The NCETM Podcast Episode 73

Using algebra tiles at KS3

Hello and welcome to this latest episode of the NCETM Maths Podcast. I'm Beth Goodliff [BG], the NCETM Senior Comms Manager, and I'm here today with Lizi [LP] and Becky [BD], and we're going to learn more about using algebra tiles for secondary maths lessons. I'm going to get Lizi and Becky to introduce themselves, and then we're going to hear all about how Lizi uses them in her lessons in her school. And hopefully you'll be able to take away from this podcast some practical tips that you'll be able to use in your lessons, too. So, Lizi, if you'd like to tell us a little bit about yourself.

LP: Hi, I'm Lizi. I am a maths teacher and a Key Stage 3 lead at Tytherington in Macclesfield.

I've been teaching roughly ten years and I'm just completing my first year as an NCETM Mastery Specialist, which has been a really amazing journey so far.

BG: Thank you. Becky will be familiar to some of our listeners, but Becky, if you could just introduce yourself.

BD: Sure. I'm Becky Donaldson. I work as one of the Assistant Directors for Secondary in the NCETM. Before I joined the team, just over two years ago, I was a maths MAT lead in Bristol and before that, I was a head of maths in London.

BG: Excellent. Thank you both very much. Over to you.

BD: So, Lizi, I'd just like to start with the big picture: why use algebra tiles? What is it that you find you and your students get out of them?

LP: Algebra is such an abstract concept for students to grapple with. I find using the manipulatives provides a really helpful scaffold not just for students that maybe struggle with it, but grappling with algebra as a concept and the full journey of algebra. It's really helpful to have something concrete and visual for students to relate the underlying structure of algebra and algebraic manipulation to.

It provides quite a good journey as well through from concrete manipulatives with the tiles, up to visuals and pictures that you can then see, and students have these images in their mind, and they can then deal with the more abstract things without the use of those diagrams, those visuals or those manipulatives. I find that if you try and teach algebra in a procedural way, the lack of understanding can lead to misconceptions and poor memorisation or poor memory of the skills moving forward.

I've found it's been really powerful to give them the tiles and ask them to rearrange them, and then they can see why the procedure comes about, or they can generalise the procedure for themselves, and that's much more powerful than me telling them how to do it.

Just having a set of steps to follow because they understand the steps and why they do what they do. It's always there to fall back on as well when you're going to do something a bit more complex, you can always go back to this.

Students are inevitably going to forget, because that's what humans do, so you can show them the visual or you can use the tiles, and normally it's just a little bit of a nudge and they can go back to the abstract again later on. So I found they transform my teaching of algebra for that reason.

BD: Amazing. You talked about a journey there. I'm really intrigued to know where you start with algebra tiles - which year group? Which topic? What is the introduction that your students get to this manipulative?

LP: I think the journey is a big part of using algebra tiles. In the past I've tried to do completing the square with Year 12, and they've never seen algebra tiles before. It's quite an abstract thing, completing the square, and the

tiles are quite abstract when you've never met them before and it didn't go down so well, so planning the journey back from that point is really important as a curriculum leader, so as Key Stage 3 leader, I'm thinking Year 7, when they're introduced to algebra.

In the first instance, getting them used to the tiles and maybe collecting like terms and then expanding and factorising, which I try and teach at the same time and to emphasise that it's equivalence, it's different ways of writing the same thing.

We do that in Year 7 and then in Year 8 you can maybe move on to expanding and factorising double brackets. Again, it's an equivalence thing, and Year 9 - maybe Year 10, depending on the class - completing the square, and that was the game-changer for me, where I thought, I can see the power in this because it is so procedural if you just teach them to halve the coefficient of *x*. Whereas if they've got the tiles in front of them, they can tell you and they can see why they've got to halve the coefficient of *x*. But, yeah, if you just start with the algebra tiles for completing the square, you're not going to get the best out of it, and I think it is important that you weave it into their journey through school and through different topics. And it's a nice common thread for them to have throughout their learning journey.

BD: I completely relate to that completing the square moment: I had that moment embarrassingly late in my teaching career. I thought 'Oh, this is why it works' because someone showed me with some algebra tiles. I can well imagine the light bulb moment that you get in your classroom.

You talk there about it being something they're really used to all the way through Years 7, 8 and 9.

I know some teachers might be less familiar with giving kids stuff in the lesson and the students actually having the manipulatives - is it something that you give them, or is it something that you demonstrate on the board? How do you make it work in your case on a practical level?

LP: For me it's been a journey as well - sorry, over-use of the word 'journey'! I started with just visuals because we didn't have any algebra tiles in the department. So I used MathsBot and even made some out of paper initially to see if it was going to work, because they are an investment, I would say, if you're buying the actual tiles, but in my opinion completely worth it.

But I started with paper and visuals initially, depending on the class size. I had a nice small Year 9 class and I did give them the paper to play with. Initially it was very visual, but since we've been on this mastery journey, we've invested in some algebra tiles and I give them out, a set each, so they've all got something to manipulate in front of them.

I often try to do it under the visualiser as well if I can: sometimes I'll write a question in whiteboard pen on my desk, go to the visualiser and show them how to do it. Initially I couldn't because I didn't have the resources there, so it's been only maybe the last three or four months, and I've been trying it with all of my classes because I'm like a kid with a new toy who wants to have a go with them in all my lessons.

I've learned a lot about how to use them effectively: there is the novelty aspect, as I'm sure you can imagine, but the tiles are really quite inoffensive - there's not a lot that they can do with the tiles. They might try and build them up but if you have the expectations of what you want them to use them for and if you make that clear and if you build it in as a part of the maths curriculum from Year 7, I don't foresee it being too much of an issue.

But kids will be kids at the end of the day, so it might even be worth giving them out and saying 'Right go nuts for two minutes, and then I'm going to trust you after that. You tried to build a tower, now the novelty is over let's do some maths with them.'

I find they're really helpful for the kids, because they can manipulate them, they can move them around and you don't get that same thing with paper - it's really hard to move around. You don't get the same thing when you're doing it with visuals on the board because then it's just you doing it. They don't physically get to do it themselves, so they are really powerful.

BD: I don't know if you can elaborate on that power a little bit more, and talk a little bit about what you've seen change in your students' understanding since you started using algebra tiles?

What have you been looking for in the classroom to evidence to yourself, that you know this is having an impact, that this is really working?

LP: There was a really nice moment with my Year 9s recently, where part of the point of using the algebra tiles is that they're not going to be needed forever. It is their first step in the journey of algebraic manipulation. You want them to be able to do it without because they won't have algebra tiles in their exam.

When my Year 9s have been doing it recently they reach a point where they're ready to move on before you tell them to, and sometimes you have to hold them back a bit and try and keep them using them a little bit longer, just to really get the concrete understanding embedded before they run with it.

And when we were recapping doing expanding, one of the students said to me – and I could see it was how he saw it, not using the tiles even though they were in front of him – 'Well, there's going to be a row of three x's and there's going to be three rows of 3, so it's going to be $3x + 9$'. That was really powerful for me because that means essentially the tiles are no longer necessary for that bit of his maths, but he can still see it in his head and therefore he can do the process in the abstract now, because he doesn't need the manipulative: that was really powerful.

BD: The representation is really embedded in his head, really, isn't it? That's there for him to access when he needs, not just when the stuff is on the table.

LP: Yeah, it's a mental 'visual' that he has there and that's how he's seeing the algebra now rather than as an abstract bunch of letters and numbers.

BD: Amazing. I wonder if you could just talk us briefly through a learning sequence that you use with algebra tiles and either one of the ones that you mentioned that's powerful, like the square, or one of the earlier ones. What is the sequence of steps that you go through with students in the lesson?

LP: The one that I'm probably more competent with because I've done it the most would be expanding and factorising. Like I said, I don't teach them necessarily as two separate things. In the first instance, it's about, how do you see this? Do you see it another way? Can you build this? Can you share this? Can you arrange it into a rectangle? The expanding and factorising is all based around arranging the tiles into a rectangle and the dimensions of the rectangle for the area would be the factorised form of the expression, and then the easier one for them to see is the area in terms of what the tiles are. So there's three x's and there's nine 1s, so it's $3x + 9$. They see that easier. So with the factorising, it's getting them to see the dimensions. For me, I'd start with some factorised expressions – 'How else could you see this? Let's build it. Can everyone see how these tiles represent that expression? What else? What other expressions can you see in there?' And then giving them the diagram or something that I've built and saying, well, what can you see with this? A lot of mini-whiteboard work. 'If you see it one way, can you see it in another way?'

And then you give them a factorised or expanded version - 'Can you build that? How? How else do you see it? Can you write it in a different way?', really embedding the fact that they are the same, it's equivalence and a lot of diagrams and varying the bits that are missing so that they're filling in different bits.

Eventually, they do become redundant when you're expanding and factorising because the tiles are limited as you've only got an x^2 and an x and a 1, so eventually you will have to take from that the process that you are applying to build up.

But I do find if it's too much of a leap for students to go straight from $3(x + 2)$ to multiply by three essentially, then you can do an area model and you can have the tiles and an area model side by side – 'How are these the same? How are they different? Can you see that they show the same thing?' And then you can even link that to the grid method afterwards, so there's always some kind of progression and scaffold away from the tiles, so it's not just 'tiles maths', there are some steps to other representations that can link really well to the tiles.

This would be maybe a week of lessons exploring the equivalence between factorised, expanded and lots of work with the tiles, lots of diagrams, and then trying to see where I need to take it from there, depending on the class. Do I need to introduce another scaffold that's not the tiles, but will help them get there? Or they're there already depending on the class, but then you do have to plan that exit from using the manipulatives, you have to plan what happens next and get away from them somehow. You maybe have a week afterwards of doing expanding and factorising where there are no tiles because they need to be able to do it without and they need to use what they've learned in different contexts. So when they've got $xy + 3x$ the tiles won't work for that question. So, what do we know about what we've done with the tiles that we can apply to those types of question?

BD: Yeah, that makes sense. It sounds that there's a real emphasis there on equivalence rather than procedures. You're spending a lot of time really dwelling on that because you could very easily do expanding and factorising super quick - here's a procedure and go - but it's worth investing that time in exploring equivalence first. How much do you find you have to plan for the abstract, and how much does the procedure fall out of playing around with the tiles? How much do the students 'get there' on their own?

LP: It depends on the class I think, and some students really want the quick wins, don't they? They want the quick procedure, and they'll try and shoehorn it in before they're ready, and you almost have to slow them down a little bit and say, hang on, keep using the tiles and we'll see if that always works.

You have to facilitate the discussion about the procedure, I would say, and guide them to seeing the procedure and how the procedure links to the tiles and what they've been doing, and it's even like sometimes alongside one another, on one side having the structure, on the other side having the abstract, and then can we all see how these two are the same?

BD: So planning ahead with that exit, like you were saying earlier, from the manipulatives, and you touched on a limitation there, where you have two unknowns like xy the algebra tiles don't really prepare students for that. Are there any other limitations that you'd warn teachers to think about if they're finalising their own lessons?

LP: Oh yeah, there's quite a few. We've only got x^2, x and 1, and the other thing is about area for factorising when there's negatives, that's assuming that you can have a negative length, which is a difficult concept for students to get their head around.

Also the fact that the x length is supposed to be an unknown and it is fixed on the tile. You see them, as soon as you hand them out, try and put the 1s next to the x and they see it's not quite 5, but it's not quite 6.

They're designed so they're not exactly the same length as any number of ones, but there is still that misconception that they might think that x is somewhere between 5 and 6, because I can't quite fit 5, and 6 is too many, so it must be between 5 and 6, and almost immediately that's what most kids will do. They'll try and work out what x is by putting them next to.

The other issue I've seen - and I've not done it myself, but I've seen other people do - where they use the tiles to try and create the right lengths, so they put an x tile there to get the right dimension, or the right tile in the right space, and they might leave it there - it's almost like a title. It's hard to explain, but there's that misconception that that's part of the answer, when it's actually there to help create the diagram. So I've seen that can be a bit of an issue.

BD: I've seen something similar - it's a bit like when you're setting up a grid method, and you're writing the numbers on the outside. You put the tiles there to be the numbers and it could be part of the solution rather than part of the question. So I guess that's something for teachers to think about, how moving from the manipulative to the abstract, how you lay it out, there are decisions you can make there that could potentially put misconceptions in the students' way.

You touched upon negatives there, and I read your blog last night where you talked about the connection between them - because they're double-sided, they're red on the one side - between the negative algebra tiles, but also between double-sided counters for negative numbers, and I wondered if you could expand a bit more on the link that you've made between those with your students.

LP: Before I started mastery development I was frustrated by how difficult students find negative numbers, and I was constantly trying to find something that made negative numbers more accessible for students. When I found the double-sided counters, that's initially how I started really making a difference with how I was teaching directed number.

They have very similar features to the *1* tiles in an algebra tile set, so there is a bit of consistency there - because the counter is red on one side, which is negative and yellow on one side, which is positive.

Then you have zero pairs where a positive counter and a negative counter sum to zero, and they're really useful in doing calculations with directed number and negative numbers.

And the connection - how often have you been teaching algebra and the algebra is not the problem, it's the directed number.

So as soon as I started teaching with the algebra tiles, I started seeing that there was a connection with the directed number and the double-sided counters. It was really powerful - I didn't even realise and so I started playing around with some tiles that I could have a zero pair that was a positive *x* and a negative *x*.

And I thought, of course: they're just more zero pairs - that's really powerful if you do the legwork with negative numbers and the directed counters, it really follows on nicely into the algebra as well. It's still not ideal - I've still got a lot of work to do on how to be better at teaching this and making it more memorable for my students. But it is incredible the difference, how their journey is impacting other areas of maths and I suppose that's what we want to see as mathematicians, that they can make the connections between different topics.

This was a really nice moment for me, nine years into teaching, saying ‘Oh, zero pairs are not just 1 and -1 are they?’

So yes, that's something that really struck me during playing around with these because before I started trying to teach with the algebra tiles, I was a novice. I read bits and I had a little bit of training with the NCETM, but the biggest learning journey for me has just been doing it with the kids and talking to them when they're using the tiles. A big takeaway message from me for anybody that's listening is to give it a go, because it is amazing and you learn more by doing it than you do by trying to research it or watch it or listen to it.

BD: That's a really nice moment to start coming back to the journey there, because it's not just been about the students’ journey in algebra, is it? You're talking about the connection with number and other aspects of the curriculum there, but also that you as a teacher, a decade in are still on a journey with this and learning. So thank you. I've really enjoyed hearing how you've been puzzling this out and making it work your students.

LP: Thank you. I've enjoyed the conversation.

BG: Thank you both very much indeed. It's been absolutely fascinating to hear about that. I'm sure our listeners to this podcast episode will have absolutely loads to take away. If people want to know more about using algebra tiles, Lizi has a blog, and there are also features and resources on the NCETM website about using algebra tiles, about completing the square, and about using other manipulatives with students in Key Stage 3. Thank you for listening and we'll have another podcast episode for you again soon.