MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.15 Division: partitioning leading to short division

Teacher guide | Year 4

Teaching point 1:

Any two-digit number can be divided by a single-digit number, by partitioning the two-digit number into tens and ones, dividing the parts by the single-digit number, then adding the partial quotients; if dividing the tens gives a remainder of one or more tens, we must exchange the remaining tens for ones before dividing the resulting ones value by the single-digit number.

Teaching point 2:

Any two-digit number can be divided by a single-digit number using an algorithm called '*short division*'; the algorithm is applied working from the most significant digit (on the left) to the least significant digit (on the right); if there is a remainder in the tens column, we must '*exchange*'.

Teaching point 3:

Any three-digit number can be divided by a single-digit number, by partitioning the two-digit number into hundreds, tens and ones, dividing the parts by the single-digit number, then adding the partial quotients; if dividing the hundreds gives a remainder of one or more hundreds, we must exchange the remaining hundreds for tens before dividing the resulting tens value by the single-digit number.

Teaching point 4:

Any three-digit number can be divided by a single-digit number using the short-division algorithm.

Overview of learning

In this segment children will:

- use informal written methods and unitising language, initially supported by the use of base-ten equipment, to divide two-digit dividends by single-digit divisors, where each digit of the dividend is a multiple of the divisor; for example:

  84 ÷ 4 = ?

  8 tens ÷ 4 = 2 tens
  4 ones ÷ 4 = 1 one

8 tens	÷	4	=	2 tens
4 ones	÷	4	=	1 one
84	÷	4	=	21

- extend the use of informal written methods to understand the origin of exchange of tens for ones for cases:
  - *without* an overall remainder; for example:

  72 ÷ 3 = ?

  7 tens ÷ 3 = 2 tens r 1 ten
  1 ten and 2 ones = 12 ones
  12 ones ÷ 3 = 4 ones

6 tens	÷	3	=	2 tens
12 ones	÷	3	=	4 ones
72	÷	3	=	24

  - *with* an overall remainder; for example:

  73 ÷ 3 = ?

  7 tens ÷ 3 = 2 tens r 1 ten
  1 ten and 3 ones = 13 ones
  13 ones ÷ 3 = 4 ones r 1 one

6 tens	÷	3	=	2 tens
13 ones	÷	3	=	4 ones r 1 one
73	÷	3	=	24 r 1

- learn to apply the short-division algorithm for each of the above examples, supported by base-ten equipment, unitising language and comparison with the informal written method, in order to understand the structure of the algorithm
- extend the informal methods to *three-digit* ÷ *single-digit* calculations, supported initially by the use of place-value counters and unitising language
- extend the short-division algorithm to *three-digit* ÷ *single-digit* calculations, including cases where the hundreds digit of the dividend is smaller than the divisor (e.g. *215* ÷ *5*).

Note that base-ten equipment should be used to expose *structure* rather than as a tool for *calculation*, and, throughout, children should be encouraged to use known multiplication facts.

For children to succeed with this segment, it is important for them to have already mastered:

- partitioning two- and three-digit numbers according to place value (*Spine 1: Number, Addition and Subtraction*, segments *1.9* and *1.18*)
- division, by skip counting according to the divisor or using known multiplication facts, for both the quotitive and partitive division structures (segment *2.6 Structures: quotitive and partitive division*)
- the origin and representation of remainders in division, and how to interpret remainders based on context (segment *2.12 Division with remainders*).

Throughout this segment, the partitive structure of division is used to develop children's understanding of the informal methods and the short-division algorithm. As in segment *2.6*, a skip-counting approach is used initially to give children a deeper understanding of the structure. Now we skip count in a *multiple* of the divisor (rather than in the divisor itself), using unitising language, and progressing to use of known multiplication facts with unitising:

'Eighty-four sticks are shared equally between four children. How many sticks does each child get?'

- *'Four tens is one ten each. That's forty.'*
- *'Eight tens is two tens each. That's eighty.'*
- *'Eight tens divided between four is equal to two tens each.'*

- *'Four ones is one each. That's four.'*
- *'Four ones divided between four is equal to one one each.'*

As implied above, the use of unitising language is a key feature of this segment (as it was in segment *2.14 Multiplication: partitioning leading to short multiplication*). Teachers should ensure that this language is used correctly and consistently; for example, when children verbalise the following calculation:

$$\begin{array}{r} 2\quad 4 \\ 3\overline{)\,7\quad {}^{1}2} \end{array}$$

they should say:

- *'Seven tens divided by three is equal to two tens remainder one ten…'*

not:

- *'Seventy divided by three is equal to sixty remainder ten…'*

Later, children can begin to simplify their language, but for now the unitising language ensures a deeper understanding of the mathematics underpinning the algorithm.

As with all formal methods taught so far (column addition and subtraction, and short multiplication), children should not 'discard' all previous strategies; they should not consider the short-division algorithm as the only resort for *two-digit ÷ single-digit* and *three-digit ÷ single-digit* calculations, rather, they should be encouraged to make sensible decisions about which is the most efficient strategy for a particular caclulation. Beyond this segment, teachers should make a continuous effort to ensure that children approach each calculation with an attitude of enquiry and flexibility.

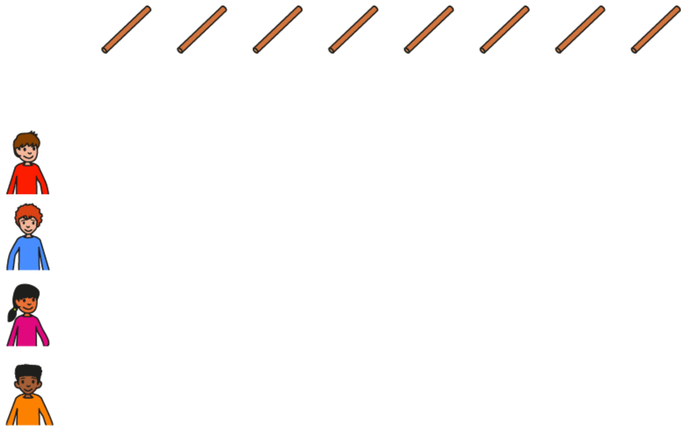
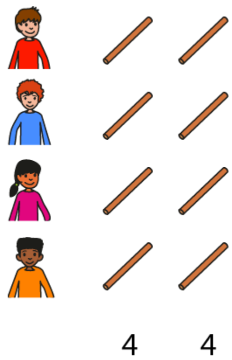
A misconception, for both teachers and children, is that short division can only be used for *single-digit* divisors. In segment *2.24 Division: dividing by two-digit divisors*, children will encounter formal methods for dividing by two-digit divisors, but it should be noted that the short-division algorithm is a valid method that also works. The two approaches are discussed further, and compared, in segment *2.24*.

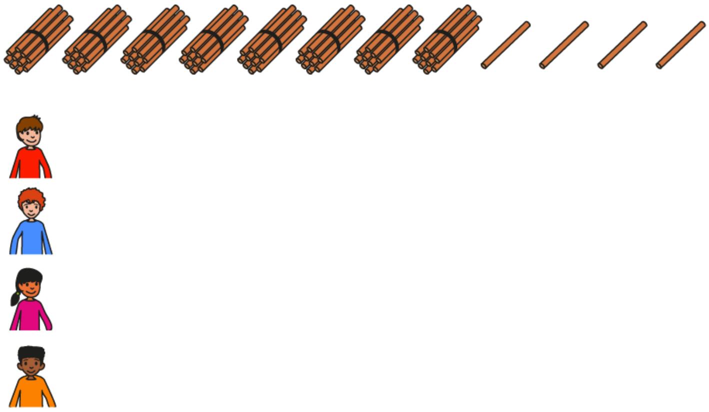
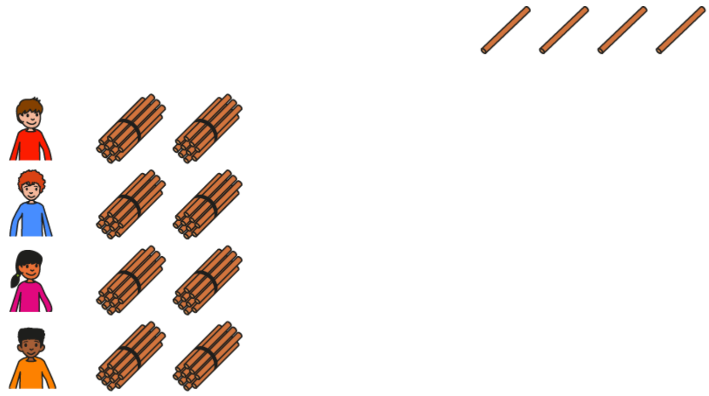
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Any two-digit number can be divided by a single-digit number, by partitioning the two-digit number into tens and ones, dividing the parts by the single-digit number, then adding the partial quotients; if dividing the tens gives a remainder of one or more tens, we must exchange the remaining tens for ones before dividing the resulting ones value by the single-digit number.

Steps in learning

	Guidance	Representations
1:1	In this teaching point, children will apply their understanding of partitioning two-digit numbers into tens and ones, to divide two-digit numbers by single-digit numbers. Children will continue to use unitising language in a similar way to segment *2.14 Multiplication: partitioning leading to short multiplication.* Before beginning, briefly review how skip counting according to the divisor can be used to solve a partitive division problem such as *'Eight sticks are shared equally between four children. How many sticks does each child get?'* Demonstrate how skip counting in the divisor represents repeatedly distributing a quantity equal to the divisor across the 'sharees', reminding children of the language used in segment *2.6 Structures: quotitive and partitive division*, as exemplified opposite.	Presenting the problem: *'Eight sticks are shared equally between four children. How many sticks does each child get?'* $8 \div 4 = ?$  Skip counting according to the divisor:  • *'One four is one each. That's four.'* • *'Two fours is two each. That's eight.'* $8 \div 4 = 2$ • *'Eight divided between four is equal to two each.'* • *'Each child gets two sticks.'*

1:2	Now present a *two-digit ÷ single-digit* partitive division context, for which each digit of the two-digit number is divisible (to give a whole number) by the dividend; for example, *'Eighty-four sticks are shared equally between four children. How many sticks does each child get?'* Working practically or pictorially: • gather 84 sticks as eight bundles of ten sticks and four individual sticks • then model sharing out the eight bundles of sticks, four bundles at a time, now using unitising language to describe how many tens are being distributed; record the resulting number of ten sticks each child gets • then model sharing out the remaining four sticks, distributing them all in one go, as in step *1:1*; record the resulting number of individual sticks each child gets • demonstrate that the total number of sticks that each child gets is the sum of the two partial quotients. As you share out the bundles of ten sticks, emphasise how we are skip counting in a multiple of the divisor (for now this is ten times the divisor, unitising in tens); you can do this by emphasising the 'four' and the 'eight' in the counting sequence. Work through several examples for which each digit is a multiple of the divisor (e.g. *96 ÷ 3* and *46 ÷ 2*) until children are confident with the language and method.	Presenting the problem: *'<u>Eighty-four</u> sticks are shared equally between <u>four</u> children. How many sticks does each child get?'* 84 ÷ 4 = ?  Skip counting according to a multiple of the divisor – sharing the tens:  • *'<u>Four</u> tens are one ten each. That's forty.'* • *'<u>Eight</u> tens are two tens each. That's eighty.'* 8 tens ÷ 4 = 2 tens • *'Eight tens divided between four is equal to two tens each.'*

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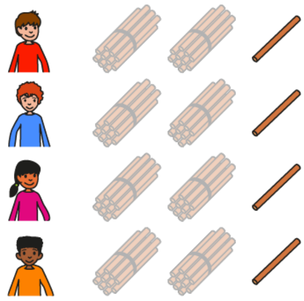
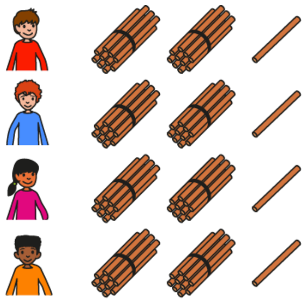
Skip counting according to a multiple of the divisor – sharing the ones:

• *'Four ones is one one each. That's four.'*
4 ones ÷ 4 = 1 one
• *'Four ones divided between four is equal to one one each.'*

Adding the partial quotients:

<table>
<tr><td>8 tens</td><td>÷</td><td>4</td><td>=</td><td>2 tens</td></tr>
<tr><td>4 ones</td><td>÷</td><td>4</td><td>=</td><td>1 one</td></tr>
<tr><td>84</td><td>÷</td><td>4</td><td>=</td><td>21</td></tr>
</table>
• *'Eight tens and four ones divided between four is equal to two tens and one one.'*
• *'Each child gets twenty-one sticks.'*
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<td>**1:3**</td>
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Before working through the next *two-digit ÷ single-digit* calculation, you may wish to briefly revisit division with a remainder using smaller numbers; for example *'Seven sticks are shared equally between three children. How many sticks does each child get?'*

Remind children that we skip count in the divisor, until we can't distribute another set of one each, and that we can express the left-over sticks in the division equation as a remainder:
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- *'One three is one each. That's three.'*
- *'Two threes are two each. That's six.'*
- *'There is one stick left over.'*

  7 ÷ 3 = 2 r 1

- *'Seven divided between three is equal to two each, with a remainder of one.'*
- *'So, the children get two sticks each; there is one stick left over.'*

(For more guidance see segment *2.12 Division with remainders*, step *1:6*.)

Now consider a *two-digit ÷ single-digit* example in which:

- the tens digit *isn't* divisible (to give a whole number) by the divisor
- the two-digit number *is* divisible by the divisor;

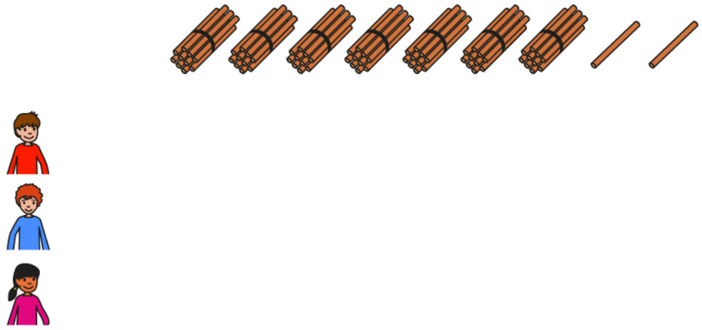
for example, '*Seventy-two sticks are shared equally between three children. How many sticks does each child get?'*

Work through the problem in the same way as in step *1:2*, now emphasising the remainder of one ten after dividing the tens, and demonstrating unbundling the left-over ten sticks and combining them with the existing ones before dividing the ones.

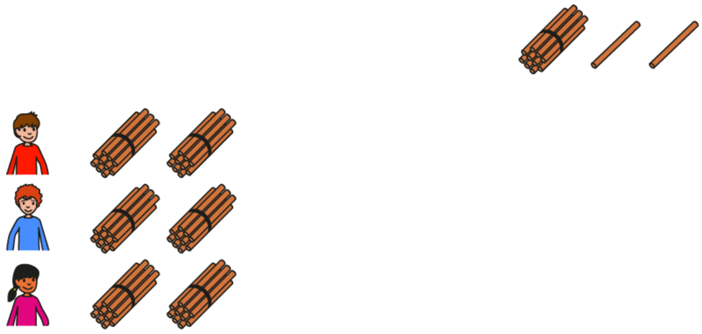
Presenting the problem:

'Seventy-two sticks are shared equally between three children. How many sticks does each child get?'

72 ÷ 3 = ?



Skip counting according to a multiple of the divisor – sharing the tens:



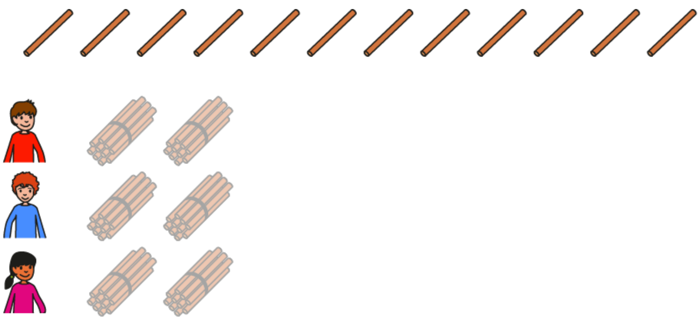
- *'Three tens are one ten each. That's thirty.'*
- *'Six tens are two tens each. That's sixty.'*
- *'There is one ten left over.'*

  7 tens ÷ 3 = 2 tens r 1 ten

- *'Seven tens divided between three is equal to two tens each, with a remainder of one ten.'*

Unbundling the remaining ten:

- *'One ten is equal to ten ones.'*



- *'So, one ten and two ones are equal to twelve ones.'*

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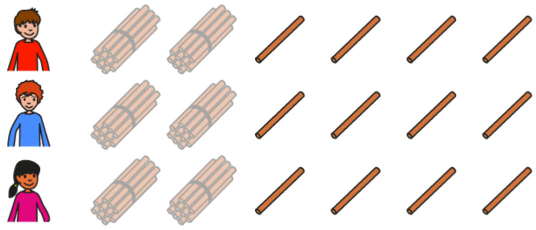
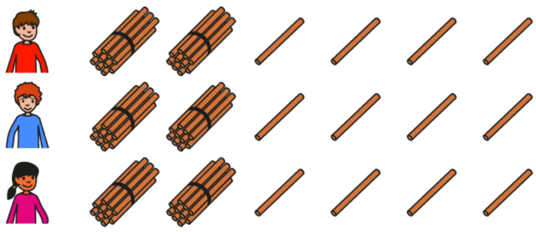
Skip counting according to a multiple of the divisor – sharing the ones:

• <u>'Three</u> ones are one each. That's three.'
• <u>'Six</u> ones are two each. That's six.'
• <u>'Nine</u> ones are three each. That's nine.'
• <u>'Twelve</u> ones are four each. That's twelve.'
12 ones ÷ 3 = 4 ones
• 'Twelve ones divided between three is equal to four ones each.'

Adding the partial quotients:

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<tr><td>6 tens</td><td>÷</td><td>3</td><td>=</td><td>2 tens</td></tr>
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<tr><td>72</td><td>÷</td><td>3</td><td>=</td><td>24</td></tr>
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• 'Each child gets twenty-four sticks.'
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<td>1:4</td>
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Work through several more examples for which:
• the tens digit isn't divisible (to give a whole number) by the divisor
• the two-digit number is divisible by the divisor.
You can use other place-value equipment, such as Dienes or place-value counters, to represent the dividend. Now, instead of unbundling the sticks, draw attention to exchanging each of the left-over tens for ten ones. Include examples, such as that shown on the next page, for which
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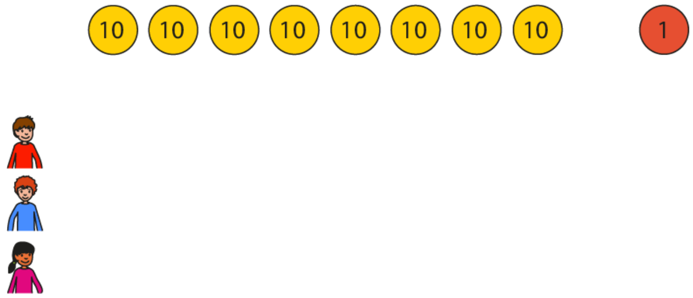
<td>more than one ten is left over.

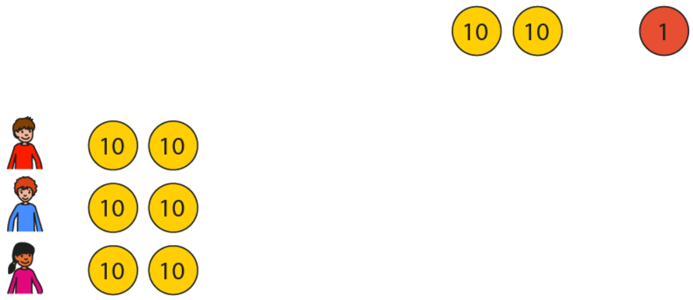
Work towards the generalisation: 'If dividing the tens gives a remainder of one or more tens, we must exchange the remaining tens for ones.'</td>
<td>Presenting the problem:

'<u>Eighty-one</u> marbles are shared equally between <u>three</u> children. How many marbles does each child get?'

81 ÷ 3 = ?



Skip counting according to a multiple of the divisor – sharing the tens:

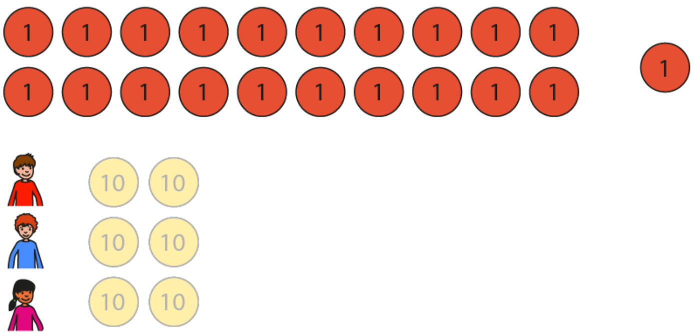


'<u>Three</u> tens are one ten each. That's thirty.'
'<u>Six</u> tens are two tens each. That's sixty.'
'There are two tens left over.'

8 tens ÷ 3 = 2 tens r 2 tens

'Eight tens divided between three is equal to two tens each, with a remainder of two tens.'

Exchanging the remaining tens for ones:



'Two tens and one one is equal to twenty-one ones.'

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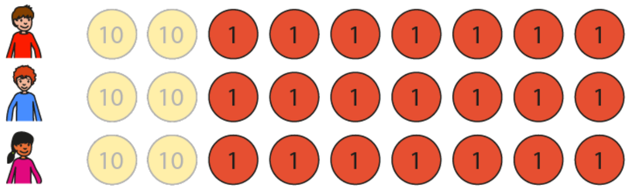
Skip counting according to a multiple of the divisor – sharing the ones:

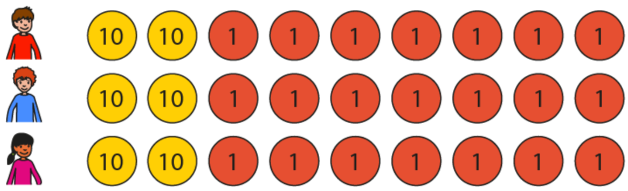


- *'Three ones are one each. That's three.'*
- *'Six ones are two each. That's six...'*
- *'Twenty-one ones is seven each. That's twenty-one.'*

  21 ones ÷ 3 = 7 ones

- *'Twenty-one ones divided between three is equal to seven ones each.'*

Adding the partial quotients:



6 tens	÷	3	=	2 tens
21 ones	÷	3	=	7 ones
81	÷	3	=	27

- *'Each child gets twenty-seven marbles.'*

1:5

Now explore a calculation that involves exchanging tens for ones and that also gives an overall remainder; for example, *'Seventy-three marbles are shared equally between three children. How many marbles does each child get?'*

Work through several more examples for which:

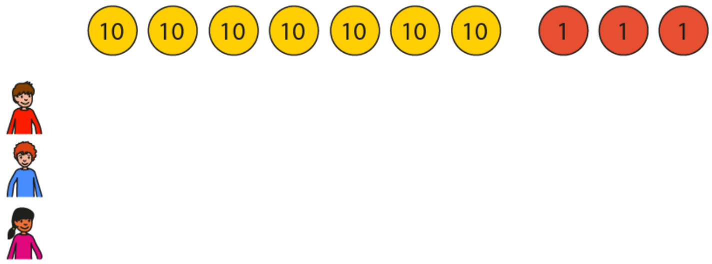
- the tens digit *isn't* divisible (to give a whole number) by the divisor
- the two-digit number *isn't* divisible by the divisor;

e.g. *91 ÷ 4* and *57 ÷ 5*.

Presenting the problem:

'Seventy-three marbles are shared equally between three children. How many marbles does each child get?'

73 ÷ 3 = ?



Summary of solution:

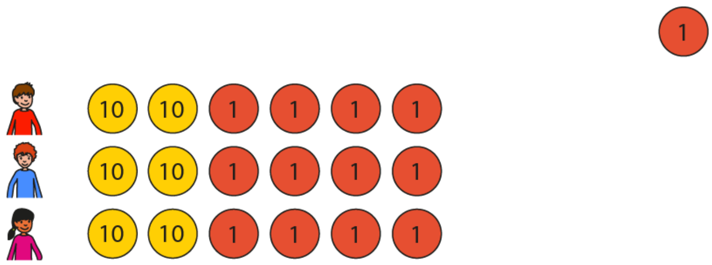
7 tens ÷ 3 = 2 tens r 1 ten

- *'Seven tens divided between three is equal to two tens each, with a remainder of one ten.'*

- *'One ten and three ones is equal to thirteen ones.'*

13 ones ÷ 3 = 4 ones r 1 one

- *'Thirteen ones divided between three is equal to four ones each, with a remainder of one one.'*



6 tens	÷	3	=	2 tens
13 ones	÷	3	=	4 ones r 1 one
73	÷	3	=	24 r 1

- *'Each child gets twenty-four marbles; there is one marble left over.'*

1:6	Finally, work through some examples without concrete or pictorial support, until children can confidently use this method to divide a two-digit number by a single digit number. Include examples of calculations for which: • both of the digits are multiples of the divisor (as in step *1:2*; see *Example 1* opposite) • the tens digit *is not* a multiple of the divisor, but the two-digit number *is* a multiple of the divisor (as in step *1:3*; see *Example 2* opposite) • neither the tens digit nor the two-digit number are multiples of the divisor (as in step *1:5*; see *Example 3* opposite).	Example 1 – both of the digits are multiples of the divisor (and no remainder): 93 ÷ 3 = ? 93 = 9 tens + 3 ones 9 tens ÷ 3 = <u>3 tens</u> 3 ones ÷ 3 = <u>1 one</u> so 93 ÷ 3 = 31 Example 2 – tens digit *is not* a multiple of the divisor (and no overall remainder): 64 ÷ 4 = ? 64 = 6 tens + 4 ones 6 tens ÷ 4 = <u>1 ten</u> r 2 tens 2 tens + 4 ones = 24 ones 24 ones ÷ 4 = <u>6 ones</u> so 64 ÷ 4 = 16 Example 3 – neither the tens digit nor the two-digit number are multiples of the divisor: 75 ÷ 2 = ? 75 = 7 tens + 5 ones 7 tens ÷ 2 = <u>3 tens</u> r 1 ten 1 ten + 5 ones = 15 ones 15 ones ÷ 2 = <u>7 ones r 1 one</u> so 75 ÷ 2 = 37 r 1

1:7

To complete this teaching point, provide children with practice dividing two-digit numbers by single-digit numbers, using the informal written methods outlined above. Children can initially use place-value counters for support, but should progress to working with equations only.

Example word problems:

- *'Eighty-three toy cars are shared equally between five children. How many toy cars does each child get? Are there any cars left over?'* (partitive division)
- *'A paddling pool holds eighty-five litres of water, how many four-litre buckets of water are needed to fill the pool?'* (quotitive division)

Matching division expressions with partial quotients:

'Draw a line to match each division expression with the correct addition expression.'

96 ÷ 3	10 + 9 r 1
96 ÷ 4	30 + 2
96 ÷ 5	20 + 4

Missing-number problems:

'Fill in the missing numbers.'

88 ÷ 4

8 tens ÷ 4 = ___ tens

8 ones ÷ 4 = ___ ones

so

88 ÷ 4 = ☐

64 ÷ 3

6 tens ÷ 3 = ___ tens

4 ones ÷ 3 = ___ ones r ___ ones

so

64 ÷ 3 = ☐ r ☐

78 ÷ 3

7 tens ÷ 3 = ___ tens r 1 ten

1 tens + 8 ones = 18 ones

18 ones ÷ 3 = ___ ones

so

78 ÷ 3 = ☐

82 ÷ 5

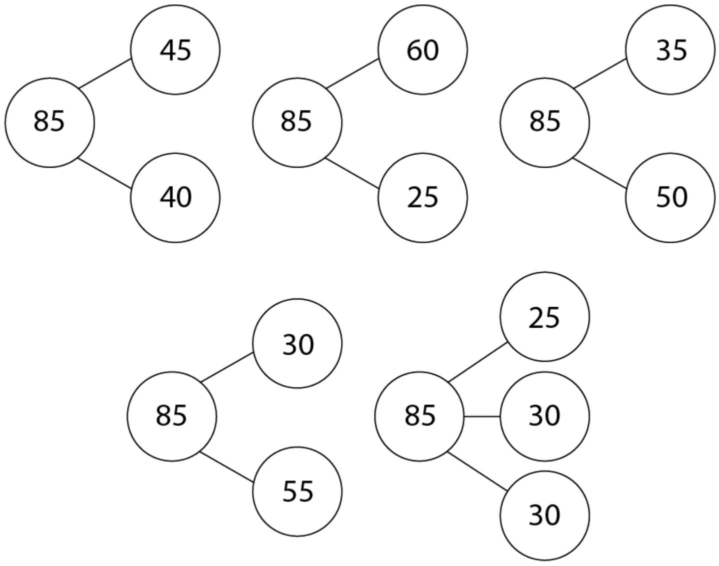
8 tens ÷ 5 = ___ tens r ___ tens

3 tens + 2 ones = 32 ones

32 ones ÷ 5 = ___ ones r ___ ones

so

82 ÷ 5 = ☐ r ☐

		$82 \div 4 =$ ☐ $92 \div 4 =$ ☐ $94 \div 4 =$ ☐ $96 \div 4 =$ ☐ Dòng nǎo jīn: *'Decide which way of partitioning the dividend is helpful for the calculation "85 ÷ 6". Explain why.'* 

Teaching point 2:

Any two-digit number can be divided by a single-digit number using an algorithm called *'short division'*; the algorithm is applied working from the most significant digit (on the left) to the least significant digit (on the right); if there is a remainder in the tens column, we must *'exchange'*.

Steps in learning

2:1 In this teaching point, the short-division algorithm is introduced, using the examples that children have seen in the previous teaching point. As in *Teaching point 1*, you may want to initially use bundles of sticks alongside introduction of the algorithm, before moving to place-value counters, and eventually removing the manipulatives (or pictorial support) entirely. Continue to use unitising language (in tens and ones) as you talk through the steps.

You can either use place-value headings above the algorithm, with the sticks/place-value counter representation alongside, *or* use the sticks/place-value counters within the algorithm without place-value headers; do not combine place-value headers and place-value equipment together within an algorithm since this is incorrect; for example placing eight ten-value counters in a column labelled '10s' represents eighty *tens* (800), not eighty.

Begin with the calculation from step *1:2*, for which both digits of the dividend are divisible by the divisor: '*Eighty-four sticks are shared equally between four children. How many sticks does each child get?*'

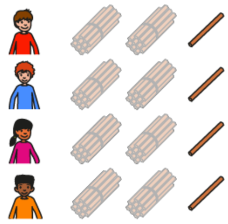
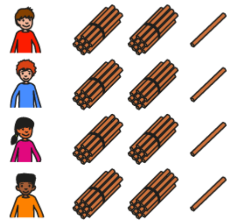
Work through the problem again, using sticks or place-value counters, and model building up the short-division calculation. Note that we now omit the skip counting and draw on multiplication/division facts and unitising.

'Eighty-four sticks are shared equally between four children. How many sticks does each child get?'

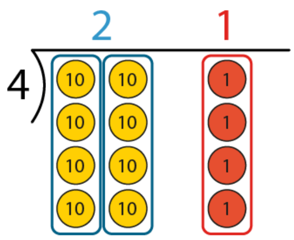
84 ÷ 4 = ?

Algorithm with place-value headings, and pictorial representation:

Step 1 – write the divisor and dividend	Step 2 – sharing the tens
	
10s 1s 4) 8 4 *'Eighty-four divided by four.'*	10s 1s 2 4) 8 4 8 tens ÷ 4 = 2 tens *'Eight tens divided by four is equal to two tens.'*

Step 3 – sharing the ones	Summary
	
10s 1s $\begin{array}{r} 2\ \ 1 \\ 4\overline{)\,8\ \ 4} \end{array}$ 8 tens ÷ 4 = 2 tens 4 ones ÷ 4 = 1 one *'Four ones divided by four is equal to one one.'*	10s 1s $\begin{array}{r} 2\ \ 1 \\ 4\overline{)\,8\ \ 4} \end{array}$ *'Each child gets twenty-one sticks.'*

Algorithm with place-value counters – summary:



8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

- *'Eight tens and four ones divided between four is equal to two tens and one one.'*
- *'Each child gets twenty-one sticks.'*

2:2 Now take a moment to examine the layout of the short-division algorithm. Compare the horizontal equation and the short-division calculation for the problem in step *2:1*, as shown below. Use the language of dividend, divisor and quotient to describe the components of the equation.

Then work as a class to lay out a range of calculations as short division (e.g. *48 ÷ 4 = 12* and *96 ÷ 3 = 32*).

Comparing horizontal equations with the short-division algorithm:

84	÷	4	=	21	$\begin{array}{r} 2\ \ 1 \\ 4\overline{)\,8\ \ 4} \end{array}$
dividend	÷	divisor	=	quotient	quotient divisor) dividend

2:3 Work through the calculation from step *2:1* again, without the support of the manipulatives and without place-value headers. In a similar way to segment *2.14 Multiplication: partitioning leading to short multiplication*, encourage children to describe the

$\begin{array}{r} 2\ \ 1 \\ 4\overline{)\,8\ \ 4} \end{array}$

- 8 tens ÷ 4 = 2 tens
  'Write "2" in the tens column.'
- 4 ones ÷ 4 = 1 one
  'Write "1" in the ones column.'

	steps as they work through the algorithm: • *'First write the divisor: "4".'* • *'Then draw the frame.'* • *'Then write the dividend: "84".'* • *'Now divide, starting with the tens: eight tens divided by four is equal to two tens; write "2" in the tens column.'* • *'Then move to the ones: four ones divided by four is equal to one one; write "1" in the ones column.'* Ask questions to ensure that children can explain what each digit (and number) represents in the algorithm; for example: • *'What does the "8" represent?'* • *'What does the "1" represent?'* • *'Which digit tells me how many tens each child gets?'* • *'Which number is the divisor?'* • *'Which number is the dividend?'* • *'Which number is the quotient?'* • *'What does the quotient represent?'*	
2:4	Give children practice laying out and completing *two-digit ÷ single-digit* calculations, keeping to examples where both digits of the dividend are multiples of the divisor.	Laying out short-division calculations: *'Write these as short-division calculations.'* 69 ÷ 3 39 ÷ 3 93 ÷ 3 66 ÷ 3 Applying the short-division algorithm: *'Complete the calculations.'* $2\overline{)8\ \ 6}$ $3\overline{)6\ \ 3}$ $4\overline{)8\ \ 8}$ Dòng nǎo jīn: *'Fill in the missing digits.'* $\begin{array}{r}2\ \ 0\\4\overline{)\square\ \ 0}\end{array}$ $\begin{array}{r}3\ \ 1\\2\overline{)\square\ \ 2}\end{array}$ $\begin{array}{r}3\ \ 1\\\square\overline{)9\ \ 3}\end{array}$

2:5

Now move to the example used in step *1:3* (the two-digit number is a multiple of the divisor, but the tens digit is not): ‘*Seventy-two sticks are shared equally between three children. How many sticks does each child get?*’

Work through the problem in a similar way to that described in steps *2:1* and *2:3*, paying particular attention to the remainder of one ten after dividing the tens. Either:

- demonstrate unbundling the left-over ten sticks and combining them with the existing ones before dividing the ones (if using sticks as in step *1:3*)

or

- demonstrate exchanging one ten-value counter for ten one-value counters and combining them with the existing one-value counters (if using place-value counters, as shown below).

In a similar way to step *2:3*, encourage children to describe the steps as they work through the algorithm, now drawing attention to the exchange:

- *‘First write the divisor: “3”.’*
- *‘Then draw the frame.’*
- *‘Then write the dividend: “72”.’*
- *‘Now divide, starting with the tens: seven tens divided by three is equal to two tens, with a remainder of one ten; write “2” in the tens column…’*
- *‘and exchange the remainder: one ten is equal to ten ones: write ‘1’ to the left of the ones digit of the dividend to make twelve ones.’*
- *‘Then move to the ones: twelve ones divided by three is equal to four ones; write “4” in the ones column.’*

Work through a variety of similar calculations (e.g. *56 ÷ 4*), gradually removing the scaffolding of sticks/place-value counters until children are confident with the language and calculation layout. Include calculations that involve exchange of more than one ten, for example:

- 84 ÷ 3 (involves exchange of 2 tens for 20 ones)
- 85 ÷ 5 (involves exchange of 3 tens for 30 ones).

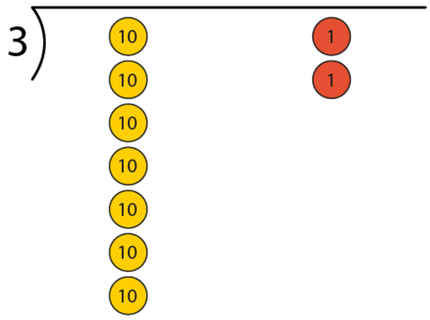
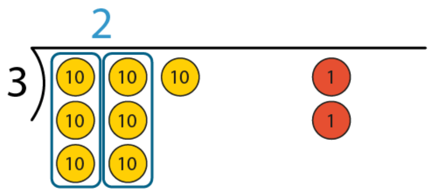
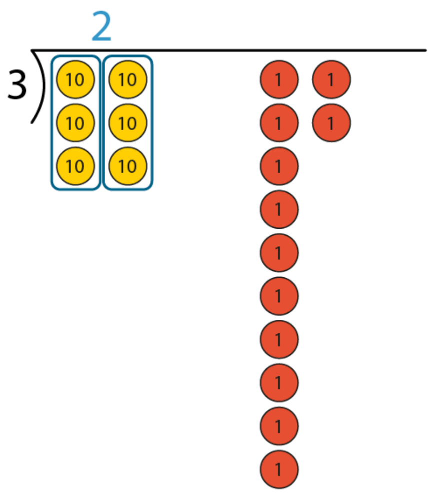
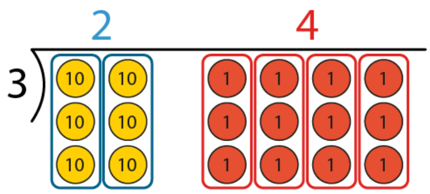
Note that, over time, children may begin to shorten the descriptive language that they use to reason through application of the algorithm; for example, for *56 ÷ 4*, children may eventually say in their heads:

- *‘Five divided by four is one remainder one.’* (writing down ‘1’ in the tens column and a small ‘1’ to the left of the ones digit of the dividend)
- *‘Sixteen divided by four is equal to four.’* (writing down ‘4’ in the ones column)

$$\begin{array}{r} 1\;\;4 \\ 4\overline{)\,5\;\;{}^{1}6} \end{array}$$

'<u>Seventy-two</u> sticks are shared equally between <u>three</u> children. How many sticks does each child get?'

72 ÷ 3 = ?

Step 1 – write the divisor and dividend	Step 2 – sharing the tens…
 $3\overline{)\,7\;\;2}$	 $\begin{array}{r} 2 \\ 3\overline{)\,7\;\;2} \end{array}$
'Seventy-two divided by three.'	7 tens ÷ 3 = 2 tens r 1 ten *'Write "2" in the tens column…'*
Step 3 – …and exchanging	**Step 4 – sharing the ones**
 $\begin{array}{r} 2\phantom{\;\;^{1}2} \\ 3\overline{)\,7\;\;^{1}2} \end{array}$	 $\begin{array}{r} 2\;\;\;4 \\ 3\overline{)\,7\;\;^{1}2} \end{array}$
1 ten = 10 ones *'…and write "1" to the left of the ones digit of the dividend to make twelve ones.'*	12 ones ÷ 3 = 4 ones *'Write "4" in the ones column.'*

2:6

Give children practice similar to that in step *2:4*, now for calculations that require exchange of tens for ones but that have no overall remainder.

Applying the short-division algorithm:

'Complete the calculations.'

$3\overline{)\,4\quad 8}$ $\qquad$ $4\overline{)\,5\quad 6}$ $\qquad$ $4\overline{)\,7\quad 2}$

Dòng nǎo jīn:

'Fill in the missing digits.'

$$\begin{array}{r} 1\quad 7 \\ 3\overline{)\,\square\quad 1} \end{array} \qquad \begin{array}{r} 1\quad 5 \\ 4\overline{)\,6\quad \square} \end{array} \qquad \begin{array}{r} 1\quad 5 \\ \square\overline{)\,7\quad 5} \end{array}$$

2:7

Now move to the example used in step *1:5* (a calculation that involves exchanging tens for ones and that also gives an overall remainder): *'Seventy-three sticks are shared equally between three children. How many sticks does each child get?'*

Work through the problem in the same way as that described in step *2:5*, encouraging children to describe the steps as they work through the algorithm, drawing attention to the remainder when dividing the ones:

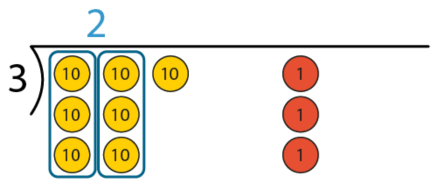
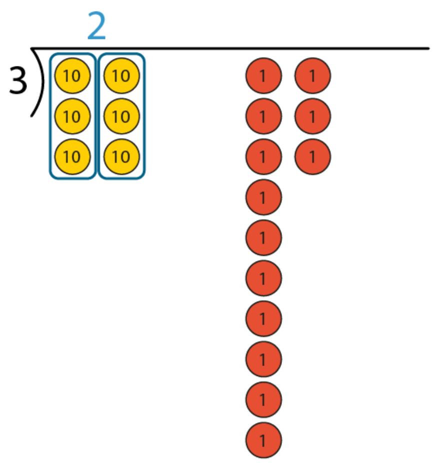
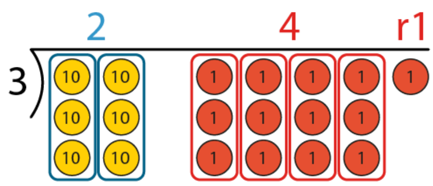
- *'First write the divisor: "3".'*
- *'Then draw the frame.'*
- *'Then write the dividend: "73".'*
- *'Now divide, starting with the tens: seven tens divided by three is equal to two tens, with a remainder of one ten; write "2" in the tens column…*
- *and exchange the remainder: one ten is equal to ten ones; write "1" to the left of the ones digit of the dividend to make thirteen ones.'*
- *'Then move to the ones: thirteen ones divided by three is equal to four ones, with a remainder of one one; write "4 r 1" in the ones column.'*

Work through a variety of similar calculations (e.g. *58 ÷ 4*), gradually removing the scaffolding of sticks/place-value counters. Include some examples where the tens digit of the dividend is smaller than the divisor, such as *83 ÷ 9*, illustrating that the calculation reduces to division of the entire two-digit number, as ones, by the divisor, and so short division is not helpful:

$$\begin{array}{r} 0\quad 9\,\text{r}\,2 \\ 9\overline{)\,8\quad {}^{8}3\quad\;\;} \end{array}$$

'<u>Seventy-three</u> sticks are shared equally between <u>three</u> children. How many sticks does each child get?'

73 ÷ 3 = ?

Step 1 – write the divisor and dividend		Step 2 – sharing the tens…	
	$3\overline{)\,7\;\;3}$		$\begin{array}{r}2\\3\overline{)\,7\;\;3}\end{array}$
'Seventy-three divided by three.'		7 tens ÷ 3 = 2 tens r 1 ten *'Write "2" in the tens column…'*	
Step 3 –…and exchanging		**Step 4 – sharing the ones**	
	$\begin{array}{r}2\phantom{\;\;^{1}3}\\3\overline{)\,7\;\;^{1}3}\end{array}$		$\begin{array}{r}2\;\;4\,r\,1\\3\overline{)\,7\;\;^{1}3}\end{array}$
1 ten = 10 ones *'…and write "1" to the left of the ones digit of the dividend to make thirteen ones.'*		13 ones ÷ 3 = 4 ones r 1 one *'Write "4 r 1" in the ones column.'*	

2:8	Children already learnt that the remainder must be less than the divisor in segment *2.12 Division with remainders*. Revisit this fact in the context of the short-division algorithm, as shown opposite. Also include some examples where the ones digit of the dividend is smaller	*'Explain the error in the following calculation.'* $\begin{array}{r}1\;\;6\,r\,6\\5\overline{)\,8\;\;^{3}6}\end{array}$

than the divisor, e.g. *83 ÷ 4*, emphasising the importance of writing a '0' before the 'r' to show that there are zero ones by comparing the incorrect answer with the correct answer:

4) 8 3 = 2 r 3 ✗ 4) 8 3 = 2 0 r 3 ✓

Dòng nǎo jīn:

'Write a short-division calculation that could have the quotient "11 r 1".'

☐) ☐ = 1 1 r 1

2:9

Give children practice similar to that in steps *2:4* and *2:6*, now for calculations that have an overall remainder. Include examples that do and do not require exchange of tens for ones.

Applying the short-division algorithm:

'Complete the calculations.'

3) 6 4 4) 7 9 5) 6 7

Dòng nǎo jīn:

'Fill in the missing digits.'

3) ☐ [1]4 = 1 4 r 2 ☐) ☐ [1]5 = 1 2 r 3

2:10

To complete this teaching point, provide children with general practice applying the short-division algorithm to *two-digit ÷ single-digit* calculations, in the form of:

- completing calculations (see examples in steps *2:4*, *2:6* and *2:9*)
- missing-number problems, to deepen children's understanding
- contextual problems, including both the partitive and quotitive structures of division, such as the examples opposite and here:
  - *'At scout-camp, six scouts can fit in each tent. How many tents will be needed for seventy-four scouts?'* (quotitive division)
  - *'If I share eighty-five stickers between seven children, how many stickers does each child get?'* (partitive division)

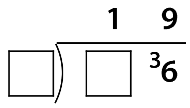
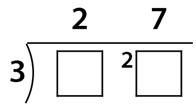
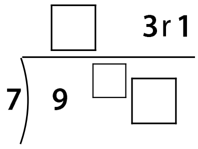
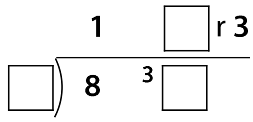
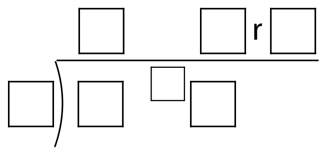
Contextual problem:

'Year 4 made seventy-two biscuits to sell at the summer fête. They want to sell the biscuits in bags. Each bag must contain the same number of biscuits. Work out how many bags they need depending on how many biscuits they put in each bag.'

Total number of biscuits: 72

Number of biscuits in each bag	Calculation	Number of bags needed
2		
3		
4		
5		
6		
7		
8		

(quotitive division)

	Children should apply their learning from segment *2.12 Division with remainders* in order to correctly interpret the quotient. Ensure that you include examples that: • require no exchange and have no remainder • require no exchange but have an overall remainder • require exchange of tens for ones and have no overall remainder • require exchange of tens for ones and also have an overall remainder.	Missing-number problems: *'Fill in the missing digits.'*   Dòng nǎo jīn: • *'Fill in the missing digits. How many solutions can you find?'*  • *'Use each of the following digits once only to correctly complete the calculation:'* 1, 2, 3, 4, 6, 7, 8 

Teaching point 3:

Any three-digit number can be divided by a single-digit number, by partitioning the two-digit number into hundreds, tens and ones, dividing the parts by the single-digit number, then adding the partial quotients; if dividing the hundreds gives a remainder of one or more hundreds, we must exchange the remaining hundreds for tens before dividing the resulting tens value by the single-digit number.

Steps in learning

	Guidance	Representations

3:1 Before extending the short-multiplication algorithm to *three-digit ÷ single-digit* calculations (*Teaching point 4*), apply the informal strategy from *Teaching point 1* to examples with three-digit dividends. Since teaching should follow a similar progression to that for informal methods for *two-digit ÷ single-digit* calculations, guidance here is brief, with full exemplars provided only for calculations with no exchange and no remainder (step *3:1*) and for exchange of hundreds for tens only, with no remainder (step *3:2*).

Begin with an example that requires no exchange and that has no remainders (e.g. *848 ÷ 4*), following similar steps to those used in *Teaching point 1* (although you can use known facts and unitising, rather than skip counting, as exemplified below). Work through the problem first using manipulatives (such as place-value counters) and then with equations only.

Practise with a range of calculations (e.g. *396 ÷ 3*; *550 ÷ 5*).

'Eight-hundred and forty-eight pencils are shared equally between four year groups. How many pencils does each year group get?'

848 ÷ 4 = ?

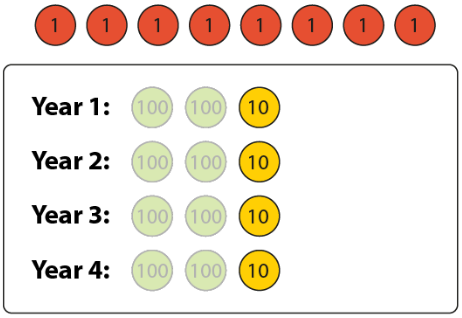
Step 1 – presenting the problem

Step 2 – sharing the hundreds

8 hundreds ÷ 4 = 2 hundreds

'Eight hundreds divided between four is equal to two hundreds each.'

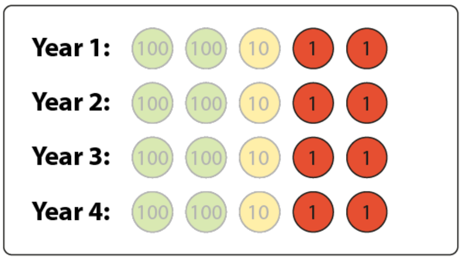
Step 3 – sharing the tens



4 tens ÷ 4 = 1 ten

'Four tens divided between four is equal to one ten each.'

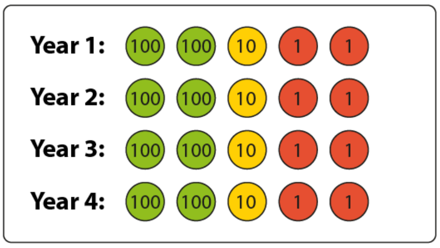
Step 4 – sharing the ones



8 ones ÷ 4 = 2 ones

'Eight ones divided between four is equal to two ones each.'

Step 5 – adding the partial quotients



8 hundreds	÷	4	=	2 hundreds
4 tens	÷	4	=	1 ten
8 ones	÷	4	=	2 ones
848	÷	4	=	212

'Each year group gets two-hundred and twelve pencils.'

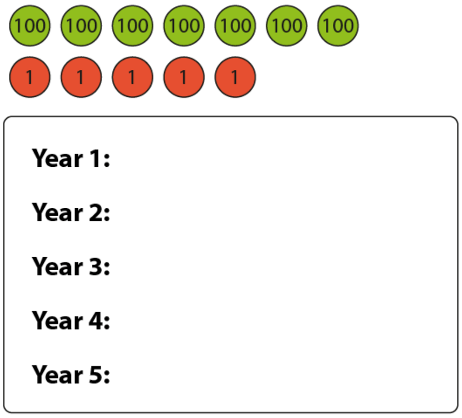
3:2 Now work through an example that requires exchange of hundreds for tens, but with no exchange of tens for ones and no overall remainder (e.g. *705 ÷ 5*). Draw particular attention to the exchange.

Use the following generalisation: ***'If dividing the hundreds gives a remainder of one or more hundreds, we must exchange the remaining hundreds for tens.'***

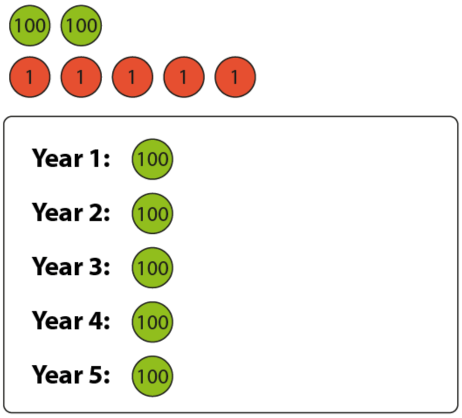
'<u>Seven hundred and five</u> exercise books are shared equally between <u>five</u> year groups. How many books does each year group get?'

705 ÷ 5 = ?

Step 1 – presenting the problem



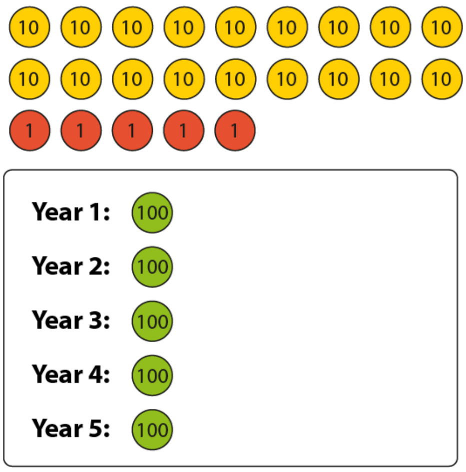
Step 2 – sharing the hundreds



7 hundreds ÷ 5 = 1 hundred r 2 hundreds

'Seven hundreds divided between five is equal to one hundred each, with a remainder of two hundreds.'

Step 3 – exchanging the remaining hundreds for tens



2 hundreds = 20 tens

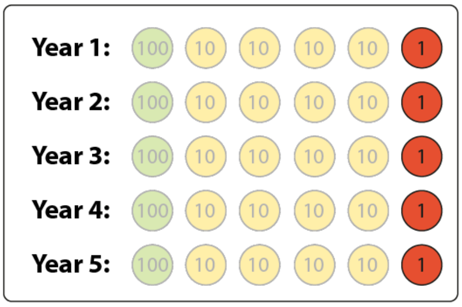
'Two hundreds is equal to twenty tens.'

Step 4 – sharing the tens

20 tens ÷ 5 = 4 tens

'Twenty tens divided between five is equal to four tens each.'

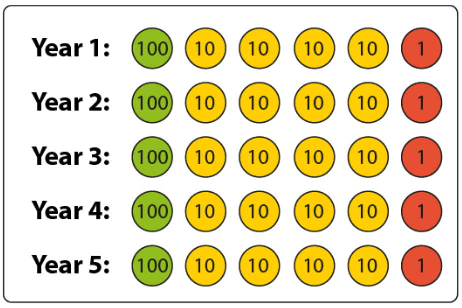
Step 5 – sharing the ones



5 ones ÷ 5 = 1 one

'Five ones divided between five is equal to one one each.'

Step 6 – adding the partial quotients



5 hundreds	÷	5	=	1 hundred
20 tens	÷	5	=	4 tens
5 ones	÷	5	=	1 one
705	÷	5	=	141

'Each year group gets one hundred and forty-one books.'

3:3

Work through a range of examples, gradually removing the support of place-value counters, until children can confidently use the partitioning method to divide any three-digit number by a single digit number. Use a variety of examples with/without exchange of hundreds for tens, and of tens for ones, and with/without an overall remainder. The example opposite can be used again later to highlight the efficiency of the short-division algorithm.

Example – exchange of hundreds for tens, tens for ones, and an overall remainder:

473 ÷ 3 = ?

473 = 4 hundreds + 7 tens + 3 ones

4 hundreds ÷ 3 = <u>1 hundred</u> r 1 hundred

1 hundred + 7 tens = 17 tens

17 tens ÷ 3 = <u>5 tens</u> r 2 tens

2 tens + 3 ones = 23 ones

23 ones ÷ 3 = <u>7 ones r 2 ones</u>

so

473 ÷ 3 = 157 r 2

3:4

To complete this teaching point, provide children with practice (similar to that described in step *1:7*) dividing three-digit numbers by single-digit numbers, using the informal written methods outlined above. Children can initially use place-value counters for support but should progress to working with equations only.

Matching division expressions with partial quotients:

'Draw a line to match each division expression with the correct addition expression.'

963 ÷ 3	200 + 40 + 1
963 ÷ 4	300 + 20 + 1
964 ÷ 4	200 + 40 r 3

	Example word problems: • *'784 marbles are shared equally between seven children. How many marbles does each child get? Are there any marbles left over?'* (partitive division) • *'A shopkeeper has £575 to spend on footballs. If each ball costs him £4, how many balls can he buy altogether? Does he have any money left over?'* (quotitive division)	Missing-number problems: *'Fill in the missing numbers.'* **849 ÷ 6** 8 hundreds ÷ 6 = ___ hundreds r ___ hundreds 2 hundreds + 4 tens = 24 tens 24 tens ÷ 6 = ___ tens 9 ones ÷ 6 = ___ ones r ___ ones so 849 ÷ 6 = ☐ r ☐ 846 ÷ 6 = ☐ 856 ÷ 6 = ☐ 858 ÷ 6 = ☐

Teaching point 4:

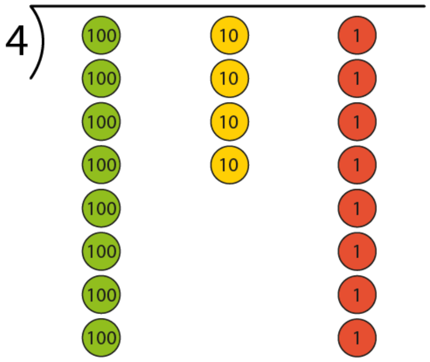
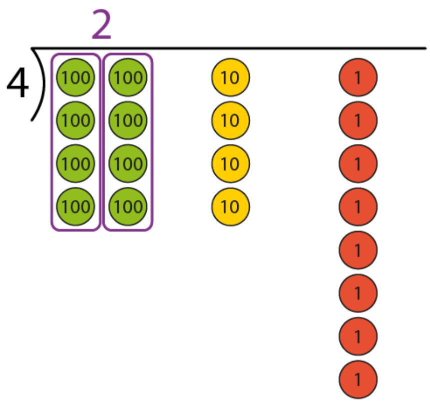
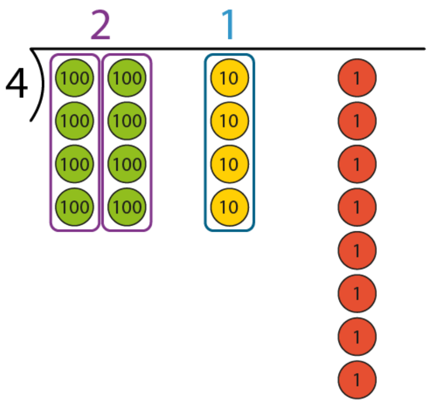
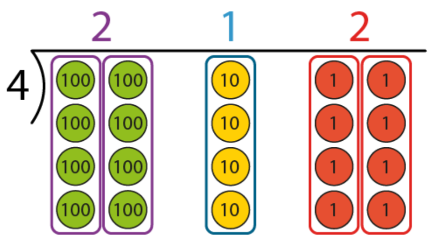
Any three-digit number can be divided by a single-digit number using the short-division algorithm.

Steps in learning

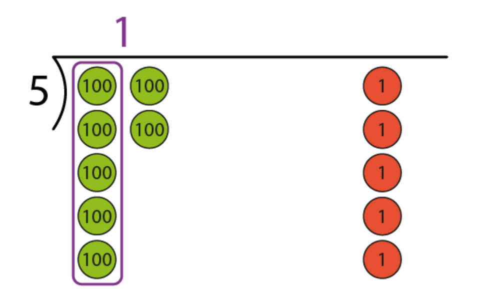
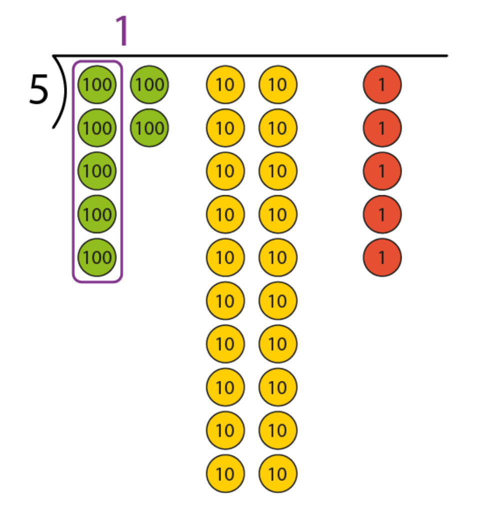
	Guidance	Representations
4:1	This teaching point is similar to *Teaching point 2*; now, the short-division algorithm is used to carry out the same *three-digit ÷ single-digit* calculations that were explored with informal methods in *Teaching point 3*. The language and approach are the same as that described in *Teaching point 2*, so guidance here is kept brief, except for new key learning points (steps *4:7–4:9*). Begin with the calculation from step *3:1*, for which all three digits are divisible by the divisor: '*Eight hundred and forty-eight pencils are shared equally between four year groups. How many pencils does each year group get?*' Work through the problem again, and model building up the short-division calculation, using place-value counters for support. In a similar way to *Teaching point 2*, encourage children to describe the steps as they work through the algorithm: • *'First write the divisor: "4".'* • *'Then draw the frame.'* • *'Then write the dividend: "848".'* • *'Now divide, starting with the hundreds: eight hundreds divided by four is equal to two hundreds; write "2" in the hundreds column.'* • *'Then move to the tens: four tens divided by four is equal to one ten; write "1" in the tens column.'* • *'Then move to the ones: eight ones divided by four is equal to two ones; write "2" in the ones column.'* Work through a variety of similar calculations (e.g. *699 ÷ 3*), gradually removing the scaffolding of place-value counters until children are confident with the language and calculation layout. Note that, over time, children will begin to shorten the descriptive language that they use to reason through application of the algorithm (see step *2:5*).	

'Eight hundred and forty-eight pencils are shared equally between four year groups. How many pencils does each year group get?'

848 ÷ 4 = ?

Step 1 – write the divisor and dividend		Step 2 – sharing the hundreds	
	$4\overline{)8\ 4\ 8}$		$\begin{array}{r}2\\4\overline{)8\ 4\ 8}\end{array}$
'Eight-hundred and forty-eight divided by four.'		8 hundreds ÷ 2 = 2 hundreds *'Write "2" in the hundreds column.'*	
Step 3 – sharing the tens		**Step 4 – sharing the ones**	
	$\begin{array}{r}2\ 1\\4\overline{)8\ 4\ 8}\end{array}$		$\begin{array}{r}2\ 1\ 2\\4\overline{)8\ 4\ 8}\end{array}$
4 tens ÷ 4 = 1 ten *'Write "1" in the tens column.'*		8 ones ÷ 4 = 2 ones *'Write "2" in the ones column.'*	

4:2	Give children practice laying out and completing *three-digit ÷ single-digit* calculations, keeping to examples where each digit of the dividend is a multiple of the divisor.	Applying the short-division algorithm: *'Complete the calculations.'* $2\overline{)4\ 6\ 4}$ $3\overline{)3\ 9\ 6}$ $5\overline{)5\ 5\ 0}$

		Dòng nǎo jīn: *'Fill in the missing digits.'* $\begin{array}{r} 1\;2\;0 \\ 4\overline{)\square\;8\;0} \end{array}$ $\qquad$ $\begin{array}{r} 2\;0\;1 \\ \square\overline{)6\;\square\;3} \end{array}$
4:3	Now move on to the calculation from step *3:2*, which requires exchange of hundreds for tens, but with no exchange of tens for ones and no overall remainder: *'Seven hundred and five exercise books are shared equally between five year groups. How many books does each year group get?'* Work in the same way as that described in step *4:1*, but now draw particular attention to the exchange: • *'First write the divisor: "5".'* • *'Then draw the frame.'* • *'Then write the dividend: "705".'* • *'Now divide, starting with the hundreds: seven hundreds divided by five is equal to one hundred, with a remainder of two hundreds; write "1" in the hundreds column…'* • *'and exchange the remainder: two hundreds is equal to twenty tens; write "2" to the left of the tens digit of the dividend to make twenty tens.'* • *'Then move to the tens: twenty tens divided by five is equal to four tens; write "4" in the tens column.'* • *'Then move to the ones: five ones divided by five is equal to one one; write "1" in the ones column.'* Note that only the middle steps – division of the hundreds and exchange – are fully illustrated opposite, but place-value counters should be used throughout, alongside the written algorithm, for the first example with exchange. Work through a variety of similar calculations (e.g. *786 ÷ 3*, *640 ÷ 4* and *576 ÷ 3*), gradually removing the	*'Seven hundred and five exercise books are shared equally between five year groups. How many books does each year group get?'* $705 \div 5 = ?$ Sharing the hundreds:  $\begin{array}{r} 1 \\ 5\overline{)7\;0\;5} \end{array}$ 7 hundreds ÷ 5 = 1 hundred r 2 hundreds *'Write "1" in the hundreds column…'* Exchanging:  $\begin{array}{r} 1\phantom{\;{}^{2}0\;5} \\ 5\overline{)7\;{}^{2}0\;5} \end{array}$ 2 hundreds = 20 tens *'…and write "2" to the left of the tens digit of the dividend to make twenty tens.'*

	scaffolding of place-value counters until children are confident with the language and calculation layout.	
4:4	As in step *4:2*, give children practice completing *three-digit ÷ single-digit* calculations, now for examples where exchange of hundreds for tens is required (but with no exchange of tens for ones, and with no overall remainder).	Applying the short-division algorithm: *'Complete the calculations.'* $2\overline{)\,5\ 4\ 6}$ $3\overline{)\,8\ 4\ 6}$ $4\overline{)\,7\ 2\ 8}$ Dòng nǎo jīn: *'Fill in the missing digits.'* $\begin{array}{r} 1\ \ 6\ \ 1 \\ 5\overline{)\,\square\ \ 0\ \ 5} \end{array}$ $\begin{array}{r} 1\ \ 5\ \ 1 \\ \square\overline{)\,9\ \ 0\ \ 6} \end{array}$

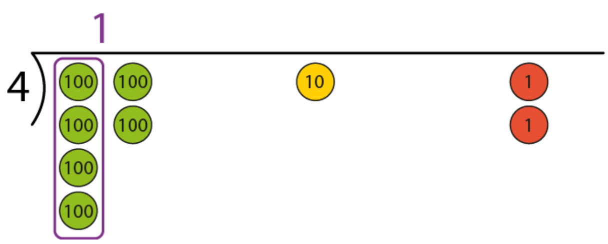
4:5 Repeat steps *4:3* and *4:4* for a range of calculations including examples involving:

- no exchange but an overall remainder (e.g. *637 ÷ 3*)
- exchange of hundreds for tens only, with an overall remainder (e.g. *727 ÷ 4*)
- exchange of tens only with (e.g. *563 ÷ 5*) or without (e.g. *570 ÷ 5*) an overall remainder
- exchange of both hundreds and tens with (e.g. *963 ÷ 7*) or without (e.g. *612 ÷ 4*) an overall remainder.

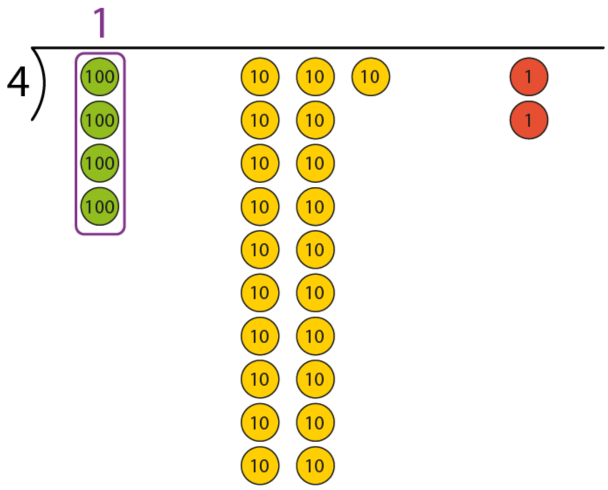
For now, keep to examples where the hundreds digit of the dividend is greater or equal to the divisor.

Example – exchange of both hundreds and tens, without an overall remainder:

612 ÷ 4 = ?

Step 1 – write the divisor and dividend	
'Six hundred and twelve divided by four.'	$4\overline{)\,6\ \ 1\ \ 2}$
Step 2 – sharing the hundreds…	
	$\begin{array}{r} 1\ \ \ \ \ \ \ \ \\ 4\overline{)\,6\ \ 1\ \ 2} \end{array}$
6 hundreds ÷ 4 = 1 hundred r 2 hundreds	*'Write "1" in the hundreds column…'*

Step 3 –…and exchanging



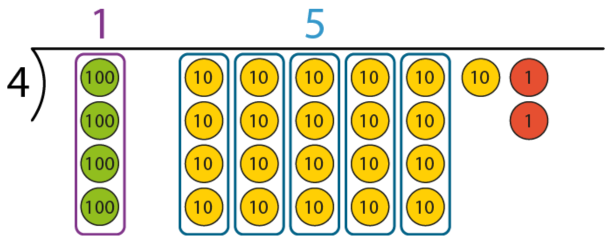
1

$4\overline{)\,6\;{}^{2}1\;\;2}$

2 hundreds = 20 tens

'…and write "2" to the left of the tens digit of the dividend to make twenty-one tens.'

Step 4 – sharing the tens…

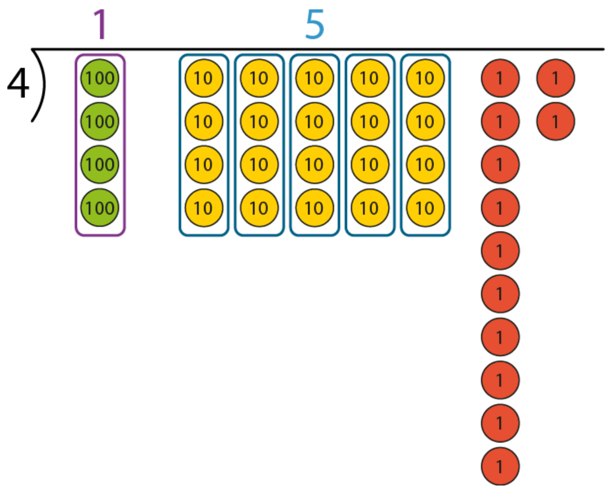


1 5

$4\overline{)\,6\;{}^{2}1\;\;2}$

21 tens ÷ 4 = 5 tens r 1 ten *'Write "5" in the tens column…'*

Step 5 –… and exchanging



1 5

$4\overline{)\,6\;{}^{2}1\;{}^{1}2}$

1 ten = 10 ones

'…and write "1" to the left of the ones digit of the dividend to make twelve ones.'

	Step 6 – sharing the ones 12 ones ÷ 4 = 3 ones *'Write "3" in the ones column.'* $\begin{array}{r} 1\ \ 5\ \ 3 \\ 4\overline{)\,6\ {}^{2}1\ {}^{1}2} \end{array}$

4:6	Provide children with practice similar to that described in step *4:4*, covering all calculation types listed in step *4:5*. Also, use sequences of calculations to revisit the fact that the remainder must be smaller than the divisor (see also step *2:8*): • 609 ÷ 4 • 610 ÷ 4 • 611 ÷ 4 • 612 ÷ 4 • 613 ÷ 4 • 614 ÷ 4 • 615 ÷ 4	Applying the short-division algorithm: *'Complete the calculations.'* $4\overline{)\,4\ 8\ 6}$ $5\overline{)\,8\ 5\ 7}$ $6\overline{)\,6\ 8\ 4}$ $7\overline{)\,8\ 7\ 2}$ Dòng nǎo jīn: *'Fill in the missing digits.'* $\begin{array}{r} 1\ \ \ 6\ \ \ 3 \\ 5\overline{)\,\square\ {}^{3}\square\ {}^{1}5} \end{array}$ $\begin{array}{r} 2\ \ 4\ \ 2\text{ r }2 \\ \square\overline{)\,7\ {}^{1}2\ \ 8} \end{array}$

4:7

Now spend some time working through a problem where the hundreds digit of the dividend is smaller than the divisor (e.g. *215 ÷ 5*). A common error that children make is to transfer an incorrect 'remainder' to the tens, equal to the difference between the divisor and the hundreds digit of the dividend, rather than exchanging all of the hundreds into tens:

$$\begin{array}{r} 0\ \ 6\ \ 3 \\ 5\overline{)\,2\ \ {}^{3}1\ \ {}^{1}5} \end{array}\ ✗ \qquad \begin{array}{r} 0\ \ 4\ \ 3 \\ 5\overline{)\,2\ \ {}^{2}1\ \ {}^{1}5} \end{array}\ ✓$$

This follows on from what children learned, in step *2:8*, regarding calculations where the ones digit of the dividend is smaller than the divisor (*83 ÷ 4*).

Emphasise the importance of writing a '0' in the hundreds column then exchanging the hundreds for tens.

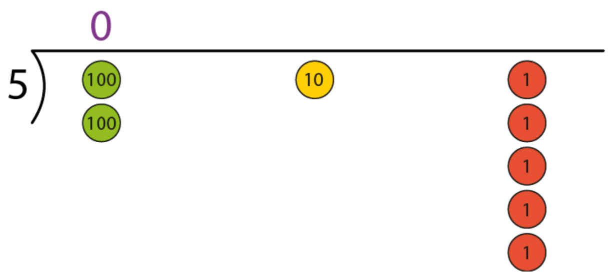
Note that, in the representations below, the counters are only illustrated for sharing and exchange of the hundreds. However, you should use counters throughout the calculation.

Work through a variety of similar calculations (e.g. *488 ÷ 8*), gradually removing the scaffolding of place-value counters until children are confident.

Hundreds digit of the dividend is smaller than the divisor:

215 ÷ 5 = ?

Sharing the hundreds

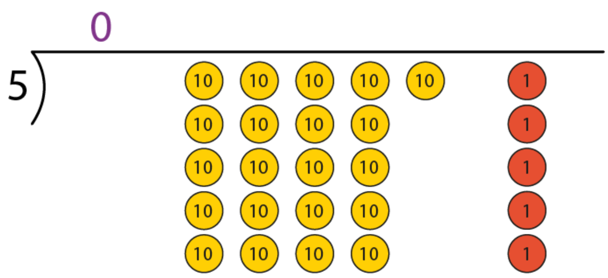


$$\begin{array}{r} 0 \\ 5\overline{)\,2\ \ 1\ \ 5} \end{array}$$

2 hundreds ÷ 5 = 0 hundreds r 2 hundreds

'Write "0" in the hundreds column…'

Exchanging



$$\begin{array}{r} 0\phantom{\ \ {}^{2}1\ \ 5} \\ 5\overline{)\,2\ \ {}^{2}1\ \ 5} \end{array}$$

2 hundreds = 20 tens

'…and write "2" to the left of the tens digit of the dividend to make twenty-one tens.'

4:8	Now explore how we can predict the number of digits in the quotient by comparing the hundreds digit of the dividend with the divisor.	Predicting the number of digits in the quotient: • **712 ÷ 4** **7 > 4** *'The quotient will be a three-digit number.'* • **426 ÷ 6** **4 < 6** *'The quotient will be a two-digit number.'* • **800 ÷ 5** **8 > 5** *'The quotient will be a three-digit number.'* • **648 ÷ 6** **6 = 6** *'The quotient will be a three-digit number.'*
4:9	Provide children with practice similar to that described in step *4:4*, now with calculations for which the hundreds digit of the dividend is smaller than the divisor. Include examples that involve: • no exchange of tens for ones, with (e.g. *423 ÷ 7*) and without (e.g. *426 ÷ 6*) an overall remainder • exchange of tens for ones, with (e.g. *645 ÷ 9*) and without (e.g. *600 ÷ 8*) an overall remainder.	Applying the short-division algorithm: *'Complete the calculations.'* 6) 4 2 6 7) 4 2 3 8) 6 0 0 9) 6 4 5 Dòng nǎo jīn: *'Fill in the missing digits.'* 0 9 1 r 2 □) □ ^{4}5 □
4:10	To complete this teaching point, provide children with general practice for *three-digit ÷ single-digit* calculations, in the form of: • completing calculations (see examples in steps *4:2*, *4:4*, *4:6* and *4:9*) • missing-digit problems, to deepen children's understanding (see the example dòng nǎo jīn problems in steps *4:2*, *4:4*, *4:6* and *4:9*) • reasoning problems, e.g. see opposite and on the next page.	Reasoning problems: • *'What digits could the diamond (◊) represent if the following calculation has a whole number quotient?'* 7) 3 2 ◊ • *'What digits could the diamond (◊) represent if the following calculation has a quotient with a remainder of three?'* 6) 3 3 ◊

- contextual problems, including both the partitive and quotitive structures of division, for example:
  - *'A farmer has 850 peaches. If she packs the peaches into boxes of six, how many whole boxes can she fill?'* (quotitive division)
  - *'A school raises £892 for new equipment. The money is shared equally between six year groups and any left-over money will be given to charity.'*
    - *'How much money does each year group get?'*
    - *'Is there any money left over for charity? If so, how much?'*

    (partitive division)
  - *'174 children are going on a trip. Four children can fit into each room in the hostel. How many rooms are needed?'* (quotitive division)

Children should apply their learning from segment *2.12 Division with remainders* in order to correctly interpret the quotient.

- *'The following calculation gives a three-digit quotient. What digit do we need to change to give a two-digit quotient? How many solutions are there?'*

5) 6 7 5

- *'Decide whether each calculation is correct or not. Explain your answers.'*

    5 5
5) 3 $^{2}7$ $^{2}5$

    1 1 9
8) 9 $^{1}5$ $^{7}2$

    1 0 8 r 7
6) 6 5 $^{5}5$

Missing-number problem:

'Fill in the missing numbers in the table.'

Dividend	Divisor	Quotient
242		121
	3	121
484		121
	5	121

Dòng nǎo jīn:

'What could the missing digits be in the following calculation? How many solutions are there?'

    2 1 4
□) □ □ □

4:11	Now that children are equipped with the short-division algorithm, they should be encouraged to make sensible choices about when it is efficient to use it. Spend some time examining a range of division calculations: • 42 ÷ 6 • 87 ÷ 3 • 248 ÷ 4 • 385 ÷ 5 • 582 ÷ 8 • 618 ÷ 6 • 804 ÷ 4 • 952 ÷ 4 Explore different strategies for finding the quotient in each case, and evaluate the efficiency of each. Note that often there is not a single 'best' approach. Beyond this segment, teachers should make a continuous effort to ensure that children approach all *two-/three-digit ÷ single-digit* calculations with an attitude of enquiry and flexibility, rather than immediately opting for short division.	