reached …!'. But that lasts a very short time, and then you have to start again!
Dr Yves Tourigny

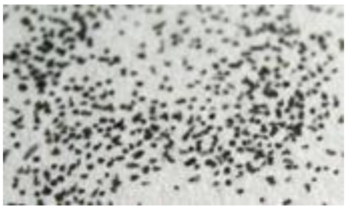
- *It's a wonderful feeling when you finally crack a problem - or you finally realise that you're on the track, and the rest is perhaps rather routine working out – that's a lovely feeling – it's one of the best feelings in life. It doesn't come often, but it does come.*
  Prof Sir John Kingman
- *Most of what we do is filled with wrong turns and dead ends and things that don't work out and ideas that just aren't right. Just every once in a while you get that thing that does work – and it's just brilliant when it happens – you get hooked on that!*
  Prof Brian Conway
- *Imagine there's something you've been working on for five years – the day comes around when you understand it – just for these great moments it's worth doing mathematics!*
  Prof Jens Eggers
- *Everything becomes clear – that's really that sensation – that it's 'Of course! It is that simple!' – it's like going from something where everything is fog, and suddenly – blue sky and sun!*
  Prof Christoph Andrieu.

Some observations above are close to aspects of what John Mason writes about expectation in Learning and Doing Mathematics:

> *It is not often possible to master anything on first exposure. It is a bit like being on board a ship, pulling into a harbour in the fog. A distant foghorn is heard intermittently, but nothing can be seen through the fog. Gradually, the foghorn gets louder, and vague shapes appear in the mist. Finally, the ship pulls up to the dock, and I can get onto firm ground.*

The Mathemapedia entry Creativity in mathematics mentions 'Aha!' moments.

You will find some provocative general statements about 'Aha!' moments, that might generate light-hearted discussion, in an article in the New York Times – the journalist, Janet Rae-Dupree, who, as technology editor of the Silicon Valley/San Jose Business Journal, has been covering technology and science in Silicon Valley since 1994, writes about Reassessing the Aha moment.



Some Random Thoughts

The fifth film of the *Mathematical Ethnographies* Project is Random Thoughts. The mathematicians share with us some thoughts about the nature of mathematics.

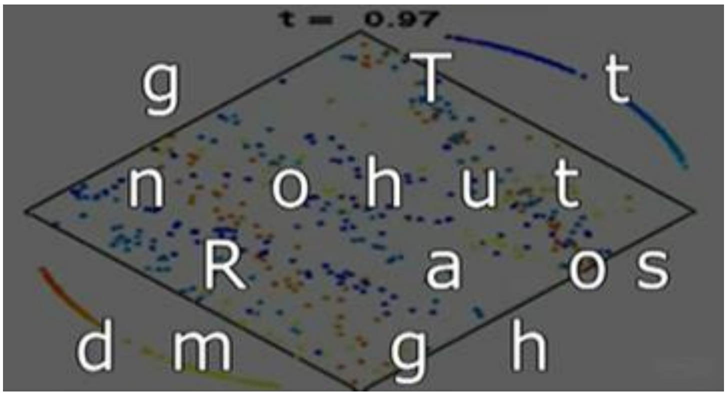


Image from the *Mathematical Ethnographies* Project

The following quotations from the film are intended to give a mere glimpse at the mathematicians' ideas. For the full impact you need to watch the film.

- *It's not so much thinking about thinking the same thing over and over again; it's – what's a new way of attacking this? What's a new thing I can bring to bear onto this problem? – so I can really get to understand what it's telling me.*
  Prof Guy Nason
- *I guess there's an argument about whether theorems are there and you're waiting to discover them, or whether you're creating them in the way that you're creating a painting. I would say more that it's a case of discovering what's true than it is creating something out of nothing – you can't write down a beautiful result and then expect that to be true. It's got to be allied to what is possible to take place within the laws of nature.*
  Dr Oliver Johnson
- *I think it's mostly discovery. I think in some sense something is either true, or it's not, and we just haven't discovered whether it is or not. On the other hand there are lots of ways of saying the same thing, and when you choose something you're making a creative choice to say OK I like this way of looking at it and not that way.*
  Dr Carl Dettmann
- *It would be unfair to say that mathematics is a-historical – it's not, but art is much more historical. The kind of art you do depends on – you can't say art would have developed the same way everywhere in the universe, or even in every culture, it just followed different pathways. Mathematics I think, we don't have comparison, but I think would have followed roughly similar trajectories anywhere in the universe.*
  Dr Ayalvadi Ganesh
- *For a mathematician the result is not as important as actually the path which leads to it. There are lots of things which we believe are true from some sort of gut feeling or computer simulations for example, but that doesn't satisfy again the mathematician. The only interest is actually how you can – the path leading to it.*
  Dr Stanislav Volkov

- *It's actually surprisingly common that you find that in some sense you've been trying to prove something too easy. You realise that there's a much more general statement that ought to be true – that implies what you're trying to prove, but implies a lot more. And once you understand what the most general statement is that ought to be true it often becomes a lot more obvious why it ought to be true.*
  Prof Jeremy Rickard
- *When I manage to turn their fear into some sort of love towards maths then I have succeeded!*
  Dr Cornelia Leitgeb.