

## Planning to teach Constructions – Supporting document

This document, which accompanies the 'Planning to teach constructions' video, gives guidance on planning lessons that allow students to appreciate the mathematical structures underpinning the various straight edge and compass constructions they need to know at Key Stage 3, and to execute them with understanding and fluency.

### Part 1 – The big idea

The central idea that students need to grasp when studying this topic is that the various constructions they will meet are all based on the **geometrical properties** of particular shapes. It is through constructing these shapes that they will be able to produce perpendicular bisectors and angle bisectors.

The following programme of study statements from the KS3 National Curriculum outline the important aspects of knowledge, skills and understanding that students will need:

- Derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- Derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line.
- Identify and construct congruent triangles.

#### What things typically go wrong?

To be proficient in all topics in mathematics, students need to have a mixture of deep conceptual understanding and procedural fluency. The teaching of constructions is really no different, but two issues seem more pertinent in this topic than most. The first is that fluency in this topic requires a significant level of manual dexterity and precision of fine motor skills, particularly in the use of a pair of compasses. The second is that, because of its inherently practical nature and the fact that constructions can be viewed as merely a series of physical actions, students can view it as nothing to do with understanding and, in particular, unrelated to any other work they have done on geometrical properties.

Common misconceptions and causes of error and confusion include:

- having difficulty in using a pair of compasses accurately
- seeing a pair of compasses as just a tool for drawing circles without appreciating that they produce a set of points all the same distance from a fixed point and so can be used to create equal lengths without the need for measuring with a ruler
- being unable to make connections with prior knowledge or with their own spatial intuition. This can lead to an unnecessary reliance on 'blind' memory which often fails
- even for those students who learn 'the steps', an inability to tackle anything other than very standard construction problems. Without a unifying conceptual framework, slight changes to orientation, position or context result in errors and confusion.

## Part 2 – Prerequisites

The following statements from the Geometry section of the Year 6 programme of study give a helpful steer on the prerequisite knowledge related to this topic:

Pupils should be taught to:

- Draw 2-D shapes using given dimensions and angles
- Compare and classify geometric shapes based on their properties

... and, in the corresponding notes and guidance section:

- Pupils draw shapes [...] accurately using measuring tools and conventional markings and labels for lines and angles.

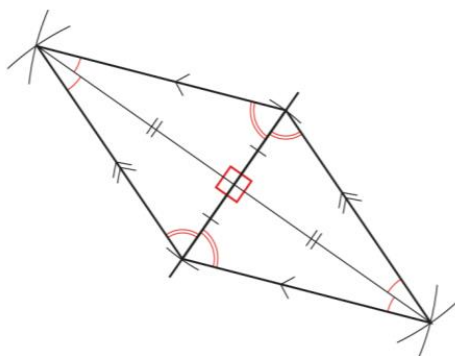
Also, in the NCETM Secondary PD Materials (page 2 of the [Core Concept document 6.4: Constructions](#)), the following guidance is offered:

“In Key Stage 2, students will have learnt about the properties of certain geometric shapes and used these properties to compare and classify shapes. They will also have had experience of drawing certain shapes with a ruler and angle measurer, but the use of compasses to construct shapes will be a new idea at Key Stage 3”.

So, in planning work on this topic, there is a need to consider the following:

- the idea of constructing equal lengths, without measuring or knowing the actual length, is new and likely to be challenging
- there is a need to revisit the properties of special shapes, for example isosceles triangles, equilateral triangles, kites, rhombuses and parallelograms
- the idea of diagonals is likely to be new (and is important for later). Students will have met the idea of a diagonal at KS2, but it will be important to bring in the idea that diagonals of certain shapes have certain key properties.

Finally, on page 6 of [Core Concept document 6.4 Constructions](#), the following diagram appears:



**Fig 2: Diagram showing how the diagonals of a rhombus bisect each other at right angles and bisect the internal angles**

and this can be a helpful diagram to show and discuss with students before beginning to teach the details of this topic. Students may well be familiar with the rhombus and some of its properties (i.e. opposite sides parallel and all sides equal). However, spending time dwelling on the detail of this diagram and drawing attention to the fact that the diagonals bisect each other at right angles and that each diagonal bisects an internal angle of the rhombus will provide a good platform for the future learning and teaching in this topic.

### Part 3 – Key teaching aspects

A central idea in this topic is the idea that a circle is the locus of all points equidistant from a fixed point, and that a pair of compasses can be used to find a point that is a specified distance away from one point and simultaneously, another specified distance from a second point.

Posing a problem like the one below and asking, “Where would the point 4cm from A and 7cm from B be?” can be an interesting and engaging way to begin teaching this idea. Students may well reach immediately for a ruler and appreciate for themselves the difficulty of coordinating these two distances to find the point. If we then introduce the idea of using a pair of compasses to **measure distances** and ask them to try to find the point using this piece of equipment, students may well come to appreciate the power and elegance of this simple construction which uses this central property of a circle.

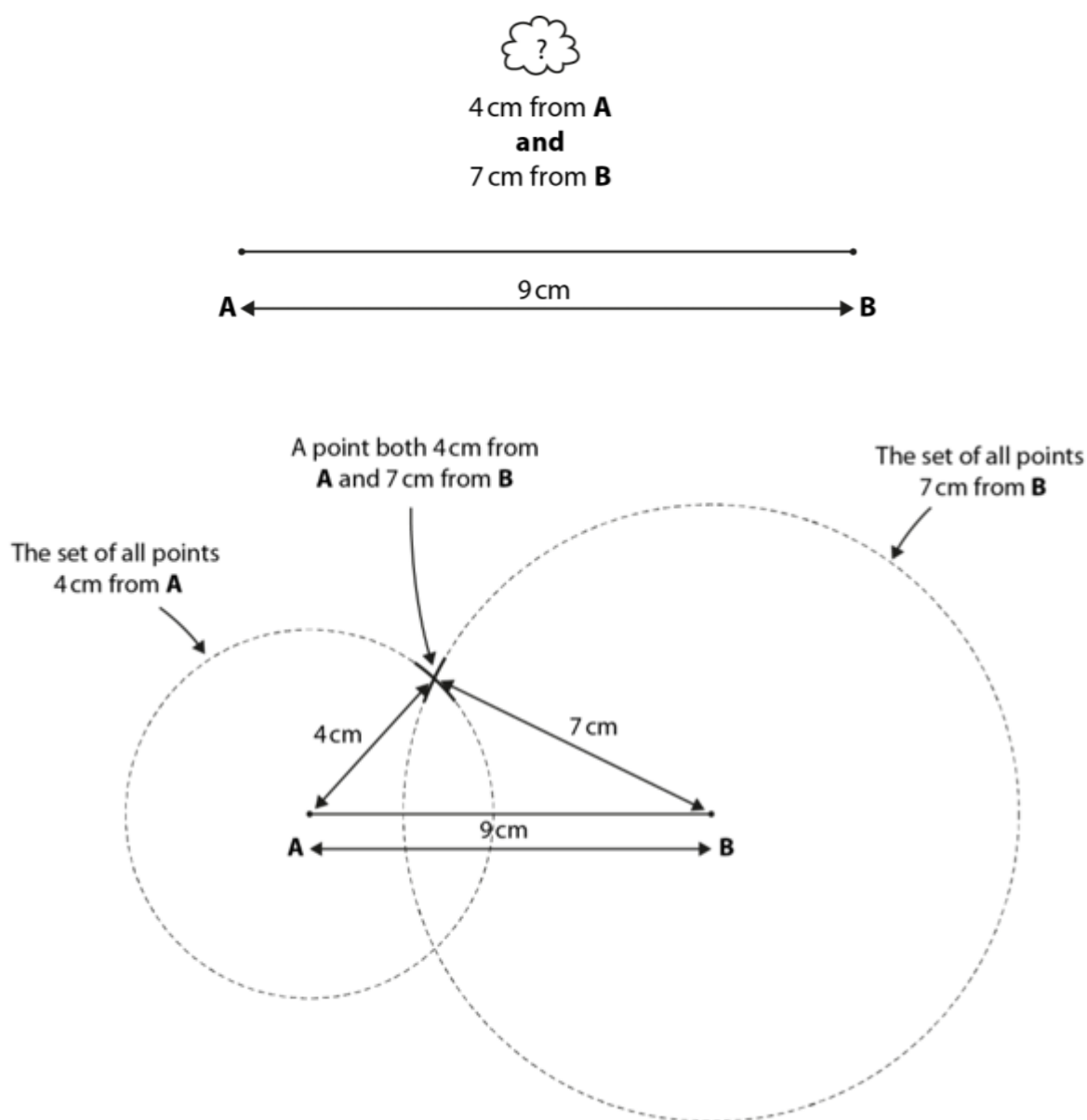
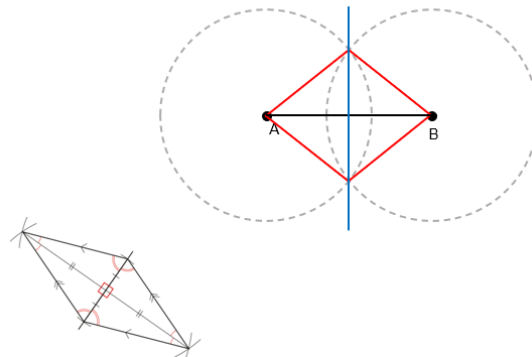


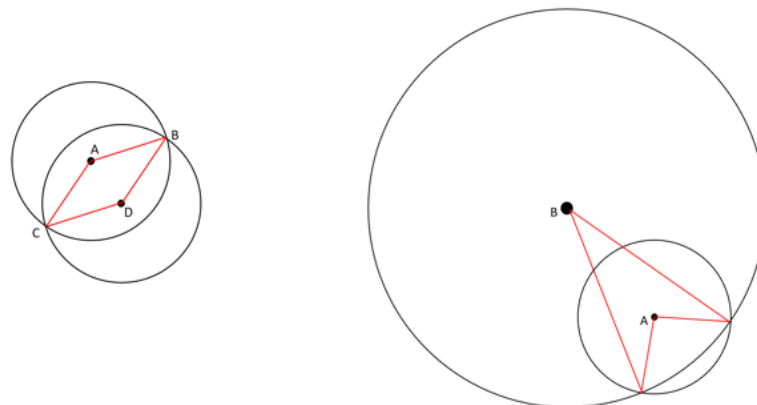
Image taken from pages 4 and 5 of the [Core Concept document 6.4 Constructions](#)

In the video, a number of activities are explained which:

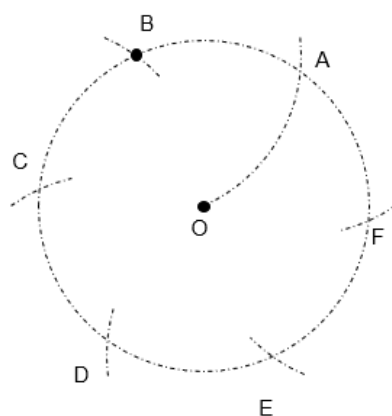
- keep the focus on reasoning and using the geometrical properties of the rhombus, all the time drawing attention to the fact that by drawing a rhombus, all the constructions you might wish to create (i.e. perpendicular bisector and angle bisector) will be present within the rhombus



- employing careful variation to make sure that non-standard as well as standard examples are experienced by the students



- support students in being precise with their use of compasses while also encouraging reasoning.



Draw the circle.  
Now draw the longer arc, and then the shorter ones

Draw AB, BC, CD, DE, EF and FA  
What shape is this?



## Part 4 – Why is this important? Where does it lead?

### How will this support future learning?

The ideas in this topic help students develop a deep understanding and support connections with learning in other areas of the KS3 curriculum including:

- loci
- establish the congruence criteria
- use congruence criteria to prove that the constructions work
- prove non-standard results using the congruence criteria, where identification of equal lengths is crucial
- strengthen prior knowledge before circle theorems; two radii and a chord make an isosceles triangle.

## Useful links

 <p>Teaching for Mastery Questions, tasks and activities to support assessment in KS3</p>	<p>NCETM Secondary Mastery Assessment materials</p> <p><a href="http://www.ncetm.org.uk/classroom-resources/assessment-materials-secondary/">www.ncetm.org.uk/classroom-resources/assessment-materials-secondary/</a></p> <p>In particular, the activity on page 38 is related to the topic of constructions</p>
 <p>Mastery Professional Development 6 Geometry</p> <p>6.4 Constructions</p> <p>Guidance document   Key Stage 3</p>	<p>NCETM Mastery Professional Development Materials (6.4 Constructions)</p> <p><a href="http://www.ncetm.org.uk/media/3viclabl/ncetm_ks3_cc_6_4.pdf">www.ncetm.org.uk/media/3viclabl/ncetm_ks3_cc_6_4.pdf</a></p>