MathsHUBS
Resources

National Centre
for Excellence in the
Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.25 Using compensation to calculate

Teacher guide | Year 6

Teaching point 1:

For multiplication, if there is a multiplicative change to one factor, the product changes by the same scale factor.

Teaching point 2:

For division, if there is a multiplicative change to the dividend and the divisor remains the same, the quotient changes by the same scale factor.

Teaching point 3:

For division, if there is a multiplicative increase to the divisor and the dividend remains the same, the quotient decreases by the same scale factor; if there is a multiplicative decrease to the divisor and the dividend remains the same, the quotient increases by the same scale factor.

Overview of learning

In this segment children will:

- explore how, for multiplication, if one factor is multiplied (or divided) by a quantity, the product will be multiplied (or divided) by the same quantity
- explore how, for division, if the dividend is multiplied (or divided) by a quantity, the quotient will be multiplied (or divided) by the same quantity
- explore how, for division, if the divisor is multiplied by a quantity, the quotient will be divided by the same quantity; if the divisor is divided by a quantity, the quotient will be multiplied by the same quantity
- learn to recognise where these concepts can be applied to make a calculation easier to solve.

In segment *2.18 Using equivalence to calculate* children learnt to manipulate multiplication and division equations while preserving the product (in multiplication) and the quotient (in division). Children will now build on this knowledge by identifying other multiplicative relationships and exploring how these relationships can be used to solve problems.

In *Teaching point 1* children will learn that if one factor in a multiplication equation increases by a scale factor, the product will increase by the same scale factor. Doubling and halving are used to begin with, building towards generalisations that can be applied to any whole number. It is important that children understand that the increase/decrease is multiplicative and is not related to addition or subtraction. This is explored in the dòng nǎo jīn problem in step *1:6*.

Children will then recap earlier learning on the relationships between the 5 and 10 times tables (segment *2.5 Commutativity (part 2), doubling and halving*), and multiplying by 10, 100 and 1,000 (segment *2.13 Calculation: multiplying and dividing by 10 or 100*) and apply this knowledge as a useful strategy to solve calculations. Children should be encouraged to look for ways in which they can use known facts to simplify problems, and to use mental calculations when working with familiar numbers and times table facts.

Teaching points 2 and *3* explore the effect on the quotient when the dividend or the divisor is increased or decreased multiplicatively. Note that children must already be secure in accurately using the language of dividend, divisor and quotient.

By the end of this segment children should be able to confidently identify and explain relationships between pairs/sequences of expressions and equations in multiplication and division contexts, and apply this knowledge as a useful strategy to solve a variety of calculations.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

For multiplication, if there is a multiplicative change to one factor, the product changes by the same scale factor.

Steps in learning

1:1 In segment *2.18 Using equivalence to calculate* children learnt to use equivalence to develop efficiency in calculation through adjusting the factors (in multiplication) and the dividend and divisor (in division). This segment builds on this knowledge. In this teaching point we consider multiplication contexts where one of the factors is increased or decreased.

Show children a problem where one of the factors is doubled. In the example below, the length of each box is double the height. Show how the boxes can be stacked in two different ways to create the equations $20 \times 3 = 60$ and $40 \times 3 = 120$. Write both equations and ask children what they notice about them. Draw out the answer that one of the factors has doubled and the product has also doubled.

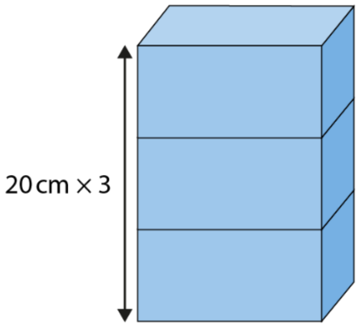
Work towards the following generalisation: ***'If I double one factor, I must double the product.'***

Explore how we can also use the following generalisation: ***'If I multiply one factor by two, I must multiply the product by two.'***

This second generalisation will be developed further in step *1:2*, when 'two' can be replaced by any number.

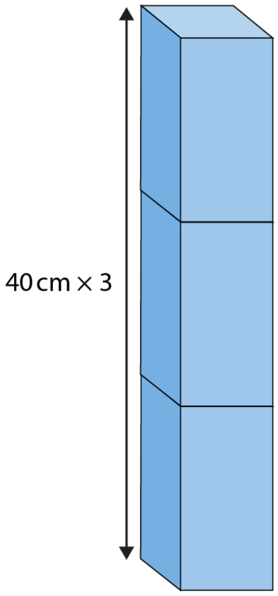
'Pip is stacking boxes that are 20 cm × 20 cm × 40 cm.'

- *'When she stacks three boxes like this, the height of all three is 60 cm.'*



20 cm × 3 = 60 cm

- *'When she stacks the same three boxes like this, the height of all three is 120 cm.'*



40 cm × 3 = 120 cm

Compare the two equations:

$$\begin{tikzcd} 20 \arrow[d, "\text{double}"] & \times & 3 & = & 60 \arrow[d, "\text{double}"] \\ 40 & \times & 3 & = & 120 \end{tikzcd}$$

- *'If I double one factor, I must double the product.'*
- *'If I multiply one factor by two, I must multiply the product by two.'*

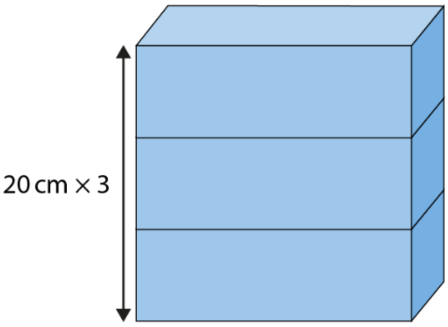
1:2

Now return to the previous example, but this time consider boxes where the length of the box is three times its height. Show how the boxes can be stacked to create the equations *20 × 3 = 60* and *60 × 3 = 180*. Ensure children understand what is different compared to the previous example (the boxes are longer).

Once again write both equations and ask children what they notice about them. Draw out the answer that one of the factors has been multiplied by three and the product has also been multiplied by three. Build on the generalisation from step *1:1* using the following stem sentence: ***'If I multiply one factor by ___, I must multiply the product by ___.'***

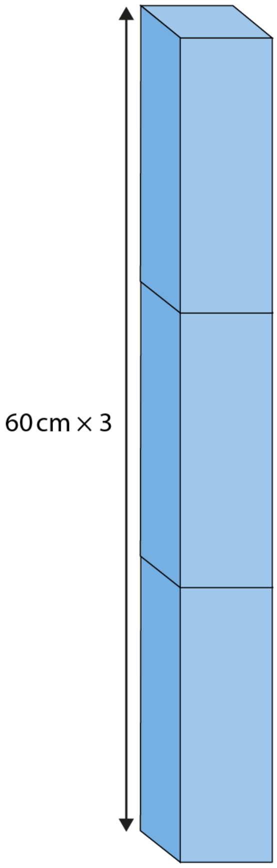
'Now Pip is stacking boxes that are 20 cm × 20 cm × 60 cm.'

- *'When she stacks three boxes like this, the height of all three is 60 cm.'*



20 cm × 3 = 60 cm

- *'When she stacks the same three boxes like this, the height of all three is 180 cm.'*



60 cm × 3 = 180 cm

Compare the two equations:

$$\begin{tikzcd} 20 \arrow[d, "\times 3"] & \times & 3 & = & 60 \arrow[d, "\times 3"] \\ 60 & \times & 3 & = & 180 \end{tikzcd}$$

- *'If I multiply one factor by three, I must multiply the product by three.'*

1:3

Next explore a context where one of the factors is decreased. To begin with, use simple numbers with products that fall within the known times tables, so that children can focus on the structure.

Show children a context where one factor is halved. In the example opposite, Josh has half as many lollipops as Ben. Write an equation for Ben (*10 × 1 = 10*) and for Josh (*5 × 1 = 5*). Compare the equations and ask children what they notice. Make the observation that one factor has been halved and the product has also been halved.

Repeat this context so that children can identify a pattern. Each time, compare the equations and encourage children to make the observation that one factor has been halved and the product has also been halved.

Work towards the following generalisation: ***'If I halve one factor, I must halve the product.'***

Explore how we can also use the following generalisation: ***'If I divide one factor by two, I must divide the product by two.'***

Now extend this to use a context where one factor is divided by a whole number other than two. In the final example on the next page, the number of lollipops is now divided by five. Use the following stem sentence: ***'If I divide one factor by ___, I must divide the product by ___.'***

Halving:

- *'Ben buys some bags of lollipops; each of his bags contains 10 lollipops. Josh also buys some bags of lollipops; each of his bags contains half as many lollipops.'*

	Ben – 10 lollipops in each bag	**Josh – 5 lollipops in each bag**
1 bag		

Ben: 10 × 1 = 10

Josh: 5 × 1 = 5

Compare the two equations:

$$\begin{tikzcd} 10 \arrow[d, "\text{half}"] & \times & 1 & = & 10 \arrow[d, "\text{half}"] \\ 5 & \times & 1 & = & 5 \end{tikzcd}$$

- *'If I halve one factor, I must halve the product.'*
- *'If I divide one factor by two, I must divide the product by two.'*

Continue to work through similar examples as a class, returning to the stem sentence throughout.

Ensure children understand that we are increasing and decreasing the factor multiplicatively, not by adding or subtracting.

Work towards the following generalisations:

- ***'If a factor increases multiplicatively, the change to the product is the same.'***
- ***'If a factor decreases multiplicatively, the change to the product is the same.'***

- *'Ben and Josh now buy two bags of lollipops each.'*

	Ben – 10 lollipops in each bag	**Josh – 5 lollipops in each bag**
2 bags		

Ben: $10 \times 2 = 20$

Josh: $5 \times 2 = 10$

Compare the two equations:

$$\begin{tikzcd} 10 \arrow[d, "half"] & \times & 2 & = & 20 \arrow[d, "half"] \\ 5 & \times & 2 & = & 10 \end{tikzcd}$$

- *'If I halve one factor, the product will be half the size.'*
- *'If I divide one factor by two, I must divide the product by two.'*

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<tr>
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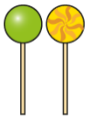
Dividing by five:

- *'Harry only has two lollipops in each bag.'*

<table>
<tr><th></th><th>Ben – 10 lollipops in each bag</th><th>Harry – 2 lollipops in each bag</th></tr>
<tr><td>1 bag</td><td></td><td></td></tr>
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Ben: $10 \times 1 = 10$
Harry: $2 \times 1 = 2$

Compare the two equations:

$$\begin{tikzcd} \textcircled{10} \arrow[d, "\div 5"] & \times & 1 & = & \textcircled{10} \arrow[d, "\div 5"] \\ \textcircled{2} & \times & 1 & = & \textcircled{2} \end{tikzcd}$$

- *'If I divide one factor by five, I must divide the product by five.'*
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<tr>
<td>1:4</td>
<td>At this point, consolidate the learning in steps <i>1:1–1:3</i> by providing children with independent practice involving increasing or decreasing one factor in a multiplication equation. You could use missing number problems such as the ones opposite.</td>
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Balancing equations:

'Fill in the missing numbers.'

$11 \times 12 = 132$

$66 \times 12 = 132 \times \square$

$25 \times 40 = 1{,}000$

$125 \times 40 = 1{,}000 \times \square$

$600 \times 15 = 9{,}000$

$600 \times 45 = 9{,}000 \times \square$
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</table>

1:5

This step looks at how to use a simple multiplication fact to solve a harder equation. It will build on the learning in steps *1:1–1:3*, increasing or decreasing (multiplicatively) one of the factors to help us work out the product.

We will begin with the useful strategy of creating a factor or a product that is a multiple of ten. In particular we will explore how to use the relationship between products when multiplying by five and by ten as a strategy when multiplying larger numbers by five.

Children have already explored the connections between the five and the ten times tables (see segment *2.5 Commutativity (part 2), doubling and halving, Teaching point 4*). Children should also already be fluent in multiplying by 10, 100 and 1,000 (see segment *2.13 Calculation: multiplying and dividing by 10 or 100*). This knowledge can now be applied in order to make calculations easier to solve.

Compare some equations using factors of five and ten. Remind children that when one factor is doubled or halved, the product is also doubled or halved, and we can use our existing knowledge that five is half of ten to help us solve difficult equations. For example, we would not normally be able to easily solve *42 × 5*, but by doubling one factor to make the equation *42 × 10*, and then halving the product, it becomes easier to solve.

Doubling one factor doubles the product; halving one factor halves the product:

- *'Solve* 42 × 5.*'*

$$\begin{tikzcd} 42 & \times & 5 \arrow[d, bend left, "\times 2"] & = & 210 \arrow[d, bend left, "\times 2"] \\ 42 & \times & 10 \arrow[u, bend left, "\div 2"] & = & 420 \arrow[u, bend left, "\div 2"] \end{tikzcd}$$

  - *'If I double one factor, the problem becomes* 42 × 10.*'*
    42 × 10 = 420
  - *'Now halve the product to find the answer to the original problem.'*
    420 ÷ 2 = 210
    42 × 5 = 210

- *'Solve* 280 × 5.*'*

$$\begin{tikzcd} 280 & \times & 5 \arrow[d, bend left, "\times 2"] & = & 1{,}400 \arrow[d, bend left, "\times 2"] \\ 280 & \times & 10 \arrow[u, bend left, "\div 2"] & = & 2{,}800 \arrow[u, bend left, "\div 2"] \end{tikzcd}$$

  - *'If I double one factor, the problem becomes* 280 × 10.*'*
    280 × 10 = 2,800
  - *'Now halve the product to find the answer to the original problem.'*
    2,800 ÷ 2 = 1,400
    280 × 5 = 1,400

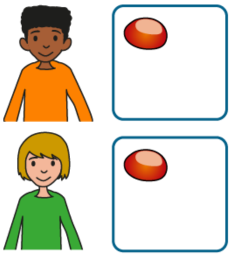
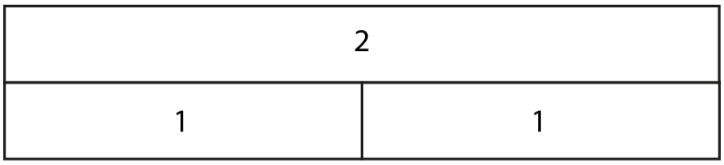
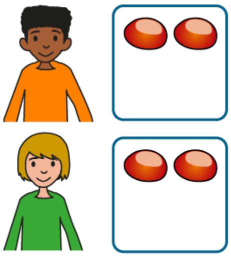
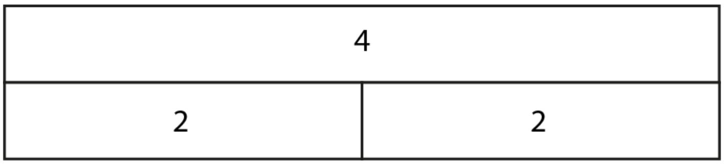
	Work through examples such as those shown opposite, ensuring children understand why they are doubling or halving numbers in each step. Refer to the generalisations from steps *1:1* and *1:3*: - ***'If I double one factor, I must double the product.'*** - ***'If I halve one factor, I must halve the product.'*** Point out that either the first factor or the second factor can be halved for the product to be halved. This contrasts with the learning in *Teaching points 2* and *3* where changes to the divisor and the dividend affect the quotient in different ways. Provide children with practice applying this strategy using missing-number problems. Include extension questions that allow children to apply the same strategy to 50 and 100. Encourage children to use mental calculations when multiplying by ten and halving. This will develop their mental maths skills and deepen their understanding of the strategy. Dòng nǎo jīn: *'Using the strategies practised in this step, solve* 480 × 25.*'*	- *'Fill in the missing numbers.'* $14 \times 5 = \boxed{}$ $14 \times 10 = 140$ $68 \times 5 = \boxed{}$ $68 \times 10 = 680$ $34 \times 5 = \boxed{}$ $34 \times 10 = 340$ $46 \times 50 = \boxed{}$ $46 \times 100 = 4{,}600$
1:6	Provide children with varied practice to apply the learning covered in this teaching point, including contextual problems: - *'Laurie buys six boxes of glue sticks, which are sold in boxes of twenty-five. Padma also buys six boxes of glue sticks, but these are sold in boxes of one hundred. Padma has six hundred glue sticks. How many glue sticks does Laurie have?'* - *'Hannah buys five times as many of the same boxes of glue sticks as Laurie. How many glue sticks does Hannah buy?'*	

	- *'Heather has some pieces of wood that are all the same length. She needs four of them to make one square. She makes fifteen squares. How many pieces of wood does she have?'*
	- *'Mel has three times as many pieces of wood as Heather. How many squares can he make?'*
	- Dòng nǎo jīn:
	'Frankie says:
	  - *"If I add two to a factor, I add two to the product."*
	$1 \times 2 = 2$
	$1 \times 4 = 4$
	  - *"If I add three to a factor, I add three to the product."*
	$1 \times 3 = 3$
	$1 \times 6 = 6$
	  - *"So, whatever I add to a factor, I add to the product."'*
	'Do you agree or disagree? Explain your answer.'	

Teaching point 2:

For division, if there is a multiplicative change to the dividend and the divisor remains the same, the quotient changes by the same scale factor.

Steps in learning

	Guidance	Representations
2:1	This teaching point looks at the relationship between the dividend, divisor and quotient, and explores what happens to the quotient when the dividend is increased or decreased (multiplicatively) by a given amount and the divisor remains the same. Begin with a context in which the dividend is doubled and the quotient is doubled, while the divisor remains the same. Use an example such as the conkers opposite and on the next page, doubling the dividend each time. Point out that the quotient also doubles each time. Bar models may be used to support understanding. Repeat this with another example, ensuring children understand that we are doubling and not adding. Use simple numbers to begin with, so that children can focus on the structure. For example: 25 ÷ 5 = 5 50 ÷ 5 = 10 100 ÷ 5 = 20 200 ÷ 5 = 40 400 ÷ 5 = 80 Work towards the following generalisation: ***'If I double the dividend and keep the divisor the same, I must double the quotient.'*** Explore how we can also use the following generalisation: ***'If I multiply the dividend by two and keep the divisor the same, I must multiply the quotient by two.'***	- *'Two conkers are shared equally between two people. They get one each.'*   - *'Four conkers are shared equally between two people. They get two each.'*  

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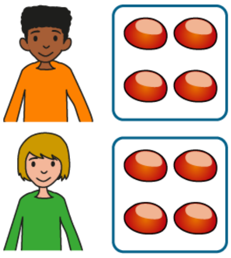
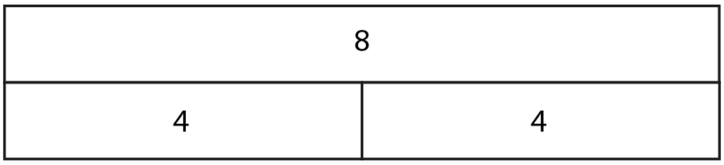
- *'Eight conkers are shared equally between two people. They get four each.'*





Compare the equations:

$$\begin{tikzcd} \textcircled{2} \arrow[d, "double"] & \div & 2 & = & \textcircled{1} \arrow[d, "double"] \\ \textcircled{4} \arrow[d, "double"] & \div & 2 & = & \textcircled{2} \arrow[d, "double"] \\ \textcircled{8} & \div & 2 & = & \textcircled{4} \end{tikzcd}$$

- *'If I double the dividend and keep the divisor the same, I must double the quotient.'*
- *'If I multiply the dividend by two and keep the divisor the same, I must multiply the quotient by two.'*

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<td>2:2</td>
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Next look at contexts where the dividend is multiplied by a number other than two, and the divisor remains the same. Use examples of partitive division, quotitive division and scaling division. Encourage children to write the pair of equations together and identify the connections between the two equations.

Use the following stem sentence: ***'If I multiply the dividend by ___ and keep the divisor the same, I must multiply the quotient by ___.'***

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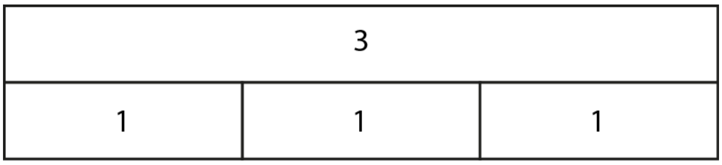
Example 1 – partitive division:

- *'£3 is shared equally between three people. How much do they get each?'*



$3 \div 3 = 1$

- *'They get £1 each.'*

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- *'£12 is shared equally between three people. How much do they get each?'*

12		
4	4	4

$12 \div 3 = 4$

- *'They get £4 each.'*

Compare the two equations:

$$\begin{tikzcd}
\textcircled{3} \arrow[d, "\times 4"] & \div & 3 & = & \textcircled{1} \arrow[d, "\times 4"] \\
\textcircled{12} & \div & 3 & = & \textcircled{4}
\end{tikzcd}$$

- *'If I multiply the dividend by four and keep the divisor the same, I must multiply the quotient by four.'*

Example 2 – quotitive division:

- *'I have five bananas that I put into bags of five. How many bags are there?'*

5
5

- *'How many fives are there in five? One.'*

$5 \div 5 = 1$

- *'There is one bag.'*

- *'I have fifteen bananas that I put into bags of five. How many bags are there?'*

15		
5	5	5

- *'How many fives are there in fifteen? Three.'*

$15 \div 5 = 3$

- *'There are three bags.'*

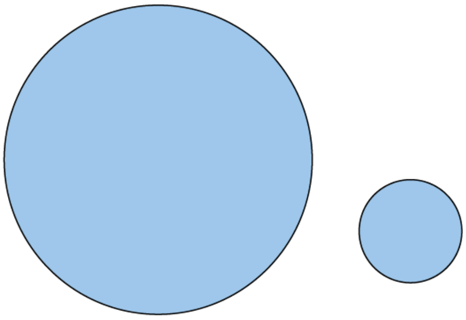
Compare the two equations:

$$\begin{tikzcd} \textcircled{5} \arrow[d, "\times 3"] & \div & 5 & = & \textcircled{1} \arrow[d, "\times 3"] \\ \textcircled{15} & \div & 5 & = & \textcircled{3} \end{tikzcd}$$

- *'If I multiply the dividend by three and keep the divisor the same, I must multiply the quotient by three.'*

Example 3 – scaling division:

- *'A circle has a perimeter (circumference) of 6 m. A smaller circle has a perimeter that is one-third the length. What is its perimeter?'*



6 m ÷ 3 = 2 m

- *'The perimeter of the smaller circle is 2 m.'*

- *'A circle has a perimeter of 24 m. A smaller circle has a perimeter that is one-third the length. What is its perimeter?'*

  24 m ÷ 3 = 8 m

  - *'The perimeter of the smaller circle is 8 m.'*

Compare the two equations:

$$\begin{tikzcd} \textcircled{6} \arrow[d, "\times 4"] & \div & 3 & = & \textcircled{2} \arrow[d, "\times 4"] \\ \textcircled{24} & \div & 3 & = & \textcircled{8} \end{tikzcd}$$

- *'If I multiply the dividend by four and keep the divisor the same, I must multiply the quotient by four.'*

2:3	Repeat steps *2:1–2:2*, but this time decreasing the size of the dividend (multiplicatively) and keeping the divisor the same. Children may wish to draw bar models to begin with, as in previous steps. Continue to include examples of partitive, quotitive and scaling division. Begin by looking at halving, using the following generalisation: ***'If I halve the dividend and keep the divisor the same, I must halve the quotient.'*** Explore how we can also use the following generalisation: ***'If I divide the dividend by two and keep the divisor the same, I must divide the quotient by two.'*** Then extend this to apply to division by other whole numbers using the following stem sentence: ***'If I divide the dividend by ___ and keep the divisor the same, I must divide the quotient by ___.'***	Example 1 – partitive division: - *'Sixteen apples are shared equally between four people. How many apples do they get each?'* 16 ÷ 4 = 4 - *'They get four apples each.'* - *'Eight apples are shared equally between four people. How many apples do they get each?'* 8 ÷ 4 = 2 - *'They get two apples each.'* Compare the two equations: $\begin{tikzcd} \textcircled{16} \arrow[d, "\div 2"] & \div & 4 & = & \textcircled{4} \arrow[d, "\div 2"] \\ \textcircled{8} & \div & 4 & = & \textcircled{2} \end{tikzcd}$ - *'If I divide the dividend by two and keep the divisor the same, I must divide the quotient by two.'* Example 2 – quotitive division: - *'I have twenty-one pencils that I put into bags of seven. How many bags are there?'* - *'How many sevens are there in twenty-one? Three.'* 21 ÷ 7 = 3 - *'There are three bags.'* - *'I have seven pencils that I put into bags of seven. How many bags are there?'* - *'How many sevens are there in seven? One.'* 7 ÷ 7 = 1 - *'There is one bag.'* Compare the two equations: $\begin{tikzcd} \textcircled{21} \arrow[d, "\div 3"] & \div & 7 & = & \textcircled{3} \arrow[d, "\div 3"] \\ \textcircled{7} & \div & 7 & = & \textcircled{1} \end{tikzcd}$ - *'If I divide the dividend by three and keep the divisor the same, I must divide the quotient by three.'*

		Example 3 – scaling division: - *'A running track is 100 m long. A shorter running track is one-fifth the length. How long is the shorter track?'* 100 m ÷ 5 = 20 m - *'The shorter track is 20 m long.'* - *'A running track is 20 m long. A shorter running track is one-fifth the length. How long is the shorter track?'* 20 m ÷ 5 = 4 m - *'The shorter track is 4 m long.'* Compare the two equations: $\begin{tikzcd} 100 \arrow[d, "\div 5"] & \div & 5 & = & 20 \arrow[d, "\div 5"] \\ 20 & \div & 5 & = & 4 \end{tikzcd}$ - *'If I divide the dividend by five and keep the divisor the same, I must divide the quotient by five.'*
2:4	Provide children with intelligent practice using missing-number problems. Encourage them to look for relationships between numbers and apply their knowledge of increasing or decreasing the dividend as a strategy to solve calculations. To begin with, use simple problems so that children can focus on the relationships between the numbers rather than on working out the quotient. Extend this to include examples where the dividend increases or decreases by multiples of ten.	*'Fill in the missing numbers.'* 3 ÷ 3 = 1 15 ÷ 3 = ☐ 16 ÷ 2 = 8 8 ÷ 2 = ☐ 6 ÷ 2 = 3 24 ÷ 2 = ☐ 60 ÷ 2 = 30 240 ÷ 2 = ☐

		810 ÷ 9 = 90 81 ÷ 9 = $\boxed{}$ 18 ÷ 2 = $\boxed{}$ 180 ÷ 2 = 90 1,800 ÷ 2 = $\boxed{}$ 2,700 ÷ 3 = $\boxed{}$ Dòng nǎo jīn: *'Mia and Sila are looking at this problem:'* 750 ÷ 50 = $\boxed{}$ 1,500 ÷ 50 = 30 • *'Mia writes:'* 750 ÷ 50 = 60 • *'Sila writes:'* 750 ÷ 50 = 15 *'Who is correct? Explain your answer.'*

Teaching point 3:

For division, if there is a multiplicative increase to the divisor and the dividend remains the same, the quotient decreases by the same scale factor; if there is a multiplicative decrease to the divisor and the dividend remains the same, the quotient increases by the same scale factor.

Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>3:1</td><td>

This teaching point explores what happens to the quotient when the *divisor* increases or decreases, but the dividend stays the same. Ensure children are confident with the terminology of dividend and divisor so that they understand how this differs from the learning in *Teaching point 2*, when the *dividend* changed and the divisor stayed the same.

Look at a context where the divisor increases (multiplicatively), but the dividend is kept the same. Compare the equations and ask children what they notice about the divisor and the quotient. Work towards the following generalisation: ***'If I double the divisor and keep the dividend the same, I must halve the quotient.'***

Explore how we can also use the following generalisation: ***'If I multiply the divisor by two and keep the dividend the same, I must divide the quotient by two.'***

Repeat using similar problems in which the divisor is multiplied by a whole number other than two. Use the following stem sentence: ***'If I multiply the divisor by ___ and keep the dividend the same, I must divide the quotient by ___.'***

Use problems involving partitive, quotitive and scaling division. To deepen understanding, ask children to explain why the quotient decreases when the divisor increases.

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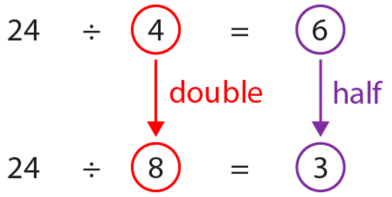
Example 1 – partitive division:

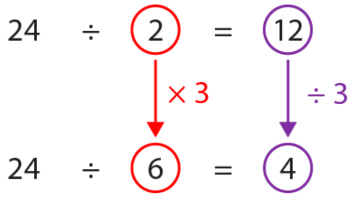
'Twenty-four meatballs are shared equally between four people. Then twenty-four meatballs are shared equally between eight people.'



- *'If I double the divisor and keep the dividend the same, I must halve the quotient.'*
- *'If I multiply the divisor by two and keep the dividend the same, I must divide the quotient by two.'*

Example 2 – quotitive division:

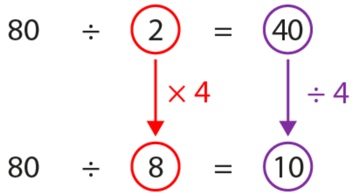
'Twenty-four marbles are put into bags of two. Then twenty-four marbles are put into bags of six.'



- *'If I multiply the divisor by three and keep the dividend the same, I must divide the quotient by three.'*

Example 3 – scaling division:

'A rope is 80 m long. It is cut to one-half the size. Another rope is 80 m long. It is cut to one-eighth the size.'



- *'If I multiply the divisor by four and keep the dividend the same, I must divide the quotient by four.'*

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3:2	Repeat step *3:1* but this time decreasing the divisor (multiplicatively) and keeping the dividend the same. Continue to include examples of partitive, quotitive and scaling division. Work towards the following generalisation: ***'If I halve the divisor and keep the dividend the same, I must double the quotient.'*** Explore how we can also use the following generalisation: ***'If I divide the divisor by two and keep the dividend the same, I must multiply the quotient by two.'*** Repeat using similar problems in which the divisor is divided by a whole number other than two. Use the following stem sentence: ***'If I divide the divisor by ___ and keep the dividend the same, I must multiply the quotient by ___.'***	Example 1 – partitive division: *'Twenty-four horses are shared equally between eight stables. Then twenty-four horses are shared equally between four stables.'* $$\begin{tikzcd} 24 & \div & 8 \arrow[d, "\text{half}"'] & = & 3 \arrow[d, "\text{double}"] \\ 24 & \div & 4 & = & 6 \end{tikzcd}$$ - *'If I halve the divisor and keep the dividend the same, I must double the quotient.'* - *'If I divide the divisor by two and keep the dividend the same, I must multiply the quotient by two.'* Example 2 – quotitive division: *'Thirty-six cherries are put into punnets of twelve. Then thirty-six cherries are put into punnets of four.'* $$\begin{tikzcd} 36 & \div & 12 \arrow[d, "\div 3"] & = & 3 \arrow[d, "\times 3"] \\ 36 & \div & 4 & = & 9 \end{tikzcd}$$ - *'If I divide the divisor by three and keep the dividend the same, I must multiply the quotient by three.'* Example 3 – scaling division: *'A running race is 400 m. However, some people do not finish the race. Some people run one-tenth the distance. Some people run one-half the distance.'* $$\begin{tikzcd} 400 & \div & 10 \arrow[d, "\div 5"] & = & 40 \arrow[d, "\times 5"] \\ 400 & \div & 2 & = & 200 \end{tikzcd}$$ - *'If I divide the divisor by five and keep the dividend the same, I must multiply the quotient by five.'*

3:3

In order to apply the knowledge learnt in steps *3:1* and *3:2*, explore equations in a systematic way so that relationships between divisors and quotients can be seen.

Work through a list of related equations, such as the one below, where the dividend stays the same but the divisor decreases or increases each time. Choose pairs of equations from the list and ask children to identify the relationship between the two equations.

24 ÷ 24 = 1
24 ÷12 = 2
24 ÷ 8 = 3
24 ÷ 6 = 4
24 ÷ 4 = 6
24 ÷ 3 = 8
24 ÷ 2 = 12
24 ÷ 1 = 24

'What is the relationship between these two equations? Refer to the divisor and the quotient in your answer.'

24 ÷ 12 = 2

24 ÷ 3 = 8

24 ÷ 1 = 24

24 ÷ 6 = 4

Encourage children to use the stem sentences from steps *3:1* and *3:2* to describe the relationships that they identify:

- ***'If I multiply the divisor by ___ and keep the dividend the same, I must divide the quotient by ___.'***
- ***'If I divide the divisor by ___ and keep the dividend the same, I must multiply the quotient by ___.'***

3:4

To conclude this segment, draw together all of the teaching points using true or false questions such as those opposite.

Encourage children to make up their own true or false questions to deepen their understanding of the relationships between multiplication and division.

'True or false?'

	True (✓) or false (✗)
Since 16 × 8 = 128 Then 32 × 8 = 256	
Since 7,344 ÷ 72 = 102 Then 7,344 ÷ 36 = 51	
Since 5,985 ÷ 63 = 95 Then 5,985 ÷ 126 = 190	
Since 1,764 ÷ 42 = 42 Then 1,764 ÷ 84 = 21	
Since 345 × 45 = 15,525 Then 345 × 90 = 15,525 × 2	
Since 45 × 20 = 900 Then 180 × 20 = 900 × 3	
Since 360 ÷ 4 = 90 Then 180 ÷ 4 = 60	
…	
…	