Planning to teach multiplicative reasoning – Supporting document

This document, which accompanies the ‘Multiplicative reasoning’ video, offers suggestions to support the process for planning to teach multiplicative reasoning.

Part 1 – The big idea

Students need to:

Understand that any two numbers can be connected by a multiplicative relationship with a single multiplier.

This includes:

- Use of ‘proportion’ as a verb, rather than a noun
- A shift from additive relationships to multiplicative relationships e.g. ‘lots of’ and scaling
- Maintaining proportionality through multiplication and division
- Eventually the use of a single (possibly fractional) multiplier.

What things typically go wrong?

Common misconceptions include:

- Viewing relationships as additive, and considering **only** differences
- Difficulty working with non-integer multipliers
- “Multiplication always makes it bigger/Division always makes it smaller”

Part 2 – Prerequisites

Expectations from KS2

Pupils should be taught to:

- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

DfE Mathematics programmes of study: key stages 1 and 2 (2013)

Prerequisite skills

It is important to check the students are secure in their ability to:

- recognise fractions of amounts in various standard representations (e.g. number line, bar model and part-part-whole area models)
- relate multiplication and division as inverses
- understand multiplication as scaling
- recognise and create equivalent fractions
- work with fractions: addition and subtraction, multiplication and division.

Part 3 – Key teaching aspects

Key message

Students need to be aware that:

Any two numbers can be linked by multiplication and division

leading to…

Any two numbers can be connected by a multiplicative relationship with a single multiplier.

KS3 expectations

Students should be taught to:

- change freely between related standard units [for example time, length, area, volume/capacity, mass]
- use scale factors, scale diagrams and maps
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part-part or part-whole ratio; express the division of a quantity into two parts as a ratio
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions.

DfE Mathematics programmes of study: key stage 3 (2013)

Key language

Students should be expected to use correct mathematical language and answer in full sentences.

Key vocabulary includes:

- whole
- part
- digit
- equivalent
- multiple
- numerator
- denominator.

Key skills

Students need to be able to:

- Use known facts and relationships to calculate new ones
- Move from one number to another multiplicatively
- Construct and use informal representations to support their reasoning.

Key representations

- bar model, to support understanding of proportional parts of a whole
- ratio tables, to support understanding in multiplicative reasoning.

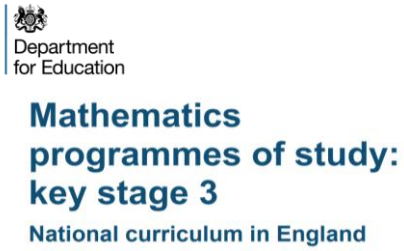
Part 4 – Why this is important

How will this support future learning?

Multiplicative reasoning underpins a huge number of elements of the mathematics curriculum, such as:

- Fraction/decimal/proportion conversions
- Equivalent fractions
- Percentages, including growth and decay problems
- Enlargement and similarity
- Trigonometry
- Compound units
- Iterative processes
- Geometric sequences
- Gradient and rate of change
- Stratified sampling

Useful links

NRICH	NRICH Mixing Lemonade https://nrich.maths.org/6870 Number Lines in Disguise https://nrich.maths.org/13452
	NCETM Mastery Professional Development Materials Key Stage 3, (3.1 Understanding multiplicative relationships) www.ncetm.org.uk/media/mqfp3xb3/ncetm_ks3_cc_3_1.pdf
	Median Don Steward https://donsteward.blogspot.com/2019/02/boxes-resources.html
	www.maths4everyone.com/resources/sats-questions.html
	DfE Mathematics programmes of study www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study
MathsBot.com Tools for Maths Teachers	MathsBot.com https://mathsbot.com/