A Departmental Workshop

Algebra

This is a suggested plan for a professional development session. It has been written to support anyone wishing to lead such a session with a group of teachers and the green 'key points' sections are intended as a support specifically for such a facilitator in guiding discussions.

N.B. These workshops have been written to provide enough professional development activity and discussion for one session of approximately one hour with the option of further activity (as outlined in the 'Possible next steps' section at the end). This final section references the NCETM Secondary Mastery Professional Development Materials which can be found here [*www.ncetm.org.uk/secondarymasterypd*](www.ncetm.org.uk/secondarymasterypd)

Overview

Algebraic thinking and the use of symbols to express generality are important ideas in students' mathematical development. For some it represents an exciting shift into the general (rather than calculating with particular numbers). But for many, it is a mystifying collection of rules governing the manipulation of symbols which bear no relationship in their minds to numbers or other mathematical ideas they have previously met and understand.

This workshop builds on the ideas introduced in the 'Structural Arithmetic' workshop and is best engaged with after it. It gives you the opportunity to work with other teachers and discuss:

- what algebra and algebraic thinking really are
- how students can be introduced to algebraic thinking and the use of symbols to express generalisations
- what implications there might be for your future practice and curriculum development.

Activity 1: Brainstorm with a group of colleagues what you understand by the word 'algebra'. Recall some activities that you use in your current practice which you feel support students' understanding of algebra. Call to mind a student of yours (past or present) who you felt was proficient at algebra. What knowledge, skills and awarenesses did/do they have?

Discussion

- What does 'algebra' mean to you? Brainstorm some words and phrases that you would use to describe algebra. Is there any categorisation that you can apply to these words and phrases? Which refer to knowledge, which to skills, and which to understanding an idea or concept?

> **Key Point**: A useful distinction to make is between thinking algebraically, and using symbols to express that thinking. Algebra is often introduced as the use of symbols rather than the thinking that is required prior to using symbols or even that which is expressed by the use of algebraic symbolism.

- Following this initial discussion, are there any other words or phrases you wish to add?

Activity 2: Using the handout (slide 1 in the PowerPoint file), discuss each of the identities. What is each one expressing? Try putting some particular numbers into the expressions (if necessary) to convince yourself that they are true. Are they true for all numbers? What is the general arithmetic structure that is being expressed? Is it possible to draw a diagram to show what is being expressed?

What do the symbols symbolise?

$5a + 6a = 11a$

$5p - 5q = 5(p - q)$

$(x + 3)(x + 5) = x^2 + 8x + 15$

$(a + b)^2 = a^2 + 2ab + b^2$

$x^2 - y^2 = (x + y)(x - y)$

Key Point: Any algebraic identity like the ones on the handout is an expression of some generality. $5x + 6x = 11x$ is an expression of the fact that 5 multiplied by ***any number*** add 6 multiplied by ***the same number*** is equal to 11 multiplied by ***that number***.

N.B. You may remember from the 'Structural Arithmetic' workshop (Activity 3) this set of calculations; $(7 \times 6) + (7 \times 4)$; $(4 \times 9) + (9 \times 6)$; 8^2 + double 8; $3^2 + (7 \times 3)$ which expressed just this property of arithmetic.

We often refer to this topic in our scheme of work by the phrase 'collecting like terms' and explain that it 'just' requires collecting the x's together. But do students really understand what is being expressed? Do they know that the letters are variables (i.e. standing for any number) and that the algebraic symbols are a shorthand for recording a fundamental arithmetic structure?

Teaching it as an algorithm such as '5 apples and 6 apples equals 11 apples' (often referred to as 'fruit salad algebra'!) can obscure that the letter stands for a number, and reinforce the idea that algebra bears no relationship to number and arithmetic.

Multiplying out brackets like $(x + 3)(x + 4)$ is really an expression of the general principle that is used when, for example, two 2-digit numbers are multiplied together and shows why there are 4 sub-products to be added.

Grid multiplication / area diagrams can often show clearly why these rules apply.

The difference of two squares can be pictured as just that: the difference between two squares.

Activity 3a: View the animated slides (slides 2-7 in the PowerPoint file) together and discuss what is being shown

Activity 3b: Do the following activity together:

- Choose any number and multiply it by itself (e.g. $7 \times 7 = 49$). Now add 1 to the first number, subtract 1 from the second and calculate the new product.
- Do this for a range of different starting numbers
- What do you notice? Does it always work? For all numbers?
- What if you add and subtract 2 (or 3, or ½, or 0.1, or …)? What happens to the new product this time?
- Use symbols to express the generalisation being exposed in these examples.

Discussion

Discuss these two ways (activities 3a and 3b) of helping students to understand the idea of the difference of two squares and the merits of each one.

Try to come up with some other examples of algebraic manipulation that you teach, and discuss what activities might allow students to see the structure behind them and understand how symbols can be used to express that structure.

Key Point: In a landmark article, Dave Hewitt makes the following point about the difference between thinking algebraically and using algebraic symbols:

"3x+1 is an algebraic statement. But let us not confuse the two words involved – algebraic and statement. The algebra is not the statement but is the work you do in order to get yourself in a position where you could make a statement."
[Hewitt, D. (1998) 'Approaching arithmetic algebraically', *Mathematics Teaching 163, 19-29*]

Much of the teaching of algebra can be done as a series of tricks and mnemonics without giving students a sense of the meaning behind the symbols, i.e. in a way that results in them doing no algebraic thinking at all.

Possible next steps

This session may have surfaced some more long-term developments that you and your department (or group of teachers you are working with) wish to take. This section offers a way of doing this at some point in a future session or series of sessions.

Have a look at:

'Core Concept 1.4: Simplifying and manipulating expressions, equations, and formula' from the NCETM Secondary Mastery PD Materials Theme 1 and *'Core Concept 2.2 Solving linear equations'* from Theme 2.

Discuss:

- how these ideas might influence your own teaching of algebra in Key Stages 3 and 4
- how these ideas might support developments in your scheme of work.