

Planning to teach division of fractions – Supporting document

This document, which accompanies the 'Planning to teach division of fractions' video, gives guidance on planning lessons that allow students to understand the structures underpinning methods for dividing by fractions.

Part 1 – The big idea

Students need to:

- Understand division as grouping (the quotative structure) as well as sharing (partitive)
- Appreciate that equivalent divisions can be generated by multiplying or dividing both the dividend and the divisor by the same amount
- Understand that fractions can be thought of as divisions and therefore that equivalent divisions and equivalent fractions are related ideas
- Understand how to find the reciprocal of any number and what it means i.e. that the product of a number and its reciprocal is 1.

What things typically go wrong?

Common misconceptions or causes of confusion include:

- Not appreciating division as anything else other than sharing
- Thinking that multiplication always makes bigger and, therefore, division always makes smaller
- As a result of these two misconceptions above, reading divisions such as $8 \div \frac{1}{2}$ as "8 divided into half" or similar and giving an answer of 4
- Following a rule like "turn the fraction upside down and multiply" without an understanding of the structures underpinning the rule.

Part 2 – Prerequisites

Expectations from KS2

Pupils should be taught to:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]
- divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places

From the DfE Mathematics programmes of study: key stages 1 and 2 (Sept 2013)

While fractions will have been taught at KS2, division involving fractions will only have involved dividing a fraction by a whole number where an image of sharing (partitive) may well have been dominant. The division of any number by a fraction is a completely new idea at KS3.

Part 3 – Key teaching aspects

Key message

Students need to interpret division as grouping (quotative) and therefore need to be able to read a division such as $15 \div 3$ as “how many threes are there in 15?”. This is important earlier on in their learning when tackling calculations such as $400 \div 25$ where a sharing model is inefficient, but it is also crucial at KS3 where calculations such as $8 \div \frac{1}{2}$ or $\frac{3}{4} \div \frac{1}{5}$ make no sense when applying an image of sharing (partitive).

Students also need to understand that equivalent divisions can be generated by multiplying and dividing both the dividend and the divisor by the same amount, and to relate this idea to what they already know about equivalent fractions.

The aim here is to build on students’ existing understanding of division and for them to see that the structures involved are exactly the same when fractions are used. When students are able to transform a division such as $\frac{3}{4} \div \frac{2}{3}$ into an equivalent division where the divisor is 1, and understand the idea that the product of reciprocals is 1, they will be well on the way to mastering the key idea behind division of fractions.

Every attempt should be made to avoid tricks and mnemonics like ‘KFC’ (keep, flip, change) or ‘turn it upside down and multiply’ and the aim should be to teach the structures that underpin them.

KS3 expectations and related ideas

Students should be taught to:

- use the four operations, including formal written methods, applied to [...] proper and improper fractions, and mixed numbers, all both positive and negative
- recognise and use relationships between operations including inverse operations
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)

From the DfE Mathematics programmes of study: key stage 3 (Sept 2013)

Part 4 – Why this is important

How will this support future learning?

The ideas in this topic helps students develop a deeper understanding of division, and support connections with learning in the following areas:

- Reinforcing links between calculations and fractions
- Equivalent fractions and changing fractions to decimals
- Reinforcing methods associated with addition and subtraction of fractions (finding equivalent denominators)
- Division with algebraic fractions
- Rearranging formulae or solving equations with fractional coefficients
- Ratio problems involving fractional scale factors.

Useful links



NCETM Secondary Mastery Assessment materials

www.ncetm.org.uk/classroom-resources/assessment-materials-secondary/

In particular, activities on page 11 are related to division of fractions



Variation Theory: Division with fractions

<https://variationtheory.com/2020/02/22/division-with-fractions/>



Median

Don Steward Mathematics Teaching 10 – 16

<https://donsteward.blogspot.com/2014/05/dividing-fraction-views.html>