

## Planning to teach directed number – Supporting document

This document, which accompanies the 'Planning to teach directed number' video, offers suggestions to support the process for planning to teach directed numbers.

### Part 1 – The big idea

#### Students need to:

- be able to perform the four operations on negative numbers.

This guidance will focus on only two of those operations: addition and subtraction.

#### What things typically go wrong?

Common misconceptions or mistakes due to incomplete understanding include:

- not understanding the structure of a calculation and relying solely on a rule or mnemonic like “Minus minus is plus” or “Two negatives make a positive”
- confusion over what larger means in the context of negative numbers. For example, some students might think that  $-7$  is greater than  $2$  because they are focusing on the magnitude of the number
- confusion caused by the fact that the same symbol is used to denote the operation of subtraction (as in  $5 - 3$ ) and the fact that a particular number is negative (as in  $-5$  or  $\cdot 5$ ). This is a key stumbling block!
- thinking (based on earlier experiences before negative numbers were introduced) that “addition ALWAYS makes bigger” and “subtraction ALWAYS makes smaller”.

## Part 2 – Prerequisites

### Expectations from the KS2 National Curriculum Programme of Study

**Year 4:** count backwards through zero to include negative numbers

**Year 5:** interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

**Year 6:** use negative numbers in context, and calculate intervals across zero

and from KS2 National Curriculum notes and guidance:

*“Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature.”*

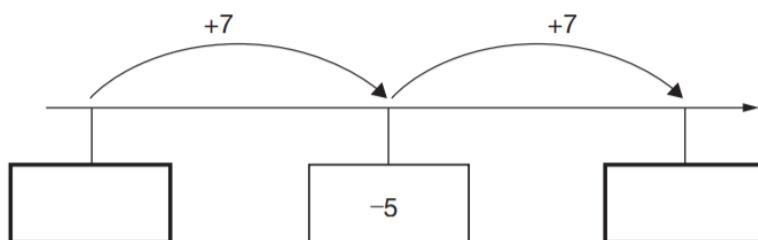
### Prerequisite skills

It is important to check that students are secure in their ability to:

- order numbers with a mixture of negative and positive numbers
- read numbers from a temperature scale
- fill in the gaps - especially backwards through zero.

Here is part of a number line.

Write the missing numbers in the boxes.



N.B. It is important to note that these KS2 experiences with negative numbers are concerned with an understanding of what negative numbers mean, how they appear in different contexts, and how they fit in the number system. As such, the number line is a powerful tool for this understanding.

In KS3, however, the emphasis shifts to working with more complex calculations with these numbers, such as  $-4 + (-2)$  and  $-5 - (-3)$ . This is where a number line can be problematic and the use of double sided counters (or algebra tiles) has proved more effective for many teachers.

## Part 3 – Key teaching aspects

### Key message

Students need to think of positive and negative numbers in a cardinal (or quantity) sense as opposed to an ordinal (or positional) sense. The double sided counters (or algebra tiles) allow for this new sense and are in contrast to the number line which emphasises the ordinal.

Students should:

- be able to represent any positive or negative number with double sided counters (or algebra tiles)
- know that any number can be represented using a mixture of positive and negative counters in multiple ways
- understand the idea of the 'zero pair' and understand that the value does not change when zero pairs are added to a collection of counters
- understand addition as adding more counters and subtraction as removing counters, and be able to create addition and subtraction number sentences with the counters using this knowledge.

### KS3 expectations

Students should be taught to:

- order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols  $=$ ,  $\neq$ ,  $<$ ,  $>$ ,  $\leq$ ,  $\geq$
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative.

N.B. The first of these statements is an extension of the earlier KS2 work. It is the second one that represents new learning for KS3 students, and this guidance focuses on this in the context of directed numbers and, in particular, addition and subtraction

### Key language

Students should be expected to use correct mathematical language and answer in full sentences.

The language we use can really help students distinguish between the operator and the symbol. They look identical in this topic but sometimes the subtraction sign is denoting an operation and sometimes to denote a negative number.

$5 + -2$

Students should read the calculation above as “positive five add negative two”, **not** “five plus minus two” and should say the full number sentence when giving an answer in class in order to embed the concepts.

### Key representations

- Double sided counters or algebra tiles

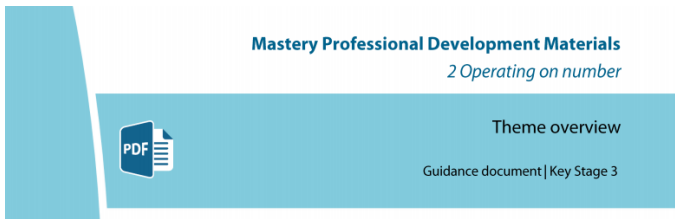
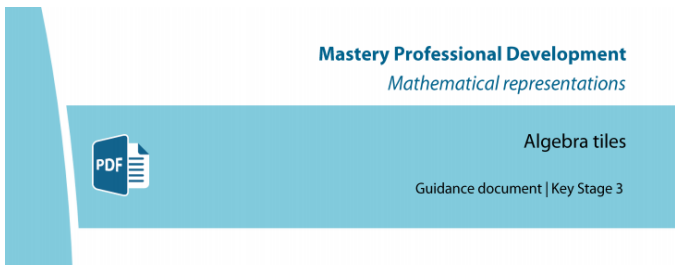

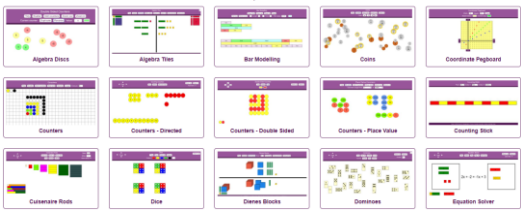
Digital representations of these and other tools can be found at [mathsbot.com](https://mathsbot.com)

## Part 4 – Why this is important

A deep understanding of directed number supports future learning in the following areas:

- Simplifying algebraic expressions

## Useful links

 <p><b>Mastery Professional Development Materials</b> <i>2 Operating on number</i></p> <p>Theme overview Guidance document   Key Stage 3</p>	<p>NCETM Secondary Mastery Professional Development Materials (2.1: Arithmetic procedures and, in particular, key idea 2.1.1.1 Understand the mathematical structures that underpin addition and subtraction of positive and negative numbers on pages 14 - 17)</p> <p><a href="https://www.ncetm.org.uk/media/xhgegzug/ncetm_ks3_cc_2_1.pdf">https://www.ncetm.org.uk/media/xhgegzug/ncetm_ks3_cc_2_1.pdf</a></p>
 <p><b>Mastery Professional Development</b> <i>Mathematical representations</i></p> <p>Algebra tiles Guidance document   Key Stage 3</p>	<p>NCETM Using mathematical representations at KS3: algebra tiles (p. 3 onwards)</p> <p><a href="http://www.ncetm.org.uk/media/8d84e790f22943a/ncetm_ks3_representations_algebra_tiles.pdf">www.ncetm.org.uk/media/8d84e790f22943a/ncetm_ks3_representations_algebra_tiles.pdf</a></p>
 <p>Department for Education</p> <p><b>Mathematics programmes of study: key stage 3</b> National curriculum in England</p>	<p>DfE Mathematics programmes of study</p> <p><a href="http://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study">www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study</a></p>
<p>Manipulatives</p> 	<p>Online virtual counters</p> <p><a href="https://mathsbot.com/#Manipulatives">https://mathsbot.com/#Manipulatives</a></p>