MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.24 Composition and calculation: hundredths and thousandths

Teacher guide | Year 4

Teaching point 1:
When one is divided into 100 equal parts, each part is one hundredth of the whole. When one tenth of a whole is divided into ten equal parts, each part is one hundredth of the whole.

Teaching point 2:
Hundredths can be expressed as decimal fractions; the number written '0.01' is one hundredth; one is one hundred times the size of 0.01; 0.1 is ten times the size of 0.01.

Teaching point 3:
We can count in hundredths up to and beyond one.

Teaching point 4:
Numbers with hundredths can be composed additively and multiplicatively.

Teaching point 5:
Numbers with tenths and hundredths are commonly used in measurement, scales and graphing contexts.

Teaching point 6:
Known facts and strategies, including column algorithms, can be applied to calculations for numbers with hundredths; the same approaches can be used for numbers with hundredths as are used for numbers with tenths.

Teaching point 7:
Numbers with hundredths can be rounded to the nearest tenth by examining the value of the hundredths digit or to the nearest whole number by examining the value of the tenths digit.

Teaching point 8:
When one is divided into 1,000 equal parts, each part is one thousandth of the whole. Knowledge and strategies for numbers with tenths and hundredths can be applied to numbers with thousandths.

Overview of learning

In this segment children will:

- extend their understanding of decimal fractions from tenths to hundredths (and beyond)
- explore the additive composition (e.g. $1.43 = 1 + 0.4 + 0.03$) and multiplicative composition (e.g. $1.43 = 143 \times 0.01$) of numbers with hundredths (and thousandths)
- explore decimal fractions in the context of measures (such as length) and graphing
- extend their understanding of calculation strategies (including unitising and column methods) to numbers with hundredths (and thousandths).

This segment follows a similar progression to segment *1.23 Composition and calculation: tenths*; the use of familiar representations such as Dienes, place-value charts, Gattegno charts and place-value counters helps children make links to their knowledge and understanding of tenths. As in segment *1.23*, one Dienes hundred square is used to represent one, and therefore a Dienes one cube represents one hundredth. If, in segment *1.23*, you chose to use a Dienes thousand cube to represent one, continue with this now, using one tens rod to represent one hundredth and a single Dienes one cube to represent one thousandth.

In this segment, children will deepen their understanding of the relationship between the different place values (for example, one tenth is equal to ten hundredths), and further develop their understanding of the base-ten structure of our number system. Children will generalise about the place value of digits on either side of the decimal point, enabling them to extend their understanding to thousandths and beyond. By the end of this segment, children will understand that they can apply the same calculation knowledge and strategies to numbers on the right of the decimal point, as for numbers on the left of the decimal point.

Children will explore how we use decimal fractions in various measures contexts, such as recording lengths/distances in metres. Decimal notation for money is covered in detail in segment *1.25 Addition and subtraction: money*.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

When one is divided into 100 equal parts, each part is one hundredth of the whole. When one tenth of a whole is divided into ten equal parts, each part is one hundredth of the whole.

Steps in learning

	Guidance	Representations

1:1

The same or similar representations should be used to introduce children to the idea of hundredths as were used to introduce them to tenths, making clear links to their previous work from segment *1.23 Composition and calculation: tenths*. In this teaching point, use the term 'hundredth' but do not represent it with numerals.

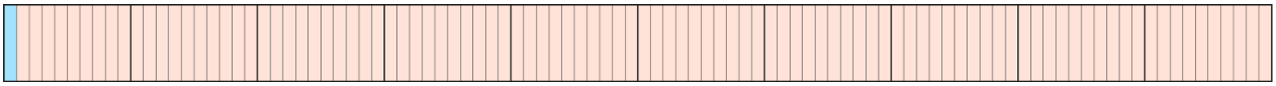
Present your first chosen manipulative or image, showing the 'whole' divided into 100 equal parts. Challenge children to guess, based on their knowledge of tenths, how we can describe each part. Use an analogous generalised statement to that used to describe tenths in segment *1.23* step *1:1*: ***'The whole is divided into one hundred equal parts; each part is one hundredth of the whole.'***

Explore one hundredth of a range of different 'wholes', using the term 'hundredth' but not representing it with numerals. Make sure that the children identify the whole in each of the representations.

Note that in segment *1.23*, one Dienes one hundred square was used to represent one, and one tens rod was used to represent one tenth. Continue with this representation, so that a single cube represents one hundredth.

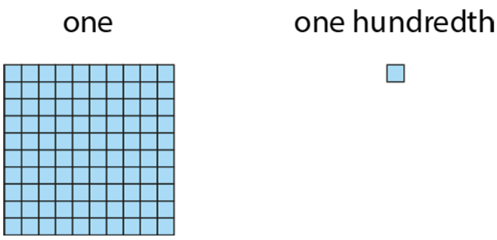
Bars:

'This is the whole.'



'The whole is divided into one hundred equal parts and one of them is shaded; each part is one hundredth of the whole; one hundred hundredths is equal to one whole.'

Dienes:



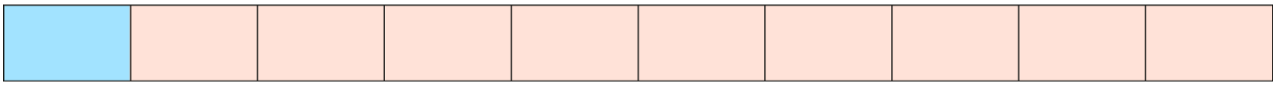
1:2 Now relate hundredths to tenths. Using the same 'wholes' as in step *1:1*:

- divide the whole into ten equal parts and ask children to describe the value of the parts (see below)
- divide each tenth into ten equal parts; ask children to identify how many parts there are in the whole and to describe the value of the parts
- focus on one of the tenths divided into ten equal parts and ask children to identify the number of parts in the tenth and the value of the parts relative to the whole.

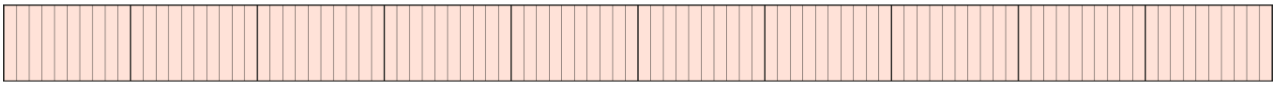
Introduce the generalised statement: ***'When one tenth is divided into ten equal parts, each part is one hundredth of the whole; ten hundredths is equal to one tenth.'***

Bars:

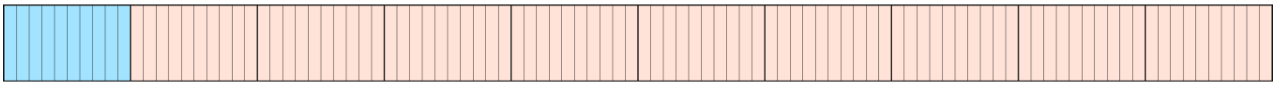
'This is the whole.'



'The whole is divided into ten equal parts, and one of them is shaded; this is one tenth of the whole.'

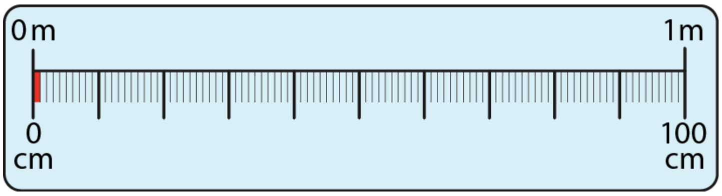


'Each tenth is divided into ten equal parts; there are one hundred parts in total and each part is one hundredth of the whole.



'One tenth is divided into ten equal parts; each part is one hundredth of the whole; ten hundredths is equal to one tenth.'

1:3 Also, include examples of measures, such as length or mass. Practical work could include, for example, dividing one kilogram of rice into ten equal parts, and then dividing one of those parts into ten again.

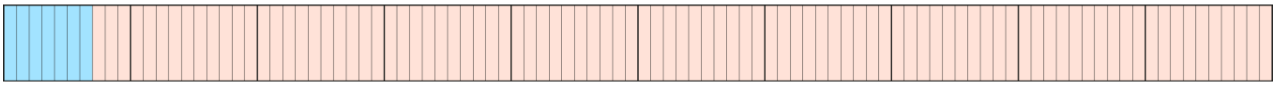


'The whole is one metre. One metre is divided into one hundred centimetres. One centimetre is equal to one hundredth of a metre.'

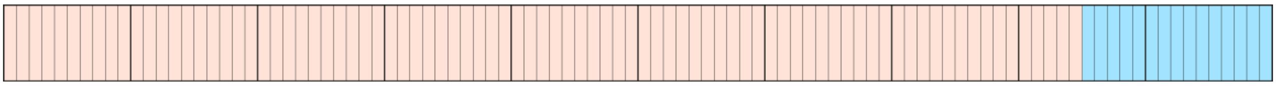
1:4 Then extend to examples with more than one hundredth identified, extending the language to the following stem sentence: ***'The whole is divided into one hundred equal parts; ___ parts is ___ hundredths.'***

Use similar language to describe measures contexts, as exemplified below.

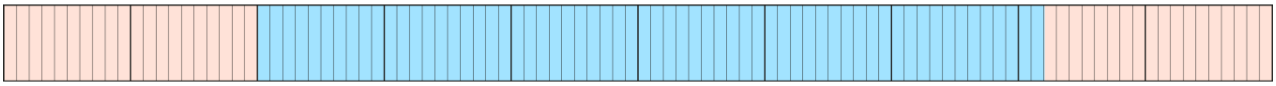
Bars:



'The whole is divided into one hundred equal parts; seven parts is seven hundredths of the whole.'

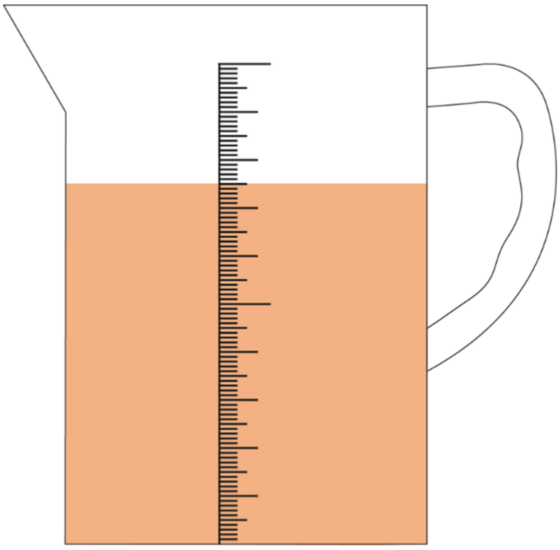


'The whole is divided into one hundred equal parts; fifteen parts is fifteen hundredths of the whole.'



'The whole is divided into one hundred equal parts; sixty-two parts is sixty-two hundredths of the whole.'

Measures context:



'The litre jug is divided into one hundred equal parts and there is water up to the seventy-fifth mark; this is seventy-five hundredths of a litre.'

Teaching point 2:

Hundredths can be expressed as decimal fractions; the number written '0.01' is one hundredth; one is one hundred times the size of 0.01; 0.1 is ten times the size of 0.01.

Steps in learning

	Guidance	Representations

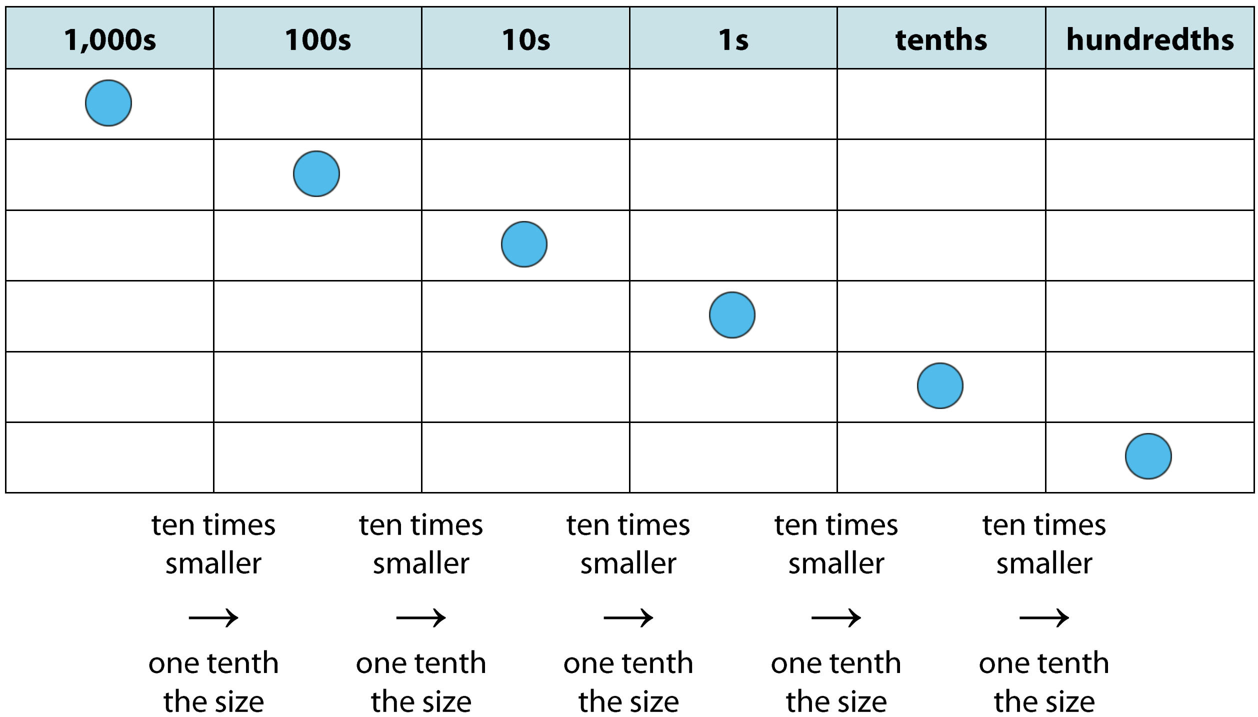
2:1 Remind children of how they recorded tenths using the decimal point and then introduce the idea that we can record hundredths in a similar way.

Build on the progression used in segment *1.23 Composition and calculation: tenths*, steps *2.1–2.2*, now adding an additional column to the place-value chart for hundredths. As in segment *1.23*, until the decimal point is introduced (step 2.2), use words to describe the place-value columns ('tenths' and 'hundredths'). Remind children what happens when a counter or digit is moved across the columns, now extending to hundredths. Use the same generalised statements as in segment *1.23*:

- ***'If a digit is moved one column to the left, the number represented becomes ten times bigger/ten times the size.'***
- ***'If a digit is moved one column to the right, the number represented becomes ten times smaller; we can also say it becomes one tenth the size.'***

Encourage children to explain relationships between different powers of ten, using the stem sentences:

- ***'___ is ten times bigger than ___.'***
- ***'___ is ten times smaller than/one tenth the size of ___.'***
- ***'___ is one hundred times bigger than ___.'***
- ***'___ is one hundred times smaller than/one hundredth the size of ___.'***



1,000s	100s	10s	1s	tenths	hundredths
1					
	1				
		1			
			1		
				1	
					1

ten times smaller	ten times smaller	ten times smaller	ten times smaller	ten times smaller
⟶	⟶	⟶	⟶	⟶
one tenth the size	one tenth the size	one tenth the size	one tenth the size	one tenth the size

2:2

You can repeat the same process as in segment *1.23 Composition and calculation: tenths*:

- Add zeros as place-holders in all places to the right of a '1'.
- Make a copy of the chart without the place-value headings and ask children to compare the number represented by each row – they should now find that in the chart without the place-value headings every row represents a number 100 times larger than intended.

Remind children that we 'solved' this problem before, by introducing the decimal point, and then add it to the chart without the place-value headings.

Draw attention to the fact that the decimal point marks the separation of the ones and digits that have a place-value smaller than one (tenths, hundredths), so that we can write these numbers without needing a place-value chart.

Then spend some time discussing where we need to include zeros:

- When we represent a number smaller than one, we write a zero in the ones place, so one tenth is written as '0.1'.
- When we write a number smaller than a tenth, we also need a zero in the tenths place, to make it clear that, for example, '1' represents one hundredth, i.e. '0.01'.

You can draw attention to the similarity between this and, for example, 304, where we have to include a place-holding zero between the '3' and the '4' to make it clear that the '3' is in the hundreds place and the '4' is in the ones place; you can ask children to consider what would happen in this case if we missed out the zero (we would be representing 34 instead of 304). Similarly, if we miss out the middle zero in '0.01' we would be representing '0.1' instead.

Including the zeros with place-value headings:

1,000s	100s	10s	1s	tenths	hundredths
1	0	0	0	0	0
	1	0	0	0	0
		1	0	0	0
			1	0	0
				1	0
					1

Including the zeros without place-value headings:

1	0	0	0	0	0
	1	0	0	0	0
		1	0	0	0
			1	0	0
				1	0
					1

Introducing the decimal point:

1	0	0	0 •	0	0
	1	0	0 •	0	0
		1	0 •	0	0
			1 •	0	0
			0 •	1	0
			•	0	1

2:3 Extend by including different numbers of counters/digits in the hundredths column and using decimal notation to write the numbers.

Begin with numbers up to and including 0.09, and then extend to numbers up to 0.99. For each number, ask children to describe how the number is written, using the stem sentence: ***'One hundredth can be written as "0.01", so ___ hundredths can be written as "0.___".'***

Note children already have experience with number names for tenths (for example, zero-point-four), so extend this language to hundredths now. Model the correct language for numbers

	with more than ten hundredths, ensuring that children say, for example, *'zero-point-three-four'* and not *'zero-point-thirty-four'*.

1,000s	100s	10s	1s	0.1s	0.01s
				3	4

			•	3	4

			0 •	3	**4**

'One hundredth can be written as "0.01", so thirty-four hundredths can be written as "0.34".'

2:4 The Gattegno chart is useful for showing the ten-times-larger/smaller relationship between the different powers of ten written with numerals. Use the Gattegno chart in the following ways:

- Point to/circle two numbers in the same column and ask children to describe the relationship between them, for example (pointing to '0.03' and '3'):
  - *'Zero-point-zero-three is ten times smaller than/one tenth the size of zero-point-three.'*
  - *'Zero-point-three is ten times the size of zero-point-zero-three.'*
- Cover up numbers on the chart and ask children to work out what they are; ask children to explain their answers by relating the covered number to the numbers above and below it in the column.

Gattegno chart:

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

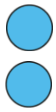
2:5 To complete this teaching point, ensure children have mastered reading and writing numbers with hundredths by presenting varied practice, including:

- presenting them with numbers represented on place-value charts and asking children to write the numbers (as numerals), and vice versa
- presenting hundred grids with some squares shaded and asking children to write the numbers (as numerals), and vice versa
- asking children to label or interpret measures contexts (as shown below)

- presenting word problems such as *'There are 100 tiles on a kitchen floor. 75 tiles are white and 25 tiles are black. Write down how much of the floor is white and how much is black.'*

Note that the divisions on the hundred grids and, for the final example below, the measuring guide, are marked in such a way as to aid subitising. Encourage children to subitise rather than count in ones from zero.

Place-value chart with counters:

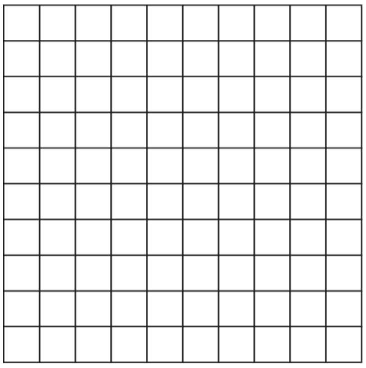
1,000s	100s	10s	1s	0.1s	0.01s
					

'Fill in the blanks.'

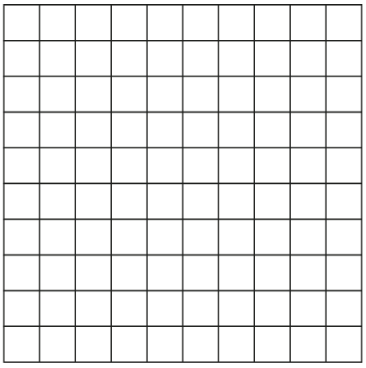
- *'This represents ___ hundredths.'*
- *'We can write this as ___.'*

Shapes:

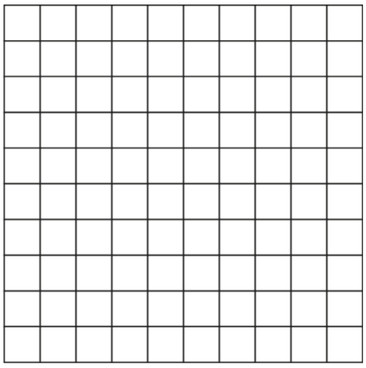
'Shade squares to represent the numbers shown.'



0.4

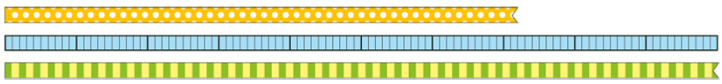


0.25



0.83

Measures context:



'Fill in the blanks.'

- *'The spotty ribbon is ___ hundredths the length of the stripy ribbon.'*
- *'We can write this as ___.'*

Teaching point 3:

We can count in hundredths up to and beyond one.

Steps in learning

3:1

Guidance

For this teaching point, follow a similar progression to segment *1.23 Composition and calculation: tenths, Teaching point 3*.

Explore counting in hundredths, initially only up to one hundred hundredths/one. You could use Dienes as a concrete representation: counting each small square on a hundred square, with children pointing as they count.

Use dual counting to emphasise unitising in hundredths:

- *'Zero-point-zero-one, zero-point-zero-two… zero-point-nine-nine, one.'*
- *'One hundredth, two hundredths… ninety-nine hundredths, one hundred hundredths.'*

Unitising in hundredths will be important when children come to apply known additive strategies to numbers with hundredths.

As well as the Dienes, practise counting with a number line and the Gattegno chart. Count from different starting points and count backwards as well as forwards, ensuring that children are confident in crossing the tenths boundaries.

Representations

Number line – counting from 0.25 to 0.35:

3:2

Guidance

Then practise counting in multiples of tenths / ten hundredths, i.e.

- *'Zero-point-one, zero-point-two… zero-point-nine, one.'*
- *'Ten hundredths, twenty hundredths… ninety hundredths, one hundred hundredths.'*
- *'One tenth, two tenths… nine tenths, ten tenths.'*

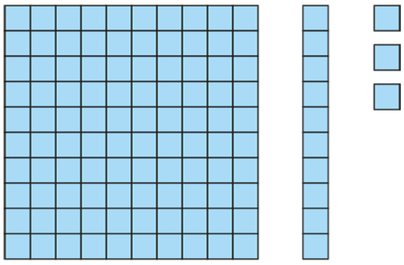
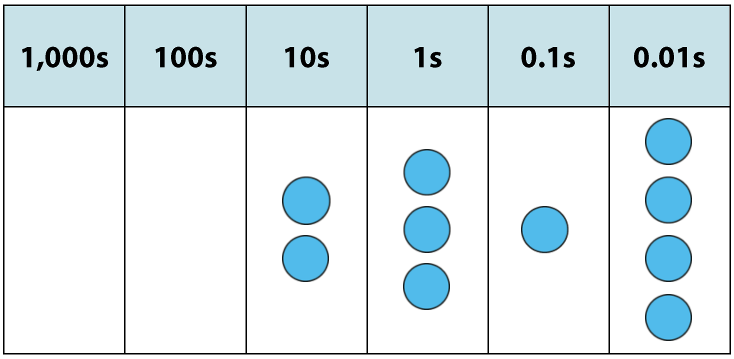
Representations

Dòng nǎo jīn:

'Choose a number, then use the Gattegno chart to count up in multiples of two hundredths (0.02). Write an addition equation to represent each step.'

Example:

1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

	Use Dienes, number lines and the Gattegno chart. Children could also use the Gattegno chart to count in other multiples of hundredths (for example multiples of 0.02, 0.03 etc.), as shown opposite.	• *'Zero-point-five-two, zero-point-five-four…'* • *'Fifty-two hundredths, fifty-four hundredths…'* 0.52 + 0.02 = 0.54
3:3	Now extend counting beyond one in hundredths. Count in three ways: • *'… one and one hundredth, one and two hundredths…'* • *'one hundred and one hundredths, one hundred and two hundredths…'* • *'one-point-zero-one, one-point-zero-two…'* Include the numbers written as numerals until children link the number names with the numerals.	Counting with Dienes:  • *'… one and thirteen hundredths…'* • *'… one hundred and thirteen hundredths…'* • *'one-point-one-three…'*
3:4	Give children practice moving between representations for numbers with hundredths that are greater than one. Also, link to work on fractions. To reinforce the link, encourage children to use the following stem sentence: ***'I say ___-point-___-___ but I think ___ and ___ hundredth(s).'***	Place-value chart with counters: *'Write the number represented as a decimal fraction.'*  Concrete/pictorial representations: *'Represent the following numbers using Dienes. Sketch your answers.'* 1.45 2.00 3.72 4.20 5.08

		Converting common fractions/mixed numbers and decimal fractions: *'Complete the table.'* <table><tr><td>$\frac{3}{100}$</td><td>0.3</td></tr><tr><td>$\frac{7}{100}$</td><td></td></tr><tr><td>$\frac{72}{100}$</td><td></td></tr><tr><td></td><td>1.0</td></tr><tr><td>$3\frac{42}{100}$</td><td></td></tr><tr><td></td><td>103.13</td></tr></table>
3:5	Now move on to the comparison of numbers with hundredths. In a similar way to segment *1.23 Composition and calculation: tenths*, present children with two numbers and ask them to identify which is larger/smaller using a cardinal representation (for example, Dienes) or ordinal representation (for example, number line) for support. They may reason, for example: • *'Four-point-three-six is greater than four-point-two-seven because four-point-three-six has nine more hundredth squares.'* (Where one small Dienes square represents one hundredth.) • *'Four-point-three-six is greater than four-point-two-seven because it is further along the number line.'*	
3:6	Then, beginning with numbers with the same number of significant digits (e.g. 3.78 and 4.32), explore how the numbers can be compared just by examining the digits. Discuss how, if the ones digits are different, we can determine the relative size of the	

numbers just by looking at the ones digits, but if the ones digits are the same, we must then examine the tenths digits, and if those are the same we must then examine the hundredths digits. Initially use Dienes for support, but then work with only the digits.

3:7

Children have already compared number pairs such as 1.7 and 2 (i.e. where the larger number has fewer digits, see segment *1.23 Composition and calculation: tenths*, step *3:12*). Now extend this idea to numbers with hundredths, comparing, for example:

- 1.69 and 2
- 1.69 and 1.7

Remind children that we need to compare like digits.

1,000s	100s	10s	1s	0.1s	0.01s
			1	6	9
			2		

'I know that one-point-six-nine is smaller than two because the largest place value is the ones, and there are more ones in two than in one-point-six-nine.'

1,000s	100s	10s	1s	0.1s	0.01s
			1	6	9
			1	7	

'I know that one-point-six-nine is smaller than one-point-seven because the largest different place value is the tenths, and there are more tenths in one-point-seven than in one-point-six-nine.'

3:8

Provide children with practice comparing numbers, using symbols or putting sequences of numbers in ascending order, as shown opposite.

Also present dòng nǎo jīn problems, such as the following example, to promote and assess depth of understanding: '*Sahirah says she is thinking of a number, between two and three, that has three digits which sum to five. What numbers could Sahirah be thinking of?*'
(Answers: 2.03, 2.12, 2.21, 2.30)

- *'Fill in the missing symbols (< > or =).'*

0.3 ○ 0.5	0.2 ○ 0.15
0.05 ○ 0.03	0.11 ○ 0.09
0.03 ○ 0.5	1.01 ○ 1.11
0.3 ○ 0.05	3 ○ 2.99
9 ○ 9.00	140 ○ 1.40

		• *'Put these numbers in order from the smallest to the largest.'* 0.21 0.2 0.01 0.1 0.2 smallest largest ☐ ☐ ☐ ☐ ☐

Teaching point 4:

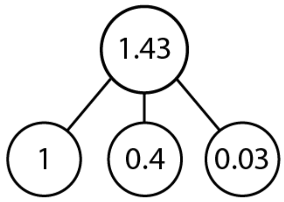
Numbers with hundredths can be composed additively and multiplicatively.

Steps in learning

	Guidance	Representations
4:1	Begin this teaching point by exploring the additive decomposition of numbers with hundredths. Use Dienes or place-value counters to represent a given number and ask children to identify how many ones (wholes), tenths and hundredths there are; ask them to represent this on a part–part(–part)–whole model and to write corresponding addition and subtraction equations. Work through a range of examples, gradually removing the scaffold of the Dienes and then the part–part(–part)–whole models, until children can write the additive equations without support. Note that the set of equations shown opposite are analogous to those for decomposition of three-digit numbers. The Gattegno chart is also a powerful representation for revealing the additive composition of these numbers. You can say a number and have children tap it out on the Gattegno chart, or vice versa.	See below

Place-value counters:

Part–part–part–whole model:



Additive equations:

1.43 = 1 + 0.4 + 0.03

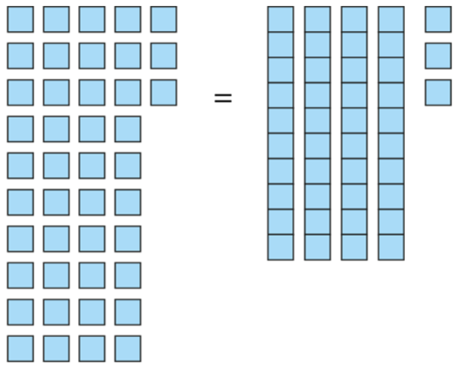
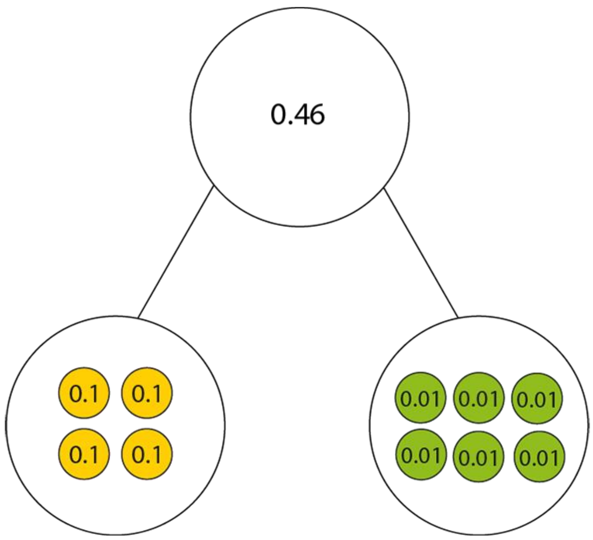
1.43 = 1 + 0.43
1.43 – 0.43 = 1
1.43 – 1 = 0.43

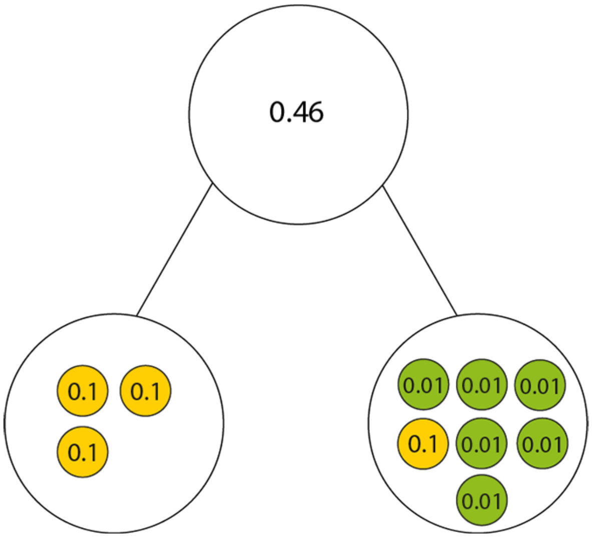
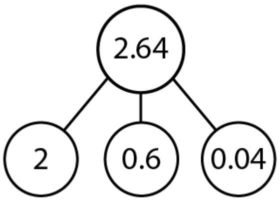
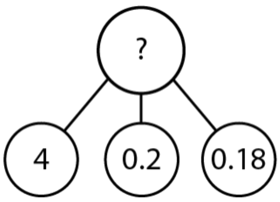
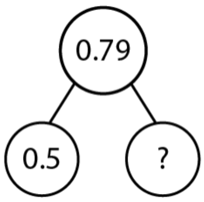
1.43 = 1.4 + 0.03
1.43 – 0.03 = 1.4
1.43 – 1.4 = 0.03

1.43 = 1.03 + 0.4
1.43 – 0.4 = 1.03
1.43 – 1.03 = 0.4

Gattegno chart:

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

4:2	Children began unitising in hundredths with the counting in *Teaching point 3*. Now link this to the multiplicative composition of numbers with hundredths. Continuing to use Dienes or place-value counters, begin with numbers smaller than one tenth, for example: $0.03 = 0.01 + 0.01 + 0.01$ $0.03 = 3 \times 0.01$ $0.01 + 0.01 + 0.01 = 3 \times 0.01$ Then move on to numbers greater than one tenth, and finally numbers greater than one. Encourage children to think flexibly about number, by combining multiplicative and additive reasoning, for example: • $0.43 = 0.4 + 3 \times 0.01$ • $0.43 = 4 \times 0.1 + 3 \times 0.01$ • $1.43 = 1 + 43 \times 0.01$	Dienes – multiplicative expression:  $43 \times 0.01 \quad = \quad 0.43$ Place-value counters – combining additive and multiplicative reasoning:  $0.43 \quad = \quad 4 \times 0.1 \quad + \quad 3 \times 0.01$ *'Zero-point-four-three is equal to four tenths and three hundredths; it is also equal to forty-three hundredths.'*
4:3	Now explore how numbers with hundredths can be partitioned in different ways (not just into the constituent ones, tenths and hundredths). This mirrors children's work on partitioning three-digit numbers (see segment *1.18 Composition and calculation: three-digit numbers, Teaching point 5*). Use place-value counters and printed part–part(–part)–whole models to work on: • Ask children to make a given number (e.g. 0.46), placing the tenths together and the hundredths together. • Then challenge children to redistribute the counters on the part–part(–part)–whole model and	Place-value counters and part–part–whole models: 

	write corresponding addition equations. Encourage children to work systematically. For each partitioning, ask children to confirm that the parts still combine to make the same whole, emphasising that the original number is conserved however we partition it.	 … 0.46 = 0.4 + 0.06 0.46 = 0.3 + 0.16 0.46 = 0.2 + 0.26 0.46 = 0.1 + 0.36
4:4	Provide varied practice based on additive and multiplicative composition of numbers with hundredths, as shown opposite.	Additive problems: *'Fill in the missing numbers.'*  2.64 = 2 + □ □ = 2.64 – 2 2.64 = 2.6 + □ □ = 2.64 – 2.6 2.64 = 2.04 + □ □ = 2.64 – 0.6  

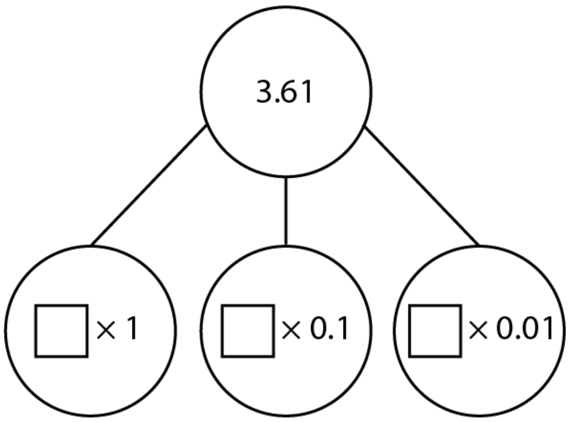
Multiplicative problems:

'Fill in the missing numbers on the left. Draw place-value counters to represent the numbers on the right.'

$1 \times 0.01 = 0.01$	0.01
$2 \times 0.01 = 0.02$	
$3 \times 0.01 = \square$	
$10 \times 0.01 = 0.1$	
$11 \times 0.01 = \square$	0.1 0.01
$23 \times 0.01 = \square$	

Combining additive and multiplicative expressions:

'Fill in the missing numbers.'



$0.09 = \square \times 0.01$

$0.59 = \square \times 0.01$

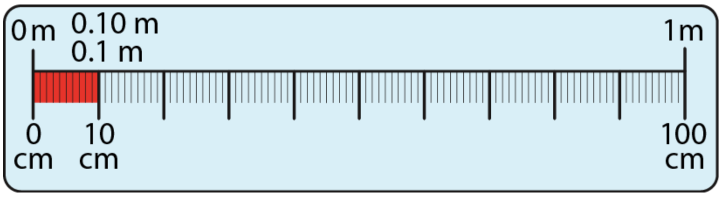
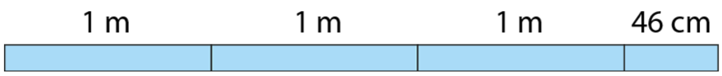
$0.59 = \square \times 0.1 + \square \times 0.01$

		$2.59 = \square \times 0.01$ $2.59 = \square \times 1 + \square \times 0.1 + \square \times 0.01$ Dòng nǎo jīn: *'Fill in the missing symbols (< > or =).'* $0.09 + 0.8 \bigcirc 0.98$ $0.07 + 4 + 0.2 \bigcirc 4.27$

Teaching point 5:

Numbers with tenths and hundredths are commonly used in measurement, scales and graphing contexts.

Steps in learning

	Guidance	Representations
5:1	Begin this teaching point by relating children's understanding of tenths and hundredths to the fact that there are 100 cm in a metre. This provides a good context for children to see the application of tenths and hundredths in the real world. Use the generalised statements: • ***'One centimetre is one hundredth of a metre, so we can write one centimetre as zero-point-zero-one.'*** • ***'Ten centimetres is one tenth of a metre, so we can write ten centimetres as zero-point-one.'*** Give children some practice expressing and interpreting lengths less than one metre using decimal notation and converting to centimetres.	100 cm in a metre:  Missing-number problems: *'Fill in the missing numbers.'* 1 cm = 0.01 m ☐ cm = 0.07 m 5 cm = ☐ m ☐ cm = 0.70 m 10 cm = ☐ m ☐ cm = 0.25 m 15 cm = ☐ m ☐ cm = 0.52 m
5:2	Now extend to measurements greater than one metre, initially using pictorial representations to support identification of the whole metres and the additional part of a metre. As in the previous step, give children some practice expressing and interpreting lengths.	Lengths greater than one metre:  3 m 46 cm = 3.46 m Missing-number problems: *'Fill in the missing numbers.'* 100 cm = ☐ m ☐ cm = 2 m 101 cm = ☐ m ☐ cm = 2.05 m 110 cm = ☐ m ☐ cm = 2.50 m 210 cm = ☐ m ☐ cm = 25 m

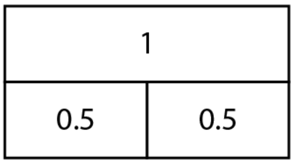
<table>
<tr>
<td>5:3</td>
<td>Give children the opportunity to order and compare different lengths, as shown opposite.

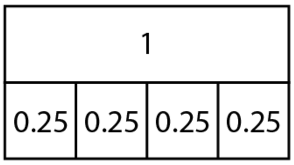
Follow up with questions in which the measurements to be ordered are in a mixture of units (centimetres and metres), for example: ‘Sarah is 1.45 m tall, her brother Francis is 182 cm tall and her cousin Eloise is 1 m 28 cm tall. Put them in order from shortest to tallest.’

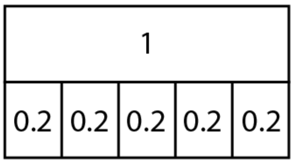
Encourage children to use the same language that they used in Teaching point 3 when ordering numbers.</td>
<td>‘The table shows how far some children jumped in a long-jump competition.’
<table>
<tr><th>Name</th><th>Distance jumped (m)</th></tr>
<tr><td>Jamal</td><td>3.04</td></tr>
<tr><td>Reyna</td><td>3.4</td></tr>
<tr><td>Faisal</td><td>2.85</td></tr>
<tr><td>Ilaria</td><td>3.19</td></tr>
<tr><td>Charlie</td><td>3.09</td></tr>
<tr><td>Kagendo</td><td>2.9</td></tr>
</table>

‘Who came third in the competition?’
‘How much further did the winner jump compared to the child who came second?’
‘What was difference between the longest and shortest jumps?’
‘How much further did Ilaria jump than Faisal?’

</td>
</tr>
<tr>
<td>5:4</td>
<td>In segment 1.17 Composition and calculation: 100 and bridging 100, Teaching Point 1 children practised counting to and from (and beyond) 100 in multiples of 20, 25 and 50. Now apply this to counting in multiples of 0.2, 0.25 and 0.5 up to one. This is important preparation for graphing and measurement contexts, where scales are often divided into these values. You can reinforce the link to counting to 100 in multiples of 20, 25 and 50 by using similar representations to those used in segment 1.17, including hundred grids and Gattegno charts. Also use number lines to make the link to scales.</td>
<td></td>
</tr>
</table>

<table>
<tr>
<td>5:5</td>
<td>As in segment 1.17 Composition and calculation: 100 and bridging 100, use bar models to reveal the related additive and multiplicative structures.</td>
<td>


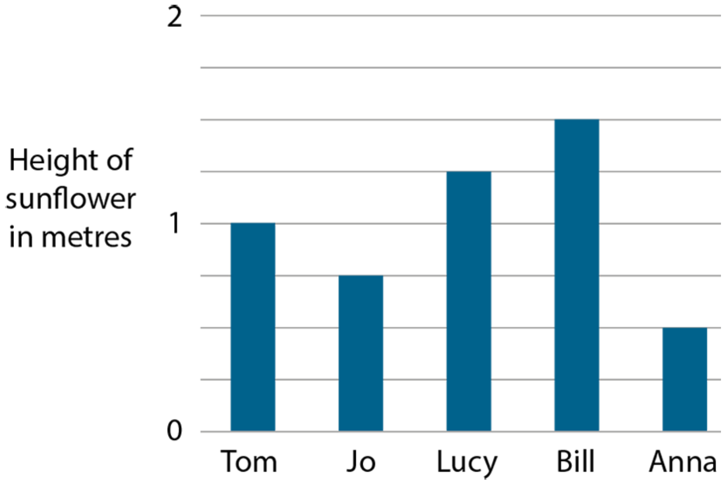
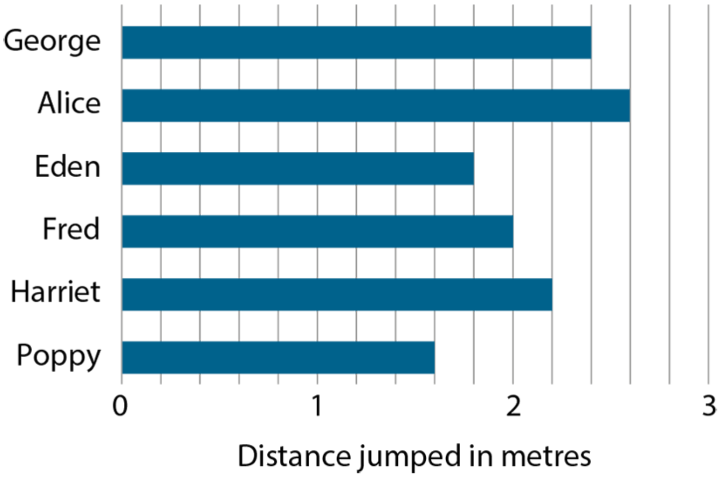
1 = 0.5 + 0.5
1 = 2 × 0.5   1 ÷ 2 = 0.5
1 = 0.5 × 2   1 ÷ 0.5 = 2


1 = 0.25 + 0.25 + 0.25 + 0.25
1 = 4 × 0.25   1 ÷ 4 = 0.25
1 = 0.25 × 4   1 ÷ 0.25 = 4


1 = 0.2 + 0.2 + 0.2 + 0.2 + 0.2
1 = 5 × 0.2   1 ÷ 5 = 0.2
1 = 0.2 × 5   1 ÷ 0.2 = 5
</td>
</tr>
<tr>
<td>5:6</td>
<td>Provide children with practice completing equations, as shown opposite.</td>
<td>Missing-number problems:
'Fill in the missing numbers.'
0.25 + 0.25 + 0.25 + 0.25 = ☐
0.25 + 0.25 = ☐
0.25 + 0.25 + 0.25 = ☐
1 – 0.25 = ☐
1 – 0.25 – 0.25 = ☐</td>
</tr>
</table>

		$1 - 0.25 - 0.25 - \square = 0.25$ $1 - 0.75 = \square$ $1 - \square = 1 - 0.25 - 0.25$ $1 - 0.2 = \square$ $1 - 0.8 = \square$ $1 - \square = 1 - 0.2 - 0.2$ $5 \times \square$ m = 1 m $4 \times \square$ m = 1 m $5 \times 0.2 = 4 \times \square$ $1 \div 5 = \square$ $1 \div 5 = 1 - \square$ Dòng nǎo jīn: • *'Fill in the missing numbers.'* 1 m – 0.25 m = $\square$ cm $\square$ m = 100 cm – 40 cm • *'What do you notice about this equation?'* $2 \times 0.5 = 5 \times 0.2$

5:7	Then extend counting in multiples of 0.2, 0.25 and 0.5 *beyond* one. Similarly, extend the related additive and multiplicative calculations beyond one. For the multiplicative problems, encourage children to use the distributive law to make one, and then deal with the 'extra lots' of, for example, 0.25	Missing-number problems. *'Fill in the missing numbers.'* $0.25 + 0.25 + 0.25 + 0.25 + 0.25 = \square$ $0.5 + 0.5 + \square = 1.5$ $1.25 = 0.25 + 0.5 + \square$ $\square \times 0.25 = 1$ $5 \times 0.25 = 4 \times 0.25 + 1 \times 0.25 = \square$ $6 \times 0.25 = 4 \times 0.25 + \square \times 0.25 = 1.5$ $0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = \square$ $1.2 - 0.2 = \square$ $1.2 - 0.2 - 0.2 = \square$ $5 \times \square = 1$ $\square \times 0.2 = 5 \times 0.2 + 1 \times 0.2 = 1.2$ $7 \times 0.2 = 5 \times 0.2 + \square \times 0.2 = \square$

5:8	Once children can confidently count and calculate in multiples of 0.2, 0.25 and 0.5, provide graphing/scaling contexts for them to apply their understanding to, as shown opposite.	• *'Five children have been growing sunflowers. The bar chart shows how tall they have grown. How tall is each flower?'*  • *'The bar chart below shows long-jump distances for six children.'*  • *'How far did the winning girl jump?'* • *'What was the difference between the two longest jumps?'*

5:9

Finally, ensure that children understand that, although written times look the same as numbers with hundredths, the notation has a different meaning. For example, in 9.15 am, the ‘1’ does not represent a tenth of an hour and the ‘5’ does not represent five hundredths of an hour. Instead, the notation represents 9 hours and 15 minutes past midnight. Emphasise this difference and demonstrate that we cannot calculate with times using the rules for calculating with numbers with a decimal point (numbers with tenths and hundredths).*

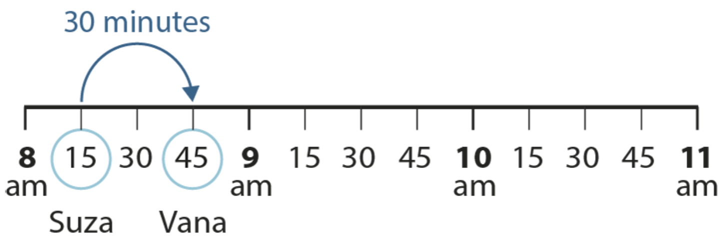
Encourage children to sketch their own time-based number lines when ordering and calculating with a set of times.

* Note that times may also be written using a colon, which is the convention when using the 24-hour clock (e.g. 09:15, 13:00).

‘This table shows the times when some children start their piano lessons. They are not in order.’

Name	Time
Dipesh	10.15 am
Vana	8.45 am
Emily	10.45 am
Suza	8.15 am
Hector	9.45 am
Tim	9.15 am

- *‘Who has their lesson first?’*
- *‘Who has their lesson last?’*
- *‘Which two children have lessons between Suza and Hector?’*
- *‘How long do you think a lesson lasts?’*



Teaching point 6:

Known facts and strategies, including column algorithms, can be applied to calculations for numbers with hundredths; the same approaches can be used for numbers with hundredths as are used for numbers with tenths.

Steps in learning

	Guidance	Representations
6:1	This teaching point extends known additive facts and strategies to numbers with hundredths, following a similar progression to segment *1.23 Composition and calculation: tenths, Teaching point 5*.	Part–part–whole models – calculation within one tenth:

6:1 Guidance

This teaching point extends known additive facts and strategies to numbers with hundredths, following a similar progression to segment *1.23 Composition and calculation: tenths, Teaching point 5*.

Children now need the opportunity to apply this understanding in calculating with hundredths in the same way that they have done with whole numbers and tenths, using the resources that they have for representing decimals in the teaching points above.

Begin by combining known facts/strategies for calculation within and across ten with unitising in hundredths to calculate, for example, *0.05 + 0.02* and *0.12 – 0.07*. As with segment *1.23*, use part–part–whole models (bar model or cherry diagram) and either Dienes or hundredth-value place-value counters on tens frames to link calculations with hundredths (e.g. *0.05 + 0.02 = 0.07*) to related single-digit facts (e.g. *5 + 2 = 7*). Use the following stem sentence: ***'___ hundredths plus/minus ___ hundredths is equal to ___ hundredths.'***

For bonds to ten, remind children that ten hundredths is equal to one tenth, incorporating this into the stem sentences:

- ***'___ hundredths plus ___ hundredths is equal to ten hundredths, which is equal to one tenth.'***

6:1 Representations

Part–part–whole models – calculation within one tenth:

7	
5	2

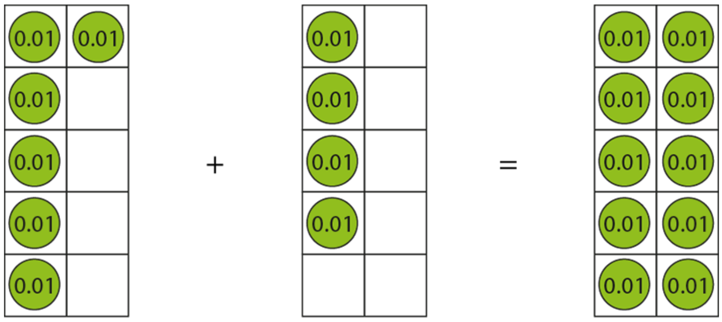
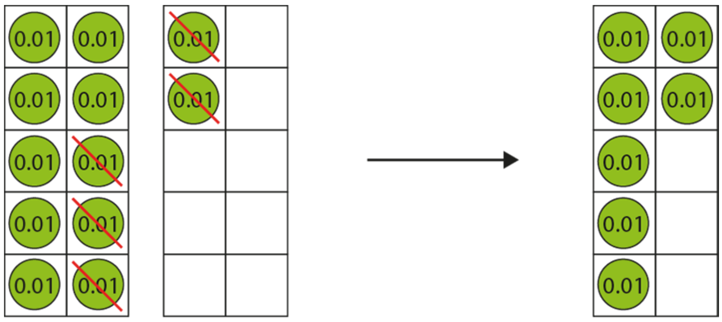
7 hundredths	
5 hundredths	2 hundredths

0.07	
0.05	0.02

- *'Five hundredths plus two hundredths is equal to seven hundredths.'*

  0.05 + 0.02 = 0.07
- *'Seven hundredths minus two hundredths is equal to five hundredths.'*

  0.07 – 0.02 = 0.05

	• ***'One tenth is equal to ten hundredths; ten hundredths minus ___ hundredths is equal to ___ hundredths.'*** When crossing the tenths boundary, a common error is to write, for example: 0.07 + 0.05 = 0.012 ✗ If this happens, go back to the place-value counters and the unitising language. Unitising in hundredths is the same as unitising in any other unit (tens, ones, tenths, etc.), so by now children should be becoming familiar with the concept. You can extend the stem sentences again, now saying, for example: *'Seven hundredths plus five hundredths is equal to twelve hundredths, which is equal to one tenth and two hundredths.'* The number line can also be used to represent calculations that bridge one tenth. You can use this opportunity to remind children of other linked facts, asking, for example, *'If we know that five plus two is equal to seven, what else do we know?'* (we can add any 'unit' after each single-digit number in the sentence, for example, tenths, tens, hundreds, etc.).	Tens frames and place-value counters – complements to one tenth:  0.06 + 0.04 = 0.1 *'Six hundredths plus four hundredths is equal to ten hundredths, which is equal to one tenth.'* Tens frames and place-value counters – bridging one tenth:  0.12 – 0.05 = 0.07 *'Twelve hundredths minus five hundredths is equal to seven hundredths.'*
6:2	Before moving on, provide children with some practice until they are secure calculating within and across one tenth. You can use intelligent practice to extend the learning about complements to one tenth, to complements to other multiples of tenths.	Missing-number problems: *'Fill in the missing numbers.'* 3 hundredths + 2 hundredths = ☐ hundredths 0.03 + 0.02 = ☐ 9 hundredths – 3 hundredths = ☐ hundredths 0.09 – 0.03 = ☐

		☐ = 0.03 + 0.04 0.07 – 0.03 = ☐ 0.08 = ☐ + 0.02 0.08 – ☐ = 0.02 8 hundredths + 3 hundredths = ☐ hundredths 8 hundredths + 3 hundredths = ☐ tenths ☐ hundredths 0.08 + 0.03 = ☐ 16 hundredths – 8 hundredths = ☐ hundredths 0.16 – 0.08 = ☐ ☐ = 0.09 + 0.04 0. 13 – 0.04 = ☐ 0.15 = ☐ + 0.08 0.15 – ☐ = 0.07 0.1 – 0.01 = ☐ 0.2 – ☐ = 0.19 0.1 – 0.02 = ☐ 0.2 – ☐ = 0.18 0.1 – 0.03 = ☐ 0.2 – ☐ = 0.17 0.1 – 0.04 = ☐ 0.2 – ☐ = 0.16 0.1 – 0.05 = ☐ 0.2 – ☐ = 0.15 0.1 – 0.06 = ☐ 0.2 – ☐ = 0.14 0.1 – 0.07 = ☐ 0.2 – ☐ = 0.13 0.1 – 0.08 = ☐ 0.2 – ☐ = 0.12 0.1 – 0.09 = ☐ 0.2 – ☐ = 0.11

Dòng nǎo jīn:

0.1 0.5 0.05 0.8 0.08 0.3

- *'Circle the numbers that sum to 0.13. How did you know?'*
- *'Which two numbers in the set sum to 1.3?'*
- *'What is the difference between these two calculations?'*
- *'What is the same?'*

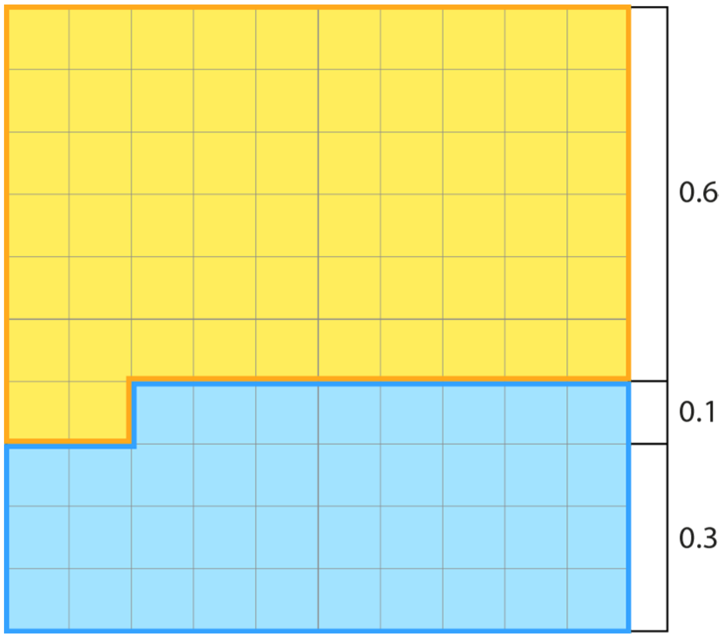
6:3

Now explore complements to one (e.g. *0.96 + 0.04 = 1* and *0.62 + 0.38 = 1*). Use the same representations and approach used for calculating complements to 100 in segment *1.17 Composition and calculation: 100 and bridging 100*. When teaching calculation of bonds to 100, care was taken to avoid the common error of 'creating an extra ten'. In the same way, here, look out for children creating an extra tenth, e.g.:

0.62 + 0.48 = 1 ✘

Provide children with practice until they can confidently calculate complements to one.

Hundred grid:



Bar model:

1	
0.62	0.38

Additive equations:

62 hundredths + 38 hundredths = 100 hundredths

0.62 + 0.38 = 1

100 hundredths − 38 hundredths = 62 hundredths

1 − 0.38 = 0.62

		100 hundredths − 62 hundredths = 38 hundredths 1 − 0.62 = 0.38
6:4	Before moving on to application of column methods to numbers with hundredths, present children with true/false questions, such as those shown opposite, to reveal and address any misconceptions around the use of zero as a place-holder, place-value and complements to 1.	*'Are these calculations correct? Mark each calculation with a ✓ or a ×'* 0.05 + 0.05 = 0.010 0.04 + 0.06 = 0.1 0.13 + 0.7 = 0.2 0.40 = 0.4 0.61 + 0.49 = 1 0.73 + 0.27 = 1 0.4 + 0.5 = 0.45

6:5	In the previous segment, children learnt to apply column addition and subtraction to numbers with tenths. Now extend to numbers with hundredths.
	Work first with column addition:
	• Begin with three-digit numbers (ones, tenths and hundredths) with no regrouping required.
	• Align the digits correctly, paying attention to the decimal point. Encourage children to articulate the value of each digit as they lay out the calculation.
	• Begin with the least significant digit (now the hundredths).
	• Emphasise the alignment of the decimal points in the addends with the decimal point in the sum.
	• Move to calculations requiring regrouping in the hundredths, then the tenths, then both. Repeat the stem sentences:
	  ◦ ***'Ten hundredths is equal to one tenth.'***
	  ◦ ***'Ten tenths is equal to one.'***
	For examples with more than two addends, add the numbers in each column in the most efficient order.
		'Complete the calculations.'
		  0 . 5 2
		+ 1 . 3 4
		  _ . _ _
		
		  4 . 2 5
		+ 3 . 3 9
		  _ . _ _
		
		  0 . 5 3
		+ 0 . 7 3
		  _ . _ _
		
		  1 . 3 6
		+ 1 . 6 8
		  _ . _ _
		
		  0 . 3 7
		  0 . 2 5
		  0 . 6 9
		+ 1 . 7 2
		  _ . _ _

	Initially, children can model the calculations with Dienes or place-value counters alongside the written calculations, to reinforce that re-grouping with hundredths is the same as regrouping with whole numbers and tenths; however, move them away from use of the manipulatives relatively quickly, so that children do not become reliant on them. As usual, the manipulatives should be used to reveal the structure, and not as a tool for 'finding the answer'. Emphasise the use of known facts for the addition of the digits in each column. Then repeat for subtraction, beginning with cases where no exchange is required, and then extending to include examples where exchange of one tenth for ten hundredths, of one one for ten tenths, or both, are required. Again, manipulatives can be used, initially, alongside the written calculations to support children's understanding of exchange. The stem sentences used for addition can be 'reversed' to support exchange: • ***'One tenth is equal to ten hundredths.'*** • ***'One is equal to ten tenths.'*** Provide children with practice until they are confident with such calculations, before progressing to the next step.	*'Complete the calculations.'* $\begin{array}{r} 1.58 \\ -\ 0.32 \\ \hline . \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 4.53 \\ -\ 3.27 \\ \hline . \\ \hline \end{array}$ $\begin{array}{r} 2.34 \\ -\ 0.72 \\ \hline . \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 5.03 \\ -\ 2.48 \\ \hline . \\ \hline \end{array}$

<table>
<tr>
<td>6:6</td>
<td>Now extend the column methods to cases where the addends/minuend/ subtrahend have different numbers of digits.

Again, encourage children to articulate the value of each digit. Draw attention to the fact that when a digit is ‘not present’ it still needs to be ‘included’ in the calculation and is treated as a zero; pay particular attention to this for subtraction. For example, for the calculation shown opposite, the minuend has no hundredths digit, but the subtrahend does.

Provide children with practice, both laying out the column calculations correctly, and completing column calculations.</td>
<td>Column subtraction – minuend has no hundredths digit:

• ‘Complete the calculation.’

$$\begin{array}{rcrcr} & 2 & 3 & . & 4 & \\ - & & 7 & . & 6 & 2 \\ \hline & & & . & & \\ \hline \end{array}$$
‘Twenty-three-point-four has no hundredths digit, but I still need to subtract two hundredths from the number.’

$$\begin{array}{rcrcr} & 2 & 3 & . & \not{4}^{3} & {}^{1}0 \\ - & & 7 & . & 6 & 2 \\ \hline & & & . & & \\ \hline \end{array}$$
• ‘Write these as column calculations.’

6.32 + 1.45
16.32 + 1.45
16.32 + 21.45
16.32 + 1.45 + 12.81

5.64 – 2.17
15.6 – 2.17
15.64 – 2.17</td>
</tr>
</table>

6:7

To complete this teaching point, present varied practice, including opportunities for children to apply their understanding of calculating for numbers with hundredths to real-world contexts, including measures and statistics, as shown opposite and below:

- *'A dressmaker had 1 m of ribbon. He used 0.65 m of it. How much does he have left?'*
- *'A gardener had 5.75 m of string. She bought another 10 m of string, and then used 9.25 m. How much string does she have now?'*

'The table shows the masses of some pets in kilograms.'

Pet	Mass (kg)
Minnie the mouse	0.02
Molly the mouse	0.04
Harvey the hamster	0.07
Harold the hamster	0.06
Chuck the chipmunk	0.13
Ronnie the rabbit	1.45
Roxie the rabbit	2.32
Larry the Labrador	32.25

- *'What is the total mass of the mice, Minnie and Molly?'*
- *'Which two pets' combined mass is equal to that of Chuck the chipmunk?'*
- *'How much greater is Roxie the rabbit's mass than Ronnie the rabbit's mass?'*
- *'Connie the cat's mass is 2.03 kg more than Roxie the rabbit's mass. What is Connie's mass?'*
- *'How much greater is Larry the Labrador's mass than Connie the cat's mass?'*

Dòng nǎo jīn

- *'What could the missing numbers be?'*
- *'What could they not be?'*
- *'How do you know?'*

$$\begin{array}{rcccc} & 5 & . & \square & 1 \\ + & 1 & . & 3 & \square \\ \hline & 7 & . & 2 & 4 \\ \hline \end{array} \qquad \begin{array}{rcccc} & 3 & . & 2 & 6 \\ + & \square & . & 4 & \square \\ \hline & 7 & . & 7 & \square \\ \hline \end{array}$$

Teaching point 7:

Numbers with hundredths can be rounded to the nearest tenth by examining the value of the hundredths digit or to the nearest whole number by examining the value of the tenths digit.

Steps in learning

7:1

Guidance

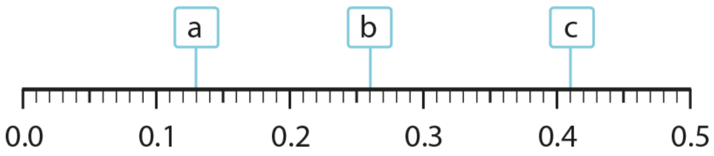
Follow a similar progression as for rounding to the nearest whole number in segment *1.23 Composition and calculation: tenths, Teaching point 6*:

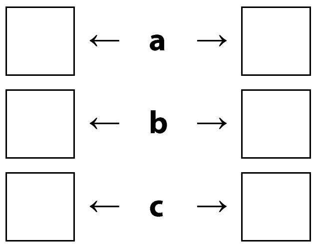
- Begin by zooming in on the number line, showing the whole numbers, then tenths and then hundredths.
- Ask children to identify which tenths a given number lies between on the number line. Use the following stem sentences to link to earlier work:
  - ***'___ is between ___ and ___.'***
  - ***'___ is the previous tenth.'***
  - ***'___ is the next tenth.'***
- Ask children to identify which of the tenths the given number is *closest* to. To support this, make sure that children can identify the half-way point between two tenths. Extend the stem sentences to include the closest tenth, as exemplified opposite.
- Build on children's previous knowledge of rounding to cover cases with five hundredths. Use the generalised statement: ***'If there are five hundredths or more round up to the next tenth; if there are fewer than five hundredths round down to the previous tenth.'***

Provide children with some practice rounding to the nearest tenth before moving on to the next step.

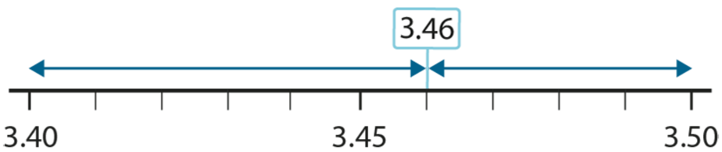
Representations

- *'For each letter on the number line, say which two tenths the value is between.'*





- *'Which tenths does 3.46 lie between?'*



- *'Three-point-four-six is between three-point-four and three-point-five.'*
- *'Three-point-four is the previous tenth.'*
- *'Three-point-five is the next tenth.'*
- *'Three-point-five is the **closest** tenth.'*

		• *'Round each of these numbers to the nearest tenth.'* 2.18 nearest tenth → ☐ 2.15 nearest tenth → ☐ 2.12 nearest tenth → ☐ 2.09 nearest tenth → ☐ 2.02 nearest tenth → ☐ 1.99 nearest tenth → ☐ 1.95 nearest tenth → ☐ 1.92 nearest tenth → ☐
7:2	Children already know how to round numbers with tenths to the nearest whole number. Now explore rounding numbers with hundredths to the nearest whole number, drawing attention to the fact that we only need to look at the tenths digit. Encourage children to explain what they are doing as they round and draw their attention to which digit(s) they are looking at. Ask the children: • *'Does the hundredth digit make a difference when you are rounding to the nearest whole number?'* • *'What happens when the tenths digit is zero?'* You can link to work in segment *1.22 Composition and calculation: 1,000 and four-digit numbers*, where children learnt that when rounding, the important digit is the one to the right of the place value they are rounding to. Encourage children to reach the same generalisation in the context of rounding numbers with tenths/hundredths.	• *'Round each of these numbers to the nearest whole number.'* 2.1 nearest whole number → ☐ 2.18 nearest whole number → ☐ 2.12 nearest whole number → ☐ 2.09 nearest whole number → ☐ 3.02 nearest whole number → ☐ • *'What do you notice about the hundredths digit?'*

7:3

Complete this teaching point by providing rounding practice, including contextual problems, for example:

- *'Poppy the python is 3.09 m long. How long is this to the nearest whole metre?'*
- *'George the gerbil has mass 34.45 g. How much is this to the nearest gram?'*
- *'Suki the snail moved 0.59 m one morning and another 0.62 m that afternoon. To the nearest whole metre, how far did Suki go?'*

Dòng nǎo jīn:

'The table shows how much ribbon a dressmaker has of different colours. Complete the table.'

Ribbon	**Length (m)**	**Length to the nearest 10 cm**	**Length to the nearest metre**	**Length to the nearest 10 m**
red	28.45	28.5		
blue	15.32		15	
yellow	12.02			10

Teaching point 8:

When one is divided into 1,000 equal parts, each part is one thousandth of the whole. Knowledge and strategies for numbers with tenths and hundredths can be applied to numbers with thousandths.

Steps in learning

	Guidance	Representations
8:1	You can follow a similar progression for thousandths: • Introduce thousandths by exploring various wholes divided into 1,000 equal parts, including measures contexts. • Relate thousandths to hundredths and tenths. • Extend children's understanding of writing tenths and hundredths as decimal fractions to writing thousandths as decimal fractions (for example, the number written '0.001' is one thousandth, the number written '0.004' is four thousandths, and so on). • Count in thousandths and compare/order numbers with thousandths, including measures contexts. • Explore the additive composition of numbers with thousandths (e.g. $1.432 = 1 + 0.4 + 0.03 + 0.02$). • Explore the multiplicative composition of numbers with thousandths, (e.g. $1.432 = 1432 \times 0.001$). • Spend some time examining various measures (kilograms, kilometres, litres, etc.) and graphing contexts that are based on thousandths. • Extend known facts and strategies, including column algorithms, mental strategies and rounding to numbers with thousandths. Work on thousandths is simply an extension of what children have already learnt about tenths and	

	hundredths, and the relationship between different place-value positions; by now they should be starting to see the emerging patterns and you may choose to spend less time using pictorial representations and manipulatives. Unless in segment *1.23 Composition and calculation: tenths* you chose to use a Dienes thousand cube to represent one, avoid the use of Dienes, otherwise children will need to shift their thinking to a different use of the manipulatives.	