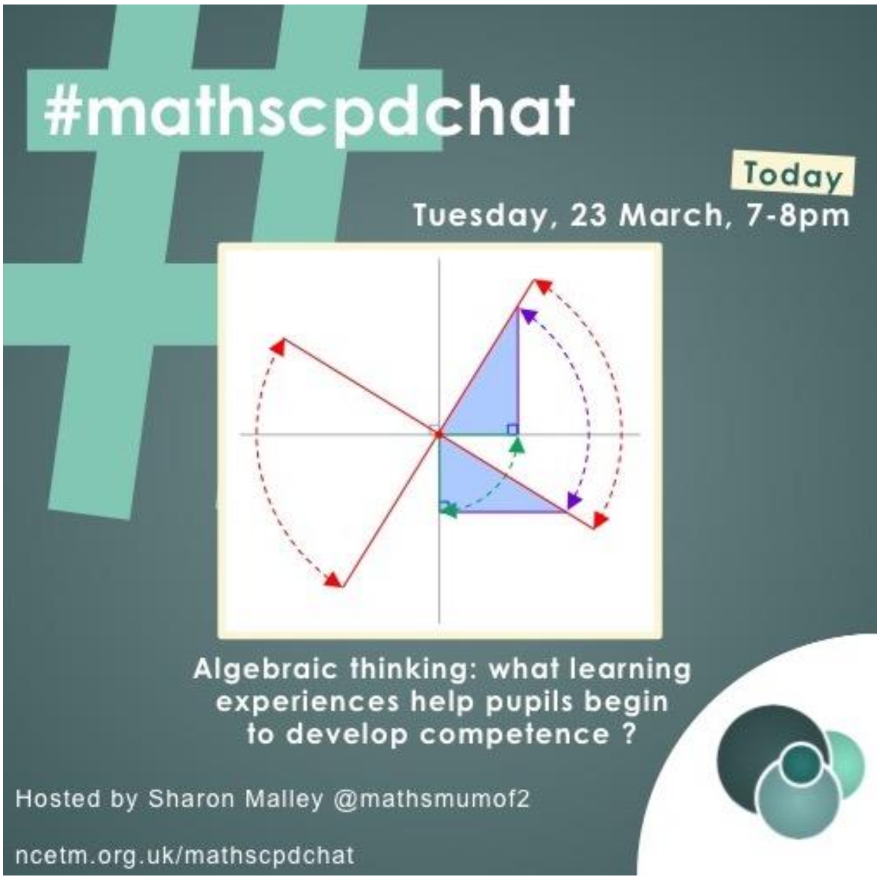
# #mathscpdchat 23 March 2021

**Algebraic thinking: what learning experiences help pupils begin to develop competence?**
Hosted by Sharon Malley

*This is a brief summary of the discussion – to see all the tweets, follow the hashtag* ***#mathscpdchat*** *in Twitter*




Among the links shared during the discussion were:

Mastery Professional Development: 1 The structure of the number system which is a Key Stage 3 guidance document from the NCETM. It addresses the first of six 'mathematical themes' identified by the NCETM as being within Key Stage 3 mathematics It was shared by Sharon Malley

Tilted Squares - Teaching Using Rich Tasks which is an article from NRICH by Charlie Gilderdale and Alison Kiddle. It contains three videos showing parts of a 75-minute lesson with a group of Year 9 students. Notes with each video draw attention to important teaching points. It was shared by Sharon Malley

Ban the equals sign which is a Mathematics Teaching 192 (ATM) article by Stephanie Prestage and Pat Perks. The authors discuss possible (mis)understandings of signs and symbols that may be a consequence of expanding their mathematical 'worlds' to include algebra It was shared by Tom Francome

Thinking with Diagrams which is a collection of essays about diagrammatic representations. The authors are all leading researchers in disciplines involved in diagrams research. The book is edited by Alan Blackwell. It was shared by Lucy

Finding and Using Patterns in Linear Generalising Problems which is an article by Kaye Stacey in Educational Studies in Mathematics. It explores responses of students aged 9 to 13 to linear generalising problems. It was shared by Lee Overy

Exploring generalization with visual patterns: tasks developed with pre-algebra students which is an article by Ana Barbosa, Isabel Vale and Pedro Palhares containing many examples of problems in which students are challenged to make generalisations about the structure of visual images. It was shared by Lee Overy

The Standards Unit: Improving Learning in Mathematics: Challenges and Strategies, (2000 - 2009), which is a book of resources developed by Dr Malcolm Swan from Nottingham University assisted by other leading maths experts in the country. It was a part of the Department of Education and Skills' response to the Smith Report, and built on research evidence strongly suggesting that learning mathematics is facilitated by actively engaging learners in mathematical thinking. It was shared by Julia Smith

Reach the peak of the pyramid which is a worksheet task. In order to complete the task pupils have to add simple algebraic expressions, and simplify the results by collecting like terms. It was shared by Julia Smith

Algebradabra - Developing a feel for school algebra which is a book by Dietmar KÜchemann, aka @ProfSmudge, published by the Association of Teachers of Mathematics (ATM). It is a collection of twenty sets of five related tasks designed to help students develop a better feel for school algebra, and to give teachers a fresh perspective on school algebra. Another collection of tasks

Algeburble - Encounters with early algebra, by the same author has recently been published. It is designed to help pupils engage with early or pre-algebra. It was shared by Mary Pardoe

The screenshots below, of chains of tweets posted during the chat, show parts of several conversations about tasks intended to help pupils begin to think algebraically both before meeting, and just as they begin to meet, conventional algebraic expressions. They include refences to arithmetical structure, structure perceivable in visual images, ways of prompting pupils, preparing pupils to understand and solve equations, ‘fruit salad’ algebra, and ‘doing and undoing’. **Click on any of these screenshots-of-a-tweet to go to that actual tweet on Twitter.**

The conversations were generated by this tweet from Sharon Malley:

**Sharon Malley** @mathsmumof2 · 20h

The discussion is moving so fast that we already have some ideas but...Q3. What activities can you use in to promote algebraic thinking before the introduction of formal symbolic work? #mathscpdchat

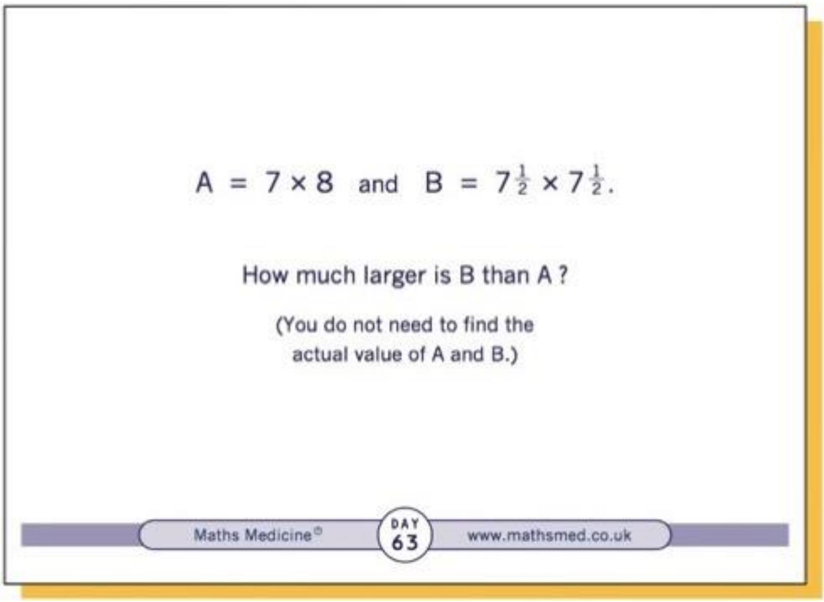
and included these from Mary Pardoe, Catherine Edwards, Sharon Malley and Lee Overy:

**Mary Pardoe** @PardoeMary · 16h

Replying to @mathsmumof2

Foundations are being built when pupils are learning to see and use structure in arithmetic ... e.g. to answer a question such as this one from one of @ProfSmudge's lovely Maths Medicine books here: mathsmed.co.uk

#mathscpdchat



**Catherine Edwards** @Edwards08C · 16h

It's that statement"You don't need to work the values" so powerful, so often ignored by students. #mathsCPDChat

**Sharon Malley** @mathsmumof2 · Mar 23

This is a superb activity that encapsulates my understanding of algebraic thinking #mathscpdchat

Which of the following can be
i) simplified
ii) simplified with a bit of manipulation
iii) cannot be simplified

$8.2 \times 10^{25} - 3.4 \times 10^{24}$

$\frac{3}{5} + \frac{1}{5}$

$\frac{9}{20} - \frac{3}{10}$

$1.75 \times 10^{-9} - 1.2 \times 10^{-9}$

$8x^2 - 4x^2$

$4x + 5x$

$3a + 2b$

$5 \times 10^3 + 3.4 \times 10^2$

$3 \times 10^2 + 6.2 \times 10^2$

$3a - a$

$\frac{7}{9} - \frac{4}{9}$

$\frac{1}{2} + \frac{1}{4}$

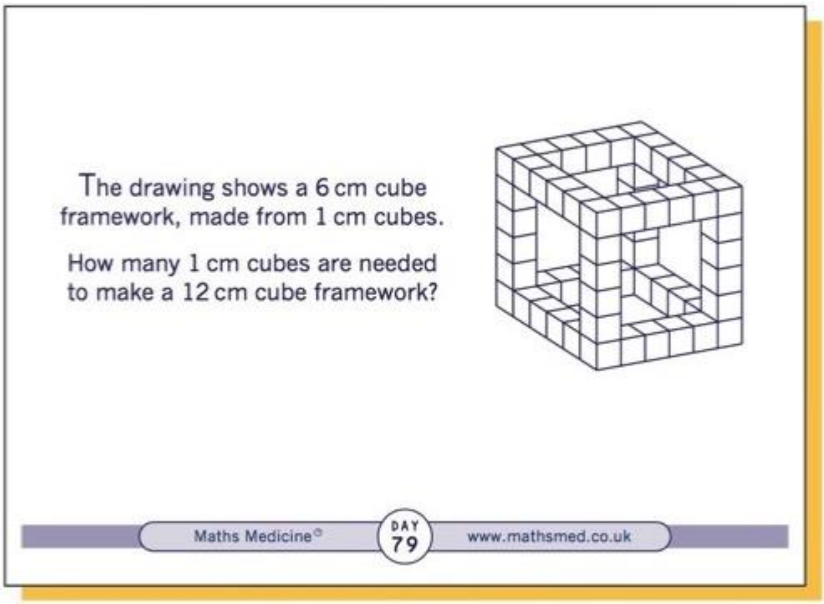
$4x + 3x^2$

**Mary Pardoe** @PardoeMary · 16h

Replying to @PardoeMary @mathsmumof2 and @ProfSmudge

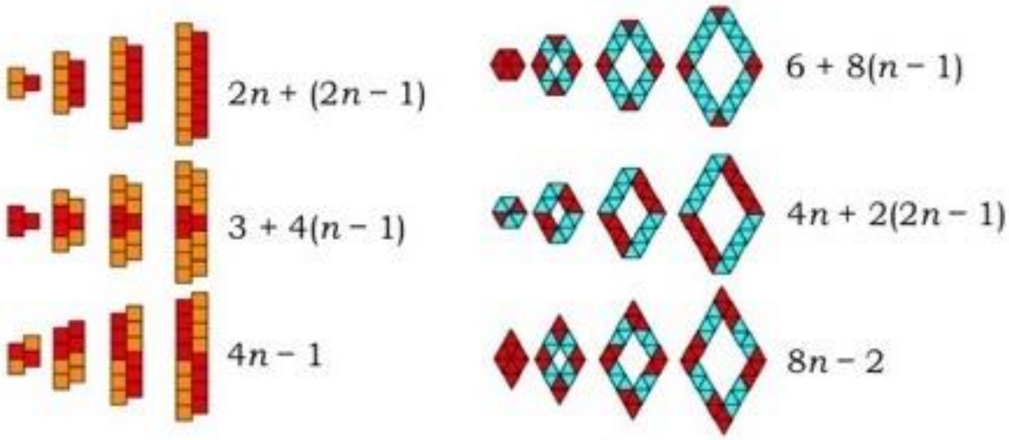
... and when learning to see structure in visual images ... this is another 'Day's dose from @ProfSmudge's Maths Medicine ... here mathsmed.co.uk

#mathsCPDchat



**Mary Pardoe** @PardoeMary · 16h

... there is masses of scope for seeing structure in visual images ... as in sequences of them ... pupils seeing them in their own idiosyncratic, individual ways ... eventually giving scope for seeing equivalence of algebraic expressions ... e.g. ...
#mathscpdchat



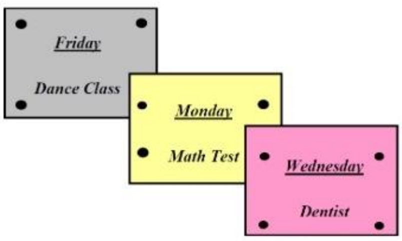

**Lee Overy** @Lwdajo · 19h

Replying to @mathsmumof2

From Exploring Generalization with Visual Patterns (Barbosa, Vale, Palhares)
#mathscpdchat

***Pins and Cards***

*Joana hangs cards on a board in her room in order to remember her appointments. She uses pins to support the cards as shown in the image.*

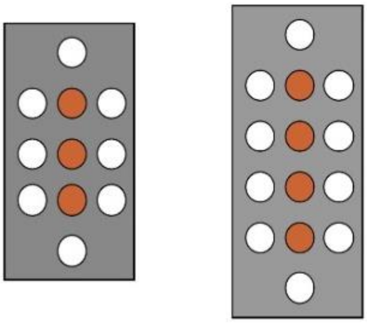



*If she continues to hang cards in her board this way:*

1. *How many pins will she need to hang 6 cards?*
2. *What if she was to hang 35 cards, how many pins would she need?*
3. *Supposing that Joana bought a box with 600 pins, how many cards can she hang in her board?*

**Sole Mio Pizzeria**

*The image shows two tables of the Sole Mio Pizzeria, one with 8 people and 3 pizzas and the other with 10 people and 4 pizzas.*



1. *How many people would sit in a table with 10 pizzas?*
2. *What if the table had 31 pizzas? How many people would be sited?*
3. *John decided to celebrate his birthday in this Pizzeria. Knowing that he invited 57 guests, how many pizzas would he have to order?*

these from Catherine Edwards, Zeenat Kokate and Sharon Malley:

**Catherine Edwards** @Edwards08C · 19h
Replying to @mathsmumof2
I'm using manipulatives more and more. Started with gateway manipulative of bar models and now using algebra tiles and double sided counters. Seems to be making a difference to students approach and understanding . Still have more thinking to do about how I use them#mathsCPDChat

**Zeenat Kokate** @zee_k20 · 20h
Replying to @MrMattock and @mathsmumof2
What's the same and what is different #mathscpdchat

**Catherine Edwards** @Edwards08C · 20h
What do you notice, what do you wonder? #mathsCPDChat

**Sharon Malley** @mathsmumof2 · 20h
I'm good at what do you notice, but need to use more what do you wonder in my lessons. #mathscpdchat

**Catherine Edwards** @Edwards08C · 20h
I'm still training my students in the what do you notice? part. They tend to be pretty competent at procedure, but less good at linking and pattern spotting #mathsCPDChat

these from Darren Elgar, Alison Hopper, Neil Almond, Brian Simber and Tom Fancome:

**Darren Elgar** @ElgarDarren · 19h
Replying to @mathsmumof2
Sorry to jump in as I don't teach algebra usually but I just happened to try out something with one of my pupils yesterday.

Numicon and a pan balance with a number 'hidden' in envelopes.

2 envelopes and 4 one side and one envelope and a 10 on the other. What is in the envelope?

**Alison Hopper** @AlisonHopperMEI · 18h
I used to do this! Kids loved it and so did I! #mathscpdchat

**Darren Elgar** @ElgarDarren · 18h
My pupil doesn't do much in the way of writing of formal lessons so this sort of thing is ideal. He had been playing around with the balance, putting things in either side.

I warmed him up with an envelope on one side, then one on each side.

**Neil Almond** 🙋 @Mr_AlmondED · 19h
Replying to @mathsmumof2
At primary. I have a bag. Do you know what's in the bag? Of course you don't. So let's write n as it means a number. I'm going to put 2 things in my bag. N+2. What if I now told you there were 15 items altogether? What would 'n' be? Discuss.

Repeat.

May be doing it wrong.

**Brian Simber** @Mr_Simbs · 18h
Neil, 'things' and 'items' don't necessarily have to be the same. You could say,"I have a number of counters in a bag, but I don't know how many, so I'll call the number N. If I then add 2 more counters the total is now 15. What is N?. Thoughts?

**Tom Francome** @TFrancome · 17h
atm.org.uk/write/MediaUpl..

Prestage and Perks: Ban the equals sign!

these from Julia Smith, Peter Mattock and Nuriye Sirinoglu:

**Tessmaths** @tessmaths · 19h

Replying to @mathsmumof2

I was always disappointed to find trainees thinking that the algebraic letters stood for fruit and veg! Algebra isn't a fruit salad! #mathscpdchat

a Apple b banana c carrot ! 🥕

**Mr Mattock FCCT NPQSL** @MrMattock · 20h

Yeah it definitely shouldn't be the way pupils are introduced to algebra.

**Nuriye Sirinoglu** @NuriyeSingh · 9h

The understanding of variable and variable vs parameter becames super important when they progress in math. I also love ' create your eqn' and 'function machines' to introduce algebra. Not apples n bananas

**Tessmaths** @tessmaths · 8h

Agreed...the 'create your equation' is one of Malcolm Swans, from The Standards Unit here...via @mrbartonmaths A superb lesson... mrbartonmaths.com/resources/stan...

and these from [Mary Pardoe]() and [Sharon Malley]():

**Mary Pardoe** @PardoeMary · 15h

Replying to @mathsmumof2

...and there is doing-and-undoing ... something even very young children think about ... this is the VERY well used example ... but there are infinitely many others encountered from an early age that can be built on ... #mathscpdchat


(to read the discussion sequence generated by any tweet look at the ‘replies’ to that tweet)

Some of the areas where discussion focussed were:

**teachers’ understandings of the nature of algebraic thinking:**

- that **the expression of generality is at the heart of algebra and algebraic thinking** … algebraic notation, and techniques for its manipulation, including conventions, should arise naturally from exploring arithmetical structures … but that there are other, possibly more interesting, structures and patterns that can be explored, some of which pupils will have encountered naturally at a young age (such as order relationships in sequences of actions that can be ‘undone’ … e.g. taking the lid off a tin, putting something in the tin, then putting the lid back on the tin) … a teacher commented that they had enjoyed an example from Dave Hewitt: that very young children are thinking algebraically when they say ‘I goed to the shop’ having generalised that you add ‘ed’ to the end of a verb when the doing is in the past;
- some people commented that **explorations that are initially geometric** can generate thinking that builds towards algebraic understanding … for example when comparing the areas of related 2-D shapes (possibly using bands stretched round pins on a geoboard) or when thinking about how the change under enlargement in volumes of 3-D objects are related to how their linear dimensions change;
- that sometimes **‘we rush into algebra when understanding of number operations is not secure’**;
- that when students start to use algebra competently they sometimes **‘”uncover” things about the number system that they never really appreciated**’;
- that algebraic **conventions need to be taught explicitly** ‘so that pupils can communicate their algebraic thinking effectively’ … and ‘pupils will encounter formulae in subjects other than maths’ … teachers agreed with this, but added the caveat ‘but their thinking and reasoning will be more effective if we can initially link it to the concrete’;
- that there is **lots of scope for seeing structure in visual images** … such as in sequences of visual images … that pupils can be encouraged to see and express general structures in sequences ‘in their own individual (and different) ways … that such different ways of seeing and expressing generalities in their own words lay foundations for later understanding of equivalence of algebraic expressions, and for seeing a need to manipulate expressions in order to convince others that they are equivalent;
- at least one teacher thinks of algebraic thinking as (including) ‘noticing underlying structure, generalising, and becoming comfortable with rearranging’ … that **multiple (various**

**different) representations can exhibit the same structure** … and some representations may or may not use written symbols … that it may be helpful to adopt a strategy of exploring representations using written symbols only after exploring other representations;

- one teacher pointed out that thinking algebraically enables us to solve problems;

**the host asked what teachers believe to be reasons why so many people express a dislike for algebra:**

- the host displayed the following section from NCETM *Mastery Professional Development* materials (link provided above)

> **Common difficulties and misconceptions**
>
> Dietmar Küchemann (1978)[i] identified the following six categories of letter usage by students (in hierarchical order):
>
> - **Letter evaluated**: the letter is assigned a numerical value from the outset, e.g. $a = 1$.
> - **Letter not used**: the letter is ignored, or at best acknowledged, but without given meaning, e.g. $3a$ taken to be 3.
> - **Letter as object**: shorthand for an object or treated as an object in its own right, e.g. $a$ = apple.
> - **Letter as specific unknown**: regarded as a specific but unknown number and can be operated on directly.
> - **Letter as generalised number**: seen as being able to take several values rather than just one.
> - **Letter as variable**: representing a range of unspecified values, and a systematic relationship is seen to exist between two sets of values.
>
> The first three offer an indication of the difficulties and misconceptions students might have. The last three outline the progression that students need to make as they develop an increasingly sophisticated view of the way letters are used to represent number.

- some teachers expressed the view that **the reasons are 'cultural unfortunately'** … that maths is seen as a 'tool for "real life" and not an end in itself' … that the purpose of maths is to find right numerical 'answers';
- other teachers believe that people 'can't see the point of it …can't see that it's a time saver';
- someone commented that even well-educated people 'intuitively understand that they are **unlikely to ever use algebra for the rest of their lives**' … that 'life can be boring if you're made to do years of algebra';
- teachers responded to the previous comments by pointing out that, although people are **'constantly generalising, pattern spotting and modelling'**, they do not see such human behaviour as having anything to do with 'algebra in school' … a short discussion about this prompted a contributor to ask whether people (students) would find algebra to be easier if they had stronger generalising, pattern spotting and modelling skills;
- another response to the comment about not 'doing algebra' beyond school days was to state that **'many everyday problems require algebraic thinking** … ask anyone who has had to write a formula for an excel spreadsheet';
- several teachers reported that **Y7/8 students reach them 'already having a fear of algebra'** … 'I had Y8 low attainers who 'hated' algebra. As soon as I showed them they had

been doing it for years, they were fine' … 'we play with unknown numbers using whatever symbol they fancy without me using the word 'algebra'';

- that 'to some people algebra encapsulates all that is difficult/mysterious about maths' when/because it is **'introduced badly, developed quickly, not co-created, not a well-designed system for representation …'** … when it is taught so that 'you "do algebra" rather than learn the benefits of working generally';
- ***Thinking With Diagrams* by Alan Blackwell** (link provided above) was mentioned and discussed briefly … it addresses reasons why people's experiences of algebra at school are often so poor;

**the host invited contributors to describe tasks that they use to generate algebraic thinking before they engage in any formal symbolic work:**

- teachers reported that they use function machines, worded formulae, many **manipulatives** including algebra tiles and double-sided counters, bar models;
- 'always, sometimes, never' tasks;
- 'convince me …' prompts/challenges;
- 'what is the same and what is different?' prompts/challenges/explorations/discussions/explanations;
- 'what do you notice, what do you wonder?' prompts/questions/explanations/…
- setting up 'pan balance' situations/challenges as **exploratory first steps in solving simple linear equations** … having unknown numbers of things inside envelopes … using missing-number-box tasks;
- tasks in which the **aim is to see and use structure and pattern in visual images, make and test conjectures, and generalise;**
- there was a discussion about the **damage to clear thinking** that is a usual consequence of **indulging in 'fruit salad' algebra** (using letters of the alphabet to represent pieces of fruit … eg 'a' stands for 'apple', 'b' stands for 'banana) … that it is hard to shift from this way of interpreting a symbol to seeing it as a variable … a discussion followed about examples of ways in which letters used 'properly' in maths do not represent variables (e.g. 'm' as an abbreviation of 'metres') … N.B. the categories of letter usage identified by Dietmar Küchemann, and quoted in the NCETM paragraph shown previously;

**finally, the host asked teachers whether they teach students to make conjectures as an essential and usual aspect of doing mathematics:**

- a teacher commented that conjecturing is not 'isolated to algebra, more part of the overall culture of a classroom' … that 'it is **not so easy to achieve in practice'**;
- **there was more discussion confirming the view that mathematical conjecturing is involved in algebraic thinking because it is an aspect of doing mathematics.**