MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.13 Addition and subtraction: two-digit and single-digit numbers

Teacher guide | Year 2

Teaching point 1:

Knowledge of the number line, and quantity values of numbers, can be applied to add/subtract one to/from a given two-digit number.

Teaching point 2:

Known facts for the numbers *within* ten can be applied to addition/subtraction of a single-digit number to/from a two-digit number.

Teaching point 3:

Knowledge of numbers which sum to ten can be applied to the addition of a single-digit number and two-digit number that sum to a multiple of ten, or subtraction of a single-digit number from a multiple of ten.

Teaching point 4:

Known strategies for addition or subtraction bridging ten can be applied to addition or subtraction bridging a multiple of ten.

Spring 2018 pilot

Overview of learning

In this segment children will:

- extend, to two-digit numbers, their understanding that one more gives the next number in the counting sequence, and one less gives the previous number in the counting sequence, and use this to add/subtract one to/from two-digit numbers
- explore how their known number facts within ten (e.g. $3 + 5 = 8$, $8 - 5 = 3$) and to ten (e.g. $7 + 3 = 10$, $10 - 3 = 7$) can be used to add/subtract a single-digit number to/from a two-digit number (e.g. $23 + 5 = 28$, $40 - 3 = 37$)
- build on the previous point (addition to make a multiple of ten; subtraction from a multiple of ten), exploring how the strategies of 'making ten' and 'subtracting through ten' can be applied, respectively, to addition and subtraction problems that bridge multiples of ten (e.g. $37 + 4 = 41$, $41 - 4 = 37$)
- identify, reason and generalise patterns highlighted by various representations of example calculations, to reveal the underlying mathematical structures.

This segment builds upon learning from previous segments, including:

- experience counting forwards and backwards for two-digit numbers, and familiarity with ordinal representations of number (such as the Gattegno chart and number lines)
- fluency in addition and subtraction within ten
- understanding of place value for two-digit numbers (partitioning and recombining, working flexibly with tens and ones)
- fluency in partitioning and recombining single-digit numbers (when looking to 'make a multiple of ten' with addition, or subtract *through* a multiple of ten).

A variety of representations have been chosen, based on their appropriateness in revealing the underlying mathematical structures. Throughout, concrete and pictorial representations should be used to expose the structures and strategies, rather than as a tool for calculation; children should be encouraged, where possible, to unitise and use known number facts, rather than to count manipulatives to 'find the answer'.

Pattern-seeking is a common thread throughout this segment. A danger in pattern-seeking, and the continuation of patterns, is the risk that the process becomes procedural; to ensure children's conceptual understanding is developed, they should always be required to explain their reasoning, expressing *why* the patterns occur and *how they know* further examples would fit within the pattern.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

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Teaching point 1:

Knowledge of the number line, and quantity values of numbers, can be applied to add/subtract one to/from a given two-digit number.

Steps in learning

1:1

Guidance

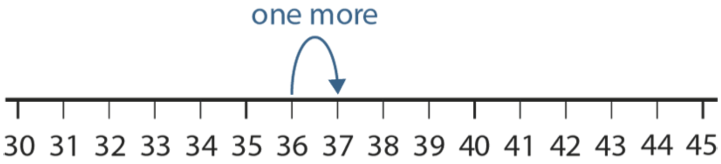
By now children should be confident in counting forwards and backwards in ones to/from 100, and be familiar with the concept that the number *after* a given number in is 'one more' and the number *before* is 'one less'. Children have practised finding one more/less than a given number in segments *1.3 Composition of numbers: 0–5* and *1.4 Composition of numbers: 6–10*; they then applied this to the formal addition and subtraction of one to/from single-digit numbers in segment *1.7 Addition and subtraction: strategies within 10.* In this teaching point, extend this knowledge to two-digit numbers (for example, $56 = 55 + 1$ because 56 is the number after 55; $54 = 55 - 1$ because 54 is the number before 55). For now, do not record the abstract equations – this will be covered from step *1:5*.

Using a number line or Gattegno chart, which both draw attention to ordinality, ask children to:

- identify the difference between two adjacent two-digit numbers (they should be familiar with this use of the term 'difference' from segment *1.12 Subtraction as difference*); draw attention to the fact that the difference is always one ('one more' or 'one less')
- ask children to identify one more or one less than a given number; for example, tap on 50 and 4 on the Gattegno chart (54), and have the children chant back one more (or one

Representations

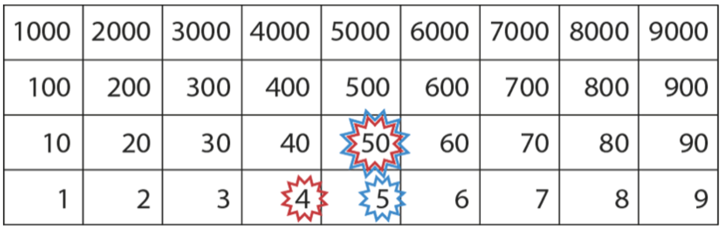
Number line:



- *'Thirty-seven is one more than thirty-six. Thirty-seven is equal to thirty-six plus one. Thirty-six plus one is equal to thirty-seven.'*
- *'Thirty-six is one less than thirty-seven. Thirty-seven minus one is thirty-six. The difference between thirty-six and thirty-seven is one.'*

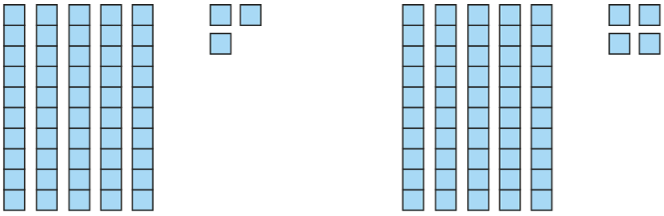
Gattegno chart:

'What is one more than fifty-four?'



1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

'Fifty-five is one more than fifty-four. Fifty-five is equal to fifty-four plus one. Fifty-four plus one is equal to fifty-five.'

	less), i.e. 'fifty-five' (or 'fifty-three') tapping the numbers as they chant. As this step uses the counting sequence, with which children should be very familiar, do not mention regrouping when working at a tens boundary (for example, adding one to 49, or subtracting one from 60). Use the stem sentences: - ***'___ is one more than ___. ___ is equal to ___ plus one. ___ plus one is equal to ___.'*** - ***'___ is one less than ___. ___ minus one is ___. The difference between ___ and ___ is one.'***	
1:2	Once children are confident in identifying one more/less on a number line and/or Gattegno chart, move on to representing the same equations using quantity-value representations. For example, if you have modelled the equation $37 = 36 + 1$ in step *1:1*, return to this equation here, using a bead bar or bead string image, or Dienes. The beads make a useful stepping stone from the number line to a purely quantity-value representation (such as Dienes). From now, until step *1:6*, avoid calculations which require regrouping (e.g. $39 + 1 = 40$, $40 - 1 = 39$). Draw attention to the fact that when we add one, the number of 'ones' increases by one, and when we subtract one, the number of 'ones' decreases by one. In preparation for using a place-value chart, lay out Dienes with the tens on the left and the ones on the right. Emphasise that the change is always happening in the ones. Show Dienes representations of two adjacent numbers and ask children to identify: - *'What's the same?'*	Bead bar / bead string:  - *'Thirty-seven has one more one than thirty-six.'* - *'Thirty-six plus one is equal to thirty-seven.'* - *'The tens have stayed the same; the ones are different.'* Dienes: - *'What's the same? What's different?'* - *'What equation could we say using the two numbers represented?'*  - *'Fifty-four is one more than fifty-three; fifty-three plus one is equal to fifty-four.'* - *'Fifty-three is one less than fifty-four; fifty-four minus one is equal to fifty-three.'* - *'The tens have stayed the same. The ones are different.'*

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- *'What's different?'*
- the addition and subtraction sentences which link the two numbers.

Children should practise making or sketching their own representations of the numbers (lines for tens Dienes; dots for ones Dienes).

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<td>1:3</td>
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Now, show the digits on a place-value chart, alongside the corresponding Dienes representation. Explore different number pairs, working towards identifying the change of one in the ones digit. At this stage also begin to model use of the equations. Note that the equations should be used to record what is being shown with the concrete/pictorial representations, and alongside the oral descriptions; children are not yet working in the abstract themselves.

To promote and assess depth of understanding, you could use a dòng nǎo jīn question, such as:

'Charlie is thinking of a number. When he subtracts one, the tens digit is double the ones digit. What number could he be thinking of? Is there more than one possibility?'

(Answers: 22, 43, 64, or 85)

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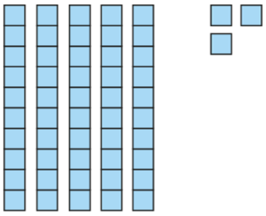

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<tr><th>10s</th><th>1s</th></tr>
<tr><td>5</td><td>3</td></tr>
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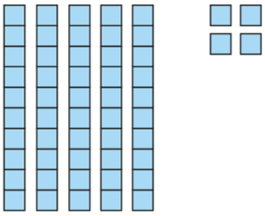

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<tr><th>10s</th><th>1s</th></tr>
<tr><td>5</td><td>4</td></tr>
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53 + 1 = 54

54 – 1 = 53

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<td>1:4</td>
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Now move on to working with the digits *without* the place-value charts, and explore missing number sequences.

Ask children to 'prove' how they know the value of the missing number, using concrete manipulatives or pictorial representations (such as the one shown opposite) to demonstrate their conceptual understanding, while describing in full sentences.

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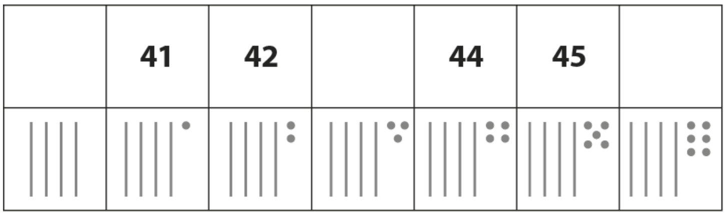
'Fill in the missing numbers.'



- *'The missing number is forty-three. I know this because forty-three is one more than forty-two; forty-two plus one is equal to forty-three.'*
- *'I also know this because forty-three is one less than forty-four. Forty-four minus one is equal to forty-three.'*

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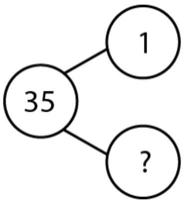
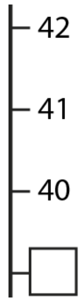
		- *'The tens digit stays the same (four); the ones digit changes (three).'*
1:5	Now children have explored 'plus one' and 'minus one' with concrete and pictorial representations, and verbally, progress to abstract representations, beginning with part–part–whole diagrams and equations alongside one another, then progressing to using just equations. Use the part–part–whole representation to support children in making the link between the inverse operations. Also, draw attention to the equivalence of, for example, *1 + 34* and *34 + 1*, and check that children are confident in applying the commutative law of addition to solve problems where the first addend is one (e.g. *1 + 34*) by thinking 'one more than'. Note that we are looking at 'one more' and 'one less', so the equations *35 – 34 = 1* (and *1 = 35 – 34*) have not been included in the example set opposite; however, these are the final two which make up the set of eight equations supported by the part–part–whole diagram, so children may mention equations such as these. Children have already learnt, in the context of single-digit numbers, that consecutive numbers have a difference of one (segments *1.7 Addition and subtraction: strategies within 10* and *1.12 Subtraction as difference*); this is one of the 'known facts' which children will draw on (in segment *1.16 Subtraction: two-digit and two-digit numbers*) to solve problems such as *35 – 34.* For missing number problems, as well as presenting related sets of equations, include isolated examples to ensure that children are secure in solving them 'out of sequence' and in the absence of	Part–part–whole diagrams and related equations: *'Fill in the missing numbers.'*  $35 = \square + 1$ $35 = 1 + \square$ $\square + 1 = 35$ $1 + \square = 35$ $35 - 1 = \square$ $\square = 35 - 1$ Missing number problems – related equations: *'Fill in the missing numbers.'* one more → 83 ☐ ← one less $42 + 1 = \square$ $\square = 26 + 1$ $\square - 1 = 42$ $26 = \square - 1$

	the related inverse operation equation. Throughout, make sure that you vary the position of the equals sign, in order to emphasise equivalence.	Missing number problems – 'isolated' examples: *'Fill in the missing numbers.'* $27 + 1 = \boxed{}$ $\qquad \boxed{} = 42 + 1$ $1 + 87 = \boxed{}$ $\qquad \boxed{} = 1 + 54$ $42 + \boxed{} = 43$ $\qquad 94 = \boxed{} + 93$ $63 - 1 = \boxed{}$ $\qquad \boxed{} = 82 - 1$
1:6	Once children can add and subtract one without crossing the tens boundary, move on to adding one to a number ending in nine, and subtracting one from a multiple of ten. Children should be able to easily cross a tens boundary, both forwards and backwards, when chanting the full number sequence, as they have had plenty of practice with the ordinal position of numbers. Now move non-sequentially between one boundary and another rather than only starting at 0 (for forward counting) or 100 (for backward counting). Initially scaffold the counting using number line and number card representations, including variation in their orientation and direction, or the Gattegno chart. Then, focusing on backward counting, remove the scaffold. Practise until you can say a multiple of ten (for example, 'forty') and the children can respond with the previous number (for example, 'thirty-nine'), and until you can write a multiple of ten on the board (e.g. 40) and children can write the previous number (e.g. 39).	Number cards: $\boxed{42}$ $\boxed{41}$ $\boxed{40}$ $\boxed{39}$ Number line:  *'Forty-two, forty-one, forty, thirty-**<u>nine</u>**.'*
1:7	Now that children are fluent in identifying 'one more' or 'one less' across a tens boundary, focus on number as a quantity, and the	

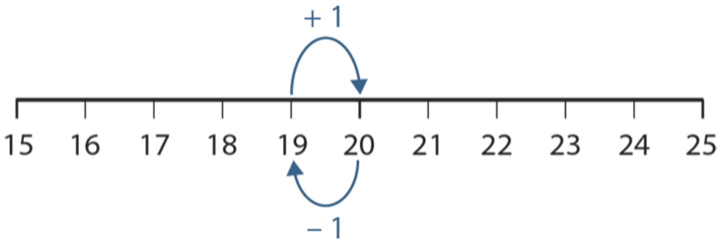
place-value aspect of adding and subtracting one.

Begin with *19 + 1*, representing it on a number line and bead bar. Then represent this with Dienes. Draw attention to the fact that if we had nine ones and then added one more one, we now have ten ones which is equal to one group of ten. Emphasise the regrouping in terms of replacing ten ones Dienes with one tens Diene, and explain that we now have two tens and zero ones, which we call twenty.

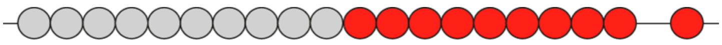
Repeat for *20 – 1*, exchanging one ten for ten ones and then subtracting a one.

The part–part–whole model is useful here, as it helps to expose the relationship between *19 + 1* and *20 – 1*.

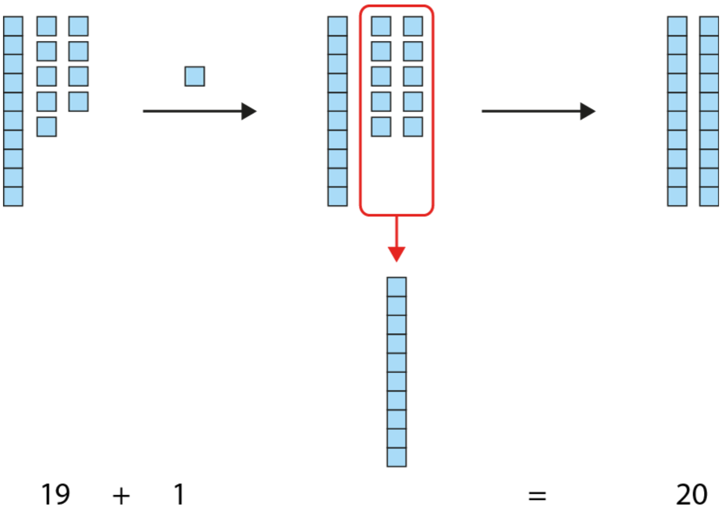
Number line:

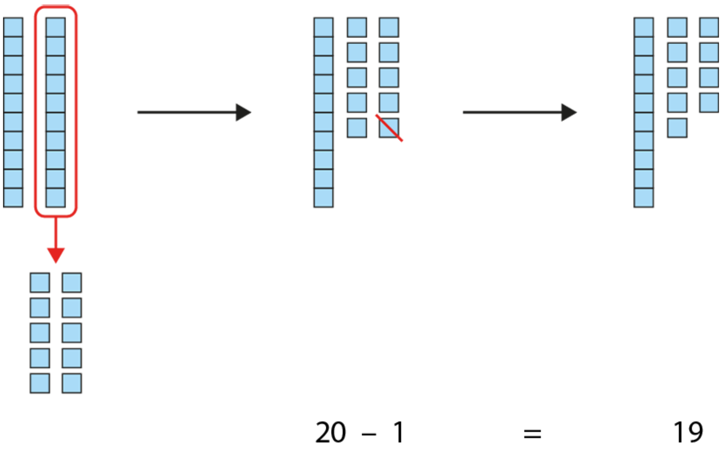


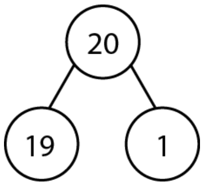
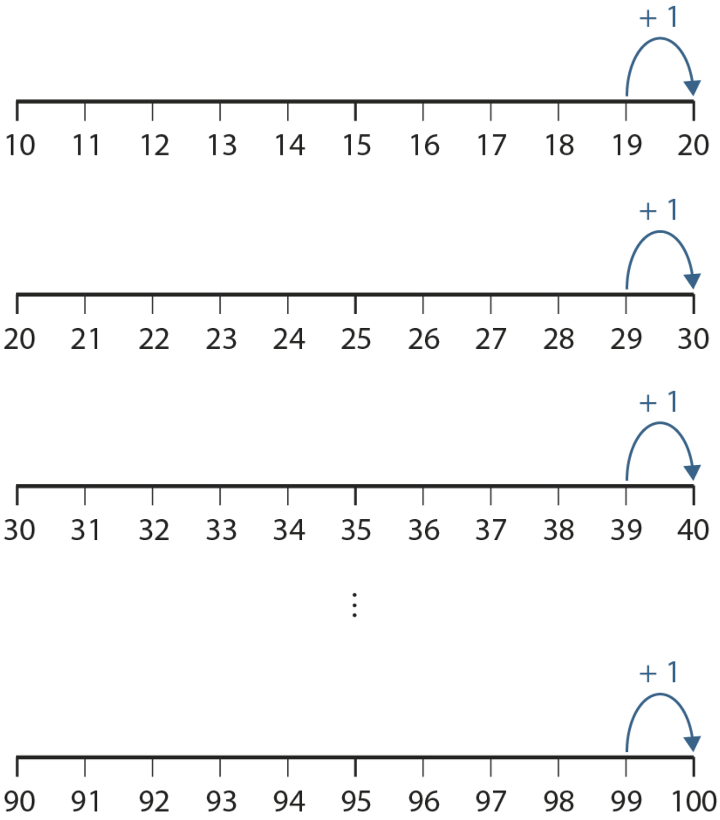
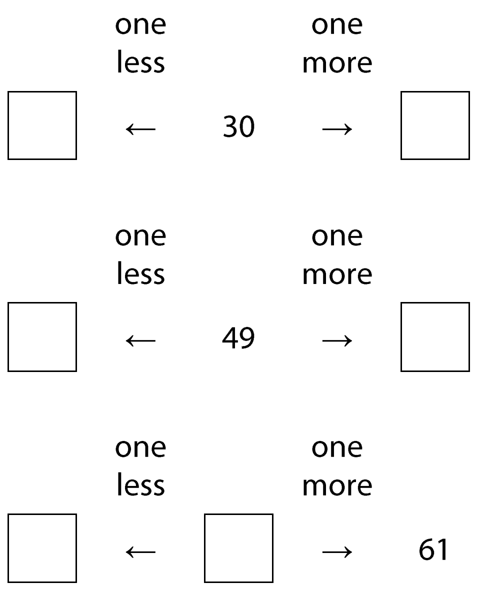
Bead bar:



Dienes:





		Part–part–whole: 
1:8	Build on the previous step by looking at *all* additions and subtractions of one that cross tens boundaries (29 and 30, 39 and 40, etc.), representing each with both an ordinal model (bead bar or number line) and with Dienes. Draw attention to what is the same in each example. In this and the next step, include the boundary between 99 and 100, since children should be secure in their understanding of 100 as ten tens from segment *1.8 Composition of numbers: multiples of 10 up to 100.*	
1:9	Provide varied missing number problems for practice, including both missing number sequences and equations with missing numbers. Children may still want to use concrete or pictorial representations for support, but eventually need to move to working just with the abstract representations through, for example, mentally evoking a number line. Begin with problems which bridge tens boundaries, then broaden the practice to include adding/subtracting one to/from any two-digit number. Children will have some experience of working with teen numbers from segment *1.10 Composition of numbers: 11–19*, so	*'Fill in the missing numbers.'* 

bring these together with the other two-digit numbers here.

To promote and assess depth of understanding, present a dòng nǎo jīn problem, such as:

'Darius writes two subtraction equations where the subtrahend (the number you are subtracting) is one. He says one problem is more difficult than the other. What could his equations have been? Why do you think this?'

Also present real-life problems involving addition and subtraction of one, for example:

- *'Saskia bought a chocolate bar for fifty-nine pence, and a sweet for one penny. How much did Saskia spend in total?'* (aggregation)
- *'Faris bought a bouncy ball that cost forty-nine pence. He gave the shop-keeper fifty pence. How much change did Faris get?'* (partitioning)
- *'At first Tiffany had seventy-four marbles; then her friend gave her another marble. How many marbles does Tiffany have now?'* (augmentation)
- *'At first the baker had twenty-five cakes; then someone bought one cake. How many cakes does the baker have now?'* (reduction)
- *'Ollie built a tower sixty-seven centimetres tall. Sarah's tower is one centimetre shorter than Ollie's tower. How tall is Sarah's tower?'* (difference)

	one less		one more	
☐	←	99	→	☐

☐ = 30 + 1

☐ = 30 – 1

49 – 1 = ☐

49 + 1 = ☐

☐ = 61 – 1

60 + 1 = ☐

24 = ☐ + 1

☐ – 1 = 23

78 = 77 + ☐

78 – ☐ = 77

	one less		one more	
☐	←	12	→	☐

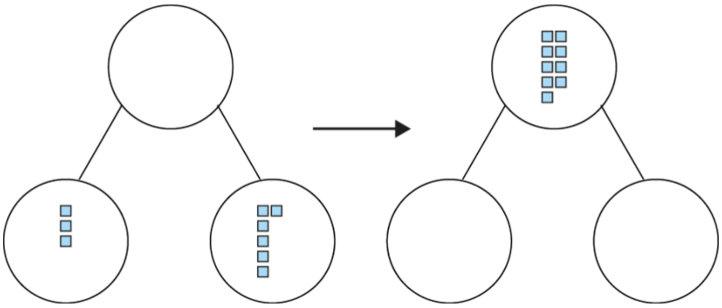
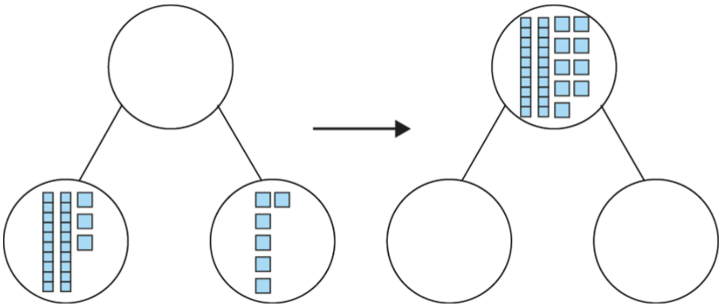
☐ = 12 – 1

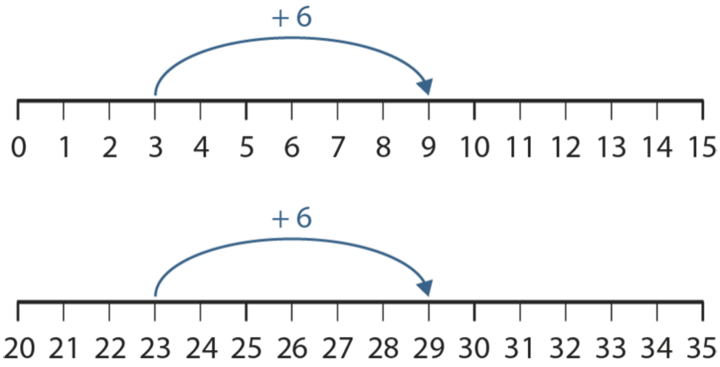
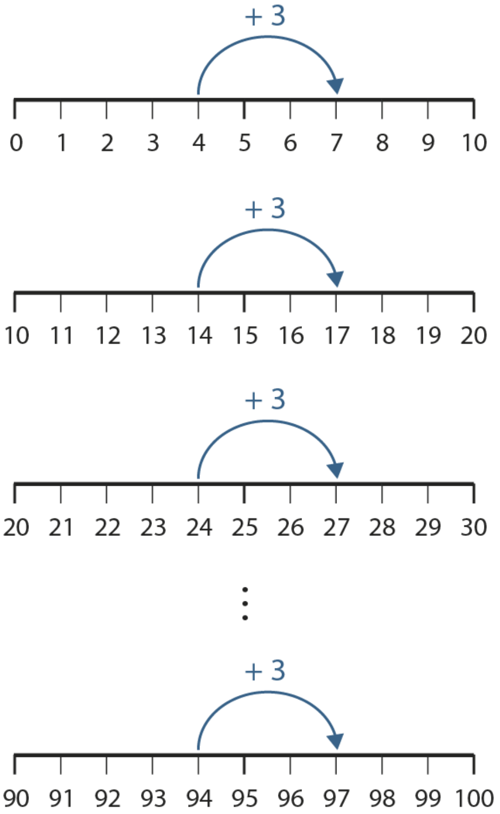
12 + 1 = ☐

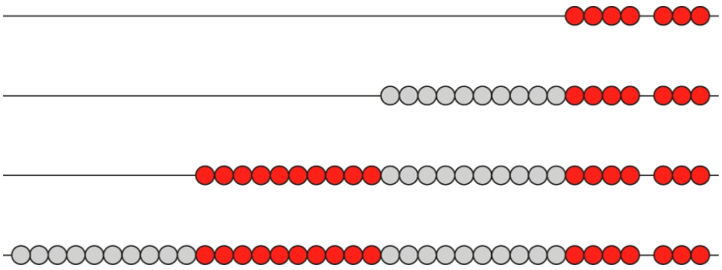
Teaching point 2:

Known facts for the numbers within ten can be applied to addition/subtraction of a single-digit number to/from a two-digit number.

Steps in learning

	Guidance	Representations
2:1	For this teaching point, do not include additions which sum to a multiple of ten (e.g. $23 + 7 = 30$), or subtractions from a multiple of ten (e.g. $30 - 7 = 23$); these will be covered in *Teaching point 3*. Begin with an addition story within ten, for example: '*Diego walked for three minutes to get to his friend's house, and then walked for another six minutes to get to school. His journey took nine minutes altogether.*' Ask children to represent the story on a part–part–whole diagram using Dienes ones, then write the corresponding equation; ask children to describe, in full sentences, what each number in the equation represents. Giving children the sum (i.e. the total journey time) ensures their attention remains on the structure, rather than on calculation. However, emphasise that we don't need to count the total number of ones to find the answer; by now many children will just know that $3 + 6 = 9$. If not, prompt them to solve $3 + 6$ by relating to their knowledge that $4 + 6 = 10$. Present a similar story, but now for addition of a two-digit and a single-digit number. Use the same ones digits as in the first story, for example: *'Dana walked for twenty-three minutes to get to her friend's house, and then walked for another six minutes to get to school. Her journey took twenty-nine minutes altogether.'* As before, ask children to represent the story on a	Dienes and part–part–whole diagrams: *'Diego walked for three minutes to get to his friend's house, and then walked for another six minutes to get to school. His journey took nine minutes altogether.'*  $3 + 6 = 9$ - *'What does the 3 represent?'* - *'What does the 6 represent?'* - *'What does the 9 represent?'* *'Dana walked for twenty-three minutes to get to her friend's house, and then walked for another six minutes to get to school. Her journey took twenty-nine minutes altogether.'*  $23 + 6 = 29$ - *'What does the 23 represent?'* - *'What does the 6 represent?'* - *'What does the 29 represent?'*

	part–part–whole diagram, now using tens and ones Dienes, and write the corresponding equation. Now compare the two stories. Ask children: - *'What's the same?'* - *'What's different?'* Draw attention to the fact that the only difference is the two additional tens in the second calculation. It is also useful to compare the calculations on a number line representation.	Number lines:  - *'What's the same?'* - *'What's different?'*
2:2	Once you are confident that children have understood these two examples, start to demonstrate that we can apply our addition facts within ten to *any* two-digit numbers. Begin by exploring a single-digit fact and the set of corresponding two-digit plus single-digit calculations to reveal the underlying pattern: for given ones addends (e.g., ***4*** + ***3***, ***14*** + ***3***, ***24*** + ***3***, etc.), the sums will have the same ones digit, regardless of the value of the tens digit. Children can continue to represent the calculations using Dienes, as in the previous step. Also, show the related calculations on a number line, ensuring that you are using the representation to expose the structure and generalise the pattern, rather than as a tool for calculation; continue to encourage children to use known facts rather than counting on in ones. As you model the calculations, record the corresponding equations. In each case, ask children: - *'What do you notice about the ones digits? Can you see any patterns?'* - *'What is changing? What is staying the same?'* - *'Which other equations would belong in this pattern? Why?'*	Related addition facts – number line:  4 + 3 = 7 14 + 3 = 17 24 + 3 = 27 94 + 3 = 97

2:3	The bead bar is a useful representation to show alongside the number line. In the example opposite, the four ones and three ones have been made from what is now an incomplete block of ten (with three beads 'discarded' to the right). This allows children to pull blocks of ten beads from the left to explore the related calculations, rather than needing to make the four and three each time; this draws attention to what is staying the same (*4* + *3*). Choose carefully whether/when to introduce this extra representation. Modelling a given concept/calculation in different ways can improve conceptual understanding by revealing underlying mathematical structures; however, using too many representations at the same time can be confusing for children.	Related addition facts – bead bar: 
2:4	Using one of the representations as a scaffold, provide children with practice writing linked facts for themselves, for example, begin with children representing *6* + *2* with Dienes, then ask them to make and write the related calculations, working forwards through the sequence (*16* + *2*, *26* + *2*, etc.). Use the stem sentence: - ***'I know that ___ plus ___ is equal to ___…'*** (single-digit fact) - ***'…so ___ plus ___ is equal to ___.'*** (related two-digit plus single-digit calculation) To encourage children to think more deeply about the patterns, ask questions such as: '*True or false – when we add five ones to a number that ends in three, the total always ends in eight?*' Ask children to explain their answers using concrete or pictorial representations to support their descriptions.	

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<p>Now work on subtraction examples, again beginning with a single-digit number fact and then exploring the related two-digit minus single-digit calculations. Use the same representations described for addition in steps <i>2:1–2:4</i>, and use the stem sentence:</p>

<i>'I know that ___ minus ___ is equal to ___…'</i>
(single-digit fact)
<i>'…so ___ minus ___ is equal to ___.'</i>
(related two-digit minus single-digit calculation)

<p>End with children practising writing linked facts, and answering questions such as: <i>'True or false – when we subtract four ones from a number that ends in nine, the total always ends in five?'</i></p>
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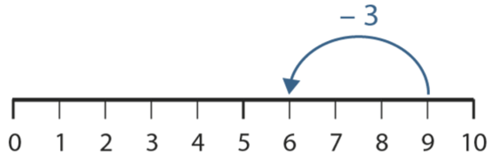
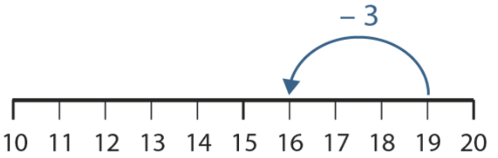
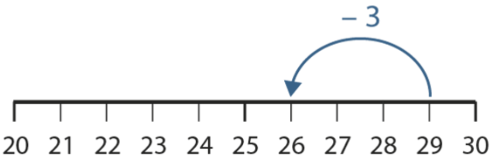
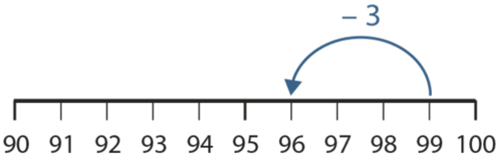
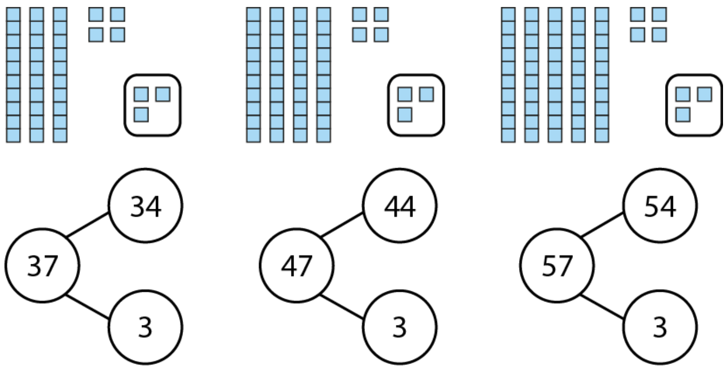
<td>
<p>Related subtraction facts – number line:</p>


<p>9 – 3 = 6</p>


<p>19 – 3 = 16</p>


<p>29 – 3 = 26</p>
<p>⋮</p>


<p>99 – 3 = 96</p>
</td>
</tr>
<tr>
<td>2:6</td>
<td>
<p>When children have mastered working with related facts as described in steps <i>2:1–2:5</i>, and can generate their own examples, introduce part–part–whole models with numerals. Begin with number sequences that you explored in detail for addition and subtraction, so that only the representation is being varied. Present the part–part–whole models alongside the familiar Dienes representations and ask children to identify similarities and differences between the representations.</p>
<p>The part–part–whole representations are particularly useful for linking addition and subtraction as inverse operations – for each diagram, ask children to identify the addition and subtraction equations. Note that, at this stage, we are looking at calculations involving addition or subtraction of a single-digit number, so</p>
</td>
<td>


<p>34 + 3 = 37
3 + 34 = 37
37 = 34 + 3
37 = 3 + 34</p>
<p>37 – 3 = 34
34 = 37 – 3</p>
<p>44 + 3 = 47
3 + 44 = 47
47 = 44 + 3
47 = 3 + 44</p>
<p>47 – 3 = 44
44 = 47 – 3</p>
<p>54 + 3 = 57
3 + 54 = 57
57 = 54 + 3
57 = 3 + 54</p>
<p>57 – 3 = 54
54 = 57 – 3</p>
</td>
</tr>
</table>

	the equations *37 – 34 = 3* (and *3 = 37 – 34*) have not been included in the example set opposite; however, these are the final two which make up the set of eight equations supported by the part–part–whole diagram, so children may mention equations such as these.	
2:7	Children must also practise working with only the abstract equations, although part–part–whole models can be used initially for support. As usual, present missing number problems with variation both in terms of the location of the missing number (sum, addend, difference, subtrahend and minuend) and in terms of the position of the equals sign (to emphasise equivalence). Encourage children to reason about their answers, for example: $57 = \boxed{54} + 3$ *'I know that fifty-seven is equal to* ***fifty-four*** *plus three because to make a seven digit in the ones, I need to add four to the three.'* Remind children to imagine visual representations of the abstract equations, such as Dienes or number line representations. Ask children to work in pairs, for example: • one child closes their eyes and describes to their partner what the numbers look like • partners take it in turns to describe the difference calculations in a sequence. Also present missing number problems in isolation, rather than as part of a sequence.	*'Fill in the missing numbers.'* $7 = 4 + 3$ $8 - 4 = 4$ $17 = 14 + 3$ $18 - 4 = 14$ $27 = 24 + \boxed{}$ $28 - \boxed{} = 24$ $37 = 34 + \boxed{}$ $38 - \boxed{} = 34$ $47 = \boxed{} + 3$ $\boxed{} - 4 = 44$ $57 = \boxed{} + 3$ $\boxed{} - 4 = 54$ $\boxed{} = 64 + 3$ $68 - 4 = \boxed{}$ $\boxed{} = 74 + 3$ $78 - 4 = \boxed{}$ $52 + 6 = \boxed{}$ $85 - 3 = \boxed{}$ $23 + \boxed{} = 29$ $65 - \boxed{} = 63$ $98 = \boxed{} + 4$ $25 = \boxed{} - 4$

2:8	To further embed the learning, and to ensure that children aren't resorting to counting on/back strategies, present children with missing number problems, and ask them to identify which single-digit fact would help them to solve the equation. Initially these could be presented in a multiple choice format, as in the example opposite. To provide further challenge use a dòng nǎo jīn problem of the form: *'Chang is sorting expressions into sets and he puts* 81 + 8 *and* 85 + 4 *together.'* • *'Why did Chang put these together?'* • *'Can you write more expressions that are part of this set?'*	Identifying related facts: ☐ = 42 + 3 *'Circle the number fact I can use to solve the equation above.'* 4 + 2 = 6 4 + 3 = 7 2 + 3 = 5 *'Explain why I can use this number fact.'*
2:9	When children are confident working in the abstract, provide them with practice using real-life problems, such as the data context on the next page or money contexts below. By choosing examples carefully, varying the structures and stories (as with the questions below), you can draw attention to the underlying mathematical structures. Ask children to identify the similarities and differences between the questions, and for each question ask: *'Which addition/subtraction fact would help you solve this problem?'* Aggregation and partitioning: • *'Omar bought a chocolate bar for thirty-two pence, and a lollipop for seven pence. How much did Omar spend in total?'* • *'Omar bought a chocolate bar for thirty-two pence, and a lollipop; he spent thirty-nine pence in total. How much did the lollipop cost?'*	

- *'Omar bought a chocolate bar, and a seven-pence lollipop; he spent thirty-nine pence in total. How much did the chocolate bar cost?'*

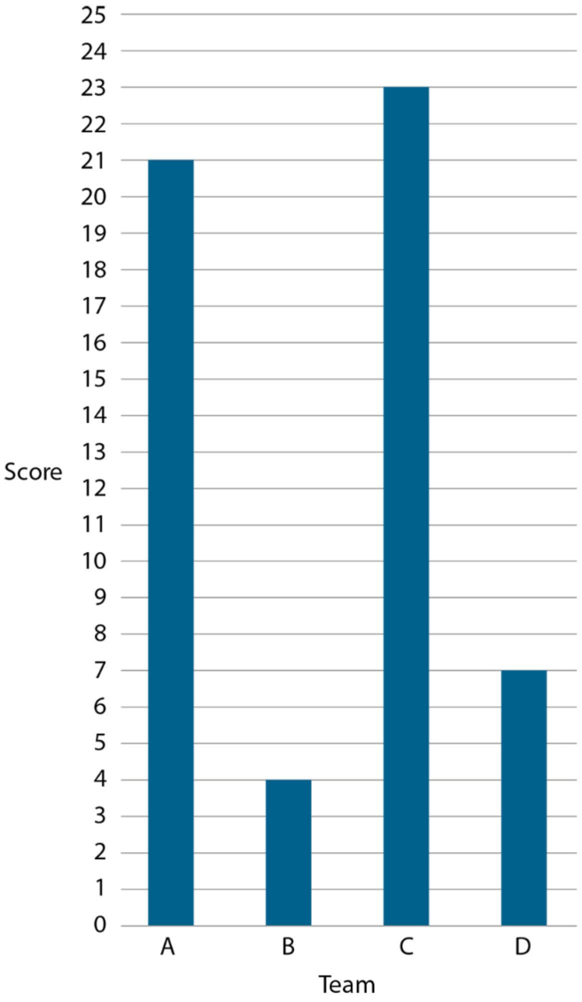
Augmentation and reduction:

- *'At first Faris had twenty-one pounds in his piggy bank; then he added his three pounds of pocket money. How much does Faris have now?'*
- *'At first Faris had twenty-one pounds in his piggy bank; then he added his pocket money. Now he has twenty-four pounds. How much pocket money did Faris add?'*
- *'At first Faris had some money in his piggy bank; then he added his three pounds of pocket money. Now he has twenty-four pounds. How much money did Faris have at first?'*

Difference:

- *'Aliona spent forty-two pounds, and James spent more than Aliona. The difference between the amounts they spent is five pounds. How much did James spend?'*
- *'Aliona spent less than James, and James spent forty-seven pounds. The difference between the amounts they spent is five pounds. How much did Aliona spend?'*
- *'Aliona spent forty-two pounds, and James spent forty-seven pounds. What is the difference between the amounts they spent?'*

Data context:



- *'What is the total of Team A and Team D's scores? Which addition fact can help us answer this? Why?'*
- *'What is the difference between Team B and Team C's scores? Which addition fact can help us answer this? Why?'*

Teaching point 3:

Knowledge of numbers which sum to ten can be applied to the addition of a single-digit number and two-digit number that sum to a multiple of ten, or subtraction of a single-digit number from a multiple of ten.

Steps in learning

Guidance

Representations

3:1

Children should already be familiar with pairs of numbers which sum to ten, and with partitioning ten. Now start to connect these known facts to calculations of the form:

two-digit number	+	single-digit number	=	multiple of ten
e.g. 16	+	4	=	20

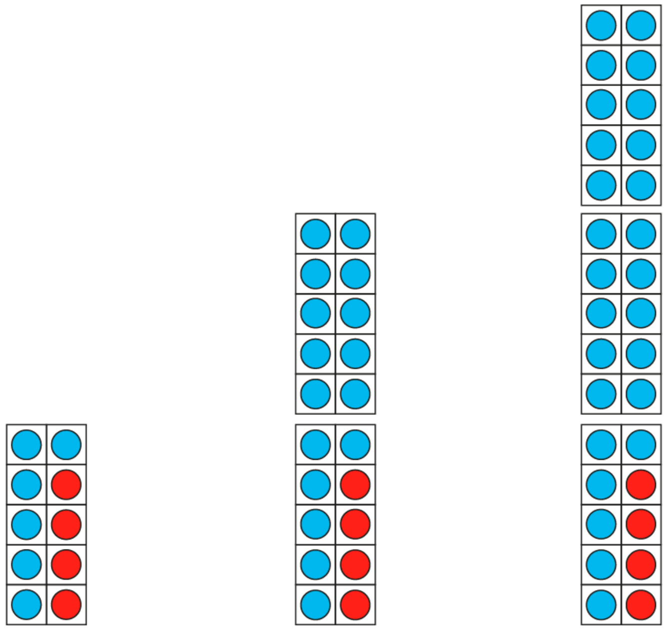
As in *Teaching point 1*, include sums to 100 (e.g. *96 + 4 = 100*).

Begin with visual representations such as 'filling' tens frames to make complete tens, and completing groups of ten on a bead bar, recording abstract equations at the same time (again, a classroom bead bar, or image of a bead string, is preferable to handing out bead strings, to ensure the structure isn't 'lost' as the bead strings curl up – see segment *1.9 Composition of numbers: 20–100*, step *3:6*).

Use the following stem sentence to emphasise the link between the calculations: ***'I know that ___ plus ___ is equal to ten, so I know that ___ plus ___ is equal to ___.'***

Note that calculations that relate to *9 + 1 = 10* and *10 – 1 = 9* were covered in detail in steps *1:6–1:8*.

Filling tens frames:

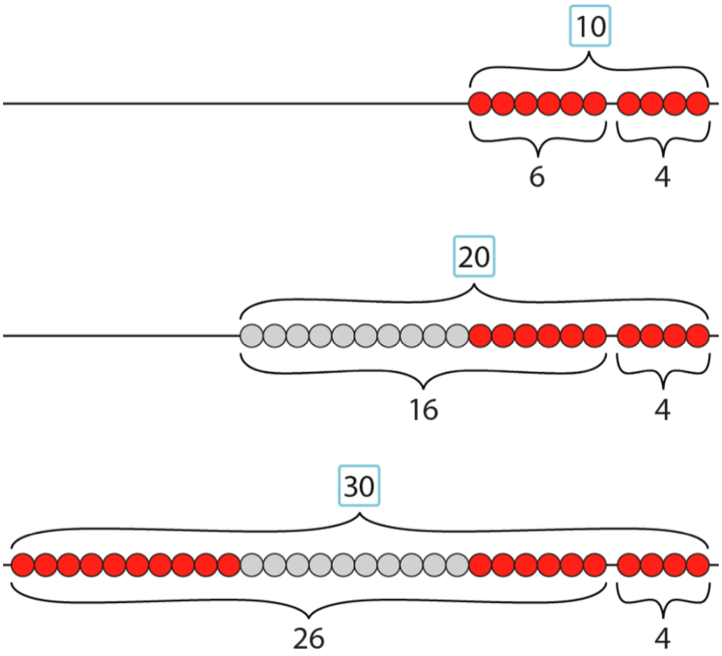


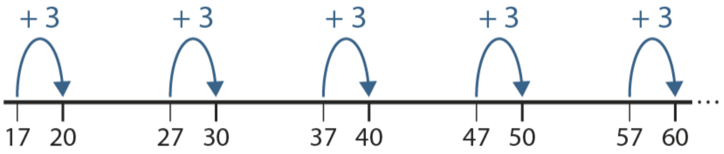
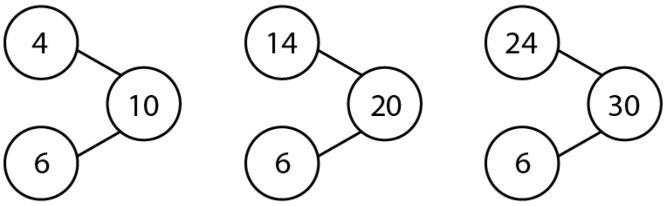
6 + 4 = 10

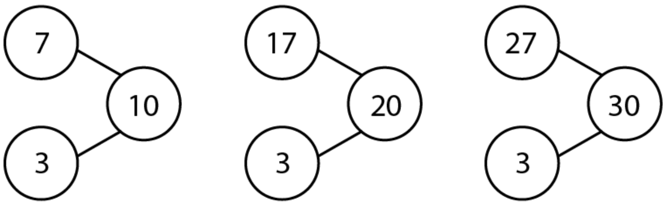
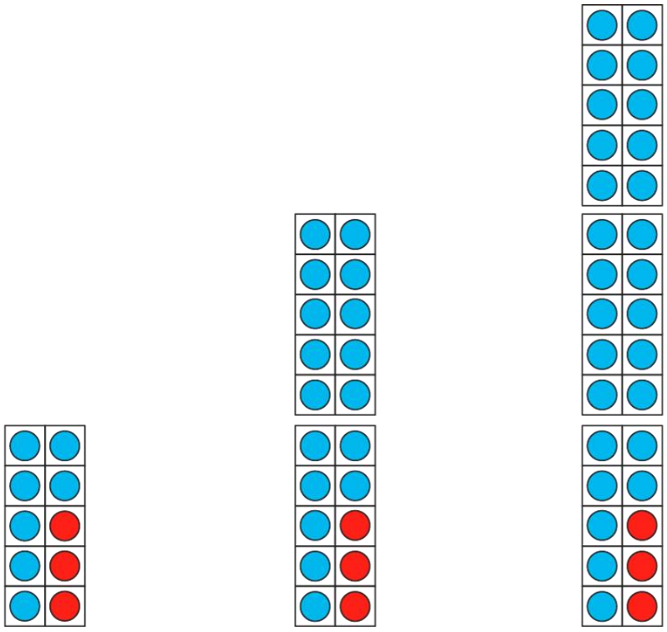
16 + 4 = **2**0

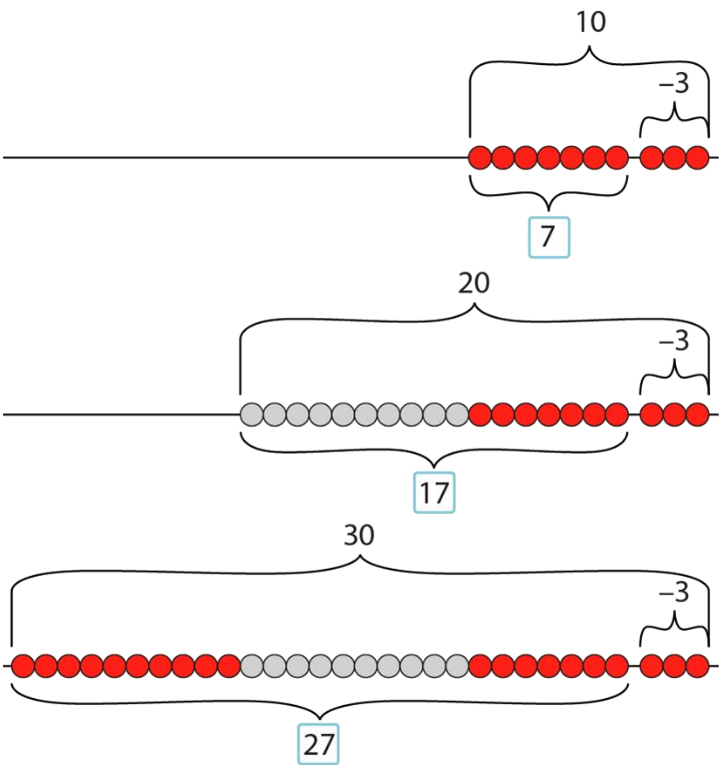
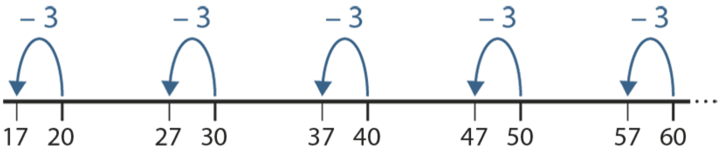
26 + 4 = **3**0

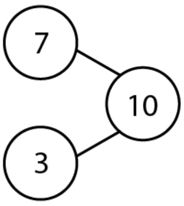
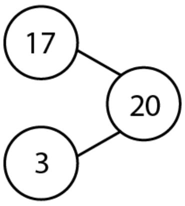
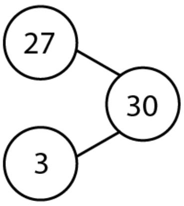
Completing 'blocks' of ten on the bead bar:



		- *'I know that six plus four is equal to ten…'* - *'…so, I also know that sixteen plus four is equal to twenty…'* - *'…and that twenty-six plus four is equal to thirty…'*
3:2	Now move on to representing relationships on a blank number line (i.e. with only the numbers involved in the calculations marked on). This is a combination of pictorial and abstract and can be easily linked to the bead bar representation in step *3:1*. Begin to draw out the pattern, by asking children questions such as: - *'What is the same each time?'* - *'What is different?'* - *'Is it true that if a number ends in a seven and we add three, we always get a multiple of ten? Why / why not?'* (Children should explain that this is because seven and three sum to ten). Present part of a pattern, then ask children to give the answer to a related calculation, for example: *'Can you use this pattern to calculate* 87 + 3*?'* To assess depth of understanding, use dòng nǎo jīn questions of the form: *'If one addend has two ones, how many ones must the other addend have for the sum to be a multiple of ten?'*	
3:3	Now provide varied practice, for a range calculations based on number bonds to ten, in the form of missing number problems. Initially children can use concrete and pictorial representations, such as those described in steps *3:1* and *3:2*, for support. They can then progress to just being supported with part–part–whole diagrams, and should finally be able to complete equations without support. The use of the part–part–whole diagrams will also help prepare children for the next step (subtraction	*'Fill in the missing numbers.'*  10 = 5 + 5 20 = 15 + 5 30 = 25 + ☐ 41 + 9 = 50 42 + 8 = 50 43 + 7 = ☐

	of a single-digit number from a multiple of ten). Continue to ask questions and discuss the patterns to avoid the exercises becoming procedural. Note that calculations related to *9 + 1* have been included in the examples opposite, bringing together all calculations related to bonds to ten.	40 = 35 + ☐ 50 = ☐ + 5 60 = ☐ + 5 ☐ = 65 + 5 ☐ = 75 + 5 ☐ = 85 + 5 ☐ = 95 + 5 44 + 6 = ☐ 45 + ☐ = 50 46 + ☐ = 50 ☐ + 3 = 50 ☐ + 2 = 50 ☐ + 1 = 50
3:4	Now, link to the inverse operation (subtraction), using part–part–whole diagrams as a scaffold. Use the same representations as in step *3:1* (tens frames and bead bars), alongside the corresponding part–part–whole diagrams. Ask children to identify the subtraction equations verbally, and record the abstract equations on the board as the children describe them. Keep the focus on using number bonds within ten: although, for example, we can correctly record the relationship *20 – 17 = 3*, focus on the equation in which the single-digit number is the subtrahend, i.e. *20 – 3 = 17*. Using the tens frames and bead bars in the manner shown opposite should help keep the focus on single-digit subtrahends. As in step *3:1*, use a stem sentence to emphasise the link between the calculations: ***'I know that ten minus ___ is equal to ___, so I know that ___ minus ___ is equal to ___.'***	Part–part–whole diagrams:  Tens frames:  10 – 3 = 7 **2**0 – 3 =**1**7 **3**0 – 3 = **2**7

		Bead bars:  - *'I know that ten minus three is equal to seven…'* - *'…so, I also know that twenty minus three is equal to seventeen…'* - *'…and that thirty minus three is equal to twenty-seven…'*
3:5	As in step *3:2*, demonstrate the pattern on a blank number line. Keep the focus on exposing the pattern, rather than using the number line to calculate. Draw attention to the patterns by asking questions such as: - *'What pattern can you see? Why is this pattern created?'* - *'What is the same each time?'* - *'What is different?'* - *'What do you notice about the tens digits in the minuend (the number from which we are subtracting) and the difference?'* (the tens digit is decreasing by one) - *'What do you notice about the ones digits in the subtrahend and the*	

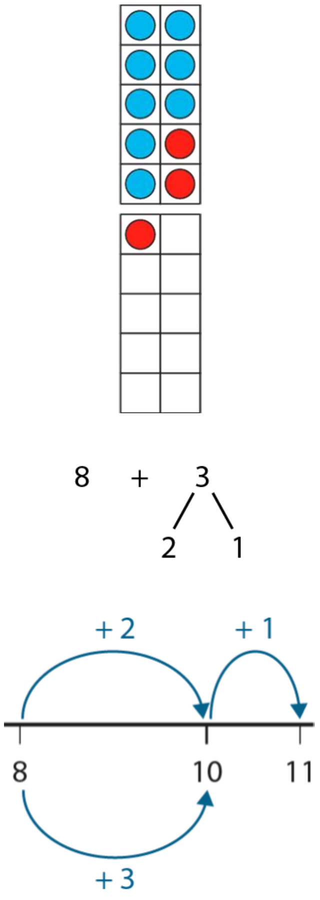
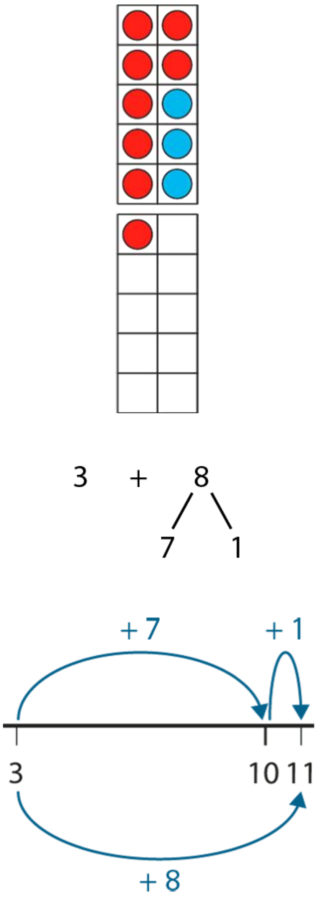
	difference?' (the ones digits sum to ten) Present part of a pattern, then ask children to give the answer to a related calculation, for example: *'Can you use this pattern to calculate* 90 – 3?*'*	
3:6	When children are able to explain the relationships shown in steps *3:4* and *3:5*, present missing number problems as shown opposite to draw further attention to the mathematical structure. As in step *3:3*, initially children can use concrete and pictorial representations for support. They can then progress to just being supported with part–part–whole diagrams, and should finally be able to complete equations without support. As before, continue to ask questions and discuss the patterns to avoid the exercises becoming procedural. Similarly to the addition practice, calculations related to *10 – 1* have been included in the examples opposite, bringing together all calculations related to bonds to ten. To promote depth of understanding, use dòng nǎo jīn questions of the form: - *'I subtract six from a multiple of ten, what can you tell me about the difference?'* - *'My minuend was a multiple of ten and my difference had three ones. What can you tell me about the subtrahend?'* - *'If I subtract four from a multiple of ten, the ones digit of the difference will always be six. True or false?'* Encourage children to explain their answers using concrete manipulatives or a picture.	*'Fill in the missing numbers.'*    10 – 3 = ☐ 20 – ☐ = 17 ☐ – 3 = 27

		10 – 1 = 9 20 – 1 = 19 30 – 1 = 29 40 – 1 = □ 50 – 1 = □ 60 – □ = 59 70 – □ = 69 □ – 1 = 79 □ – 1 = 89 □ – 1 = 99 49 = 50 – 1 48 = 50 – 2 47 = 50 – 3 46 = 50 – □ 45 = 50 – □ 44 = □ – 6 43 = □ – 7 □ = 50 – 8 □ = 50 – 9

Teaching point 4:

Known strategies for addition or subtraction bridging ten can be applied to addition or subtraction bridging a multiple of ten.

Steps in learning

	Guidance	Representations
4:1	Children already learnt strategies for addition and subtraction across ten in segment *1.11 Addition and subtraction: bridging 10*. This teaching point builds on the 'making ten' strategy for addition (segment *1.11*, *Teaching point 5*) and the 'subtraction *through* ten' strategy for subtraction (segment *1.11*, step *6:1*). Revise these strategies now, and ensure all children can apply them flexibly, drawing on an understanding of the commutativity of addition (for example, we can calculate *8 + 3* via *8 + 2 + 1* or via *3 + 7 + 1* as shown opposite). As before, use the tens frames, number lines and abstract representations alongside one another, drawing attention to the fact that each representation shows the same thing. Here, link addition and subtraction by showing them side-by-side. Encourage children to describe the calculations in full sentences, for example: • *'First I partition the three into two plus one.* • *'Then eight plus two is equal to ten…'* • *'…and ten plus one is equal to eleven.'* Note that, unlike in segment *1.11*, the calculations opposite have been recorded in a 'continuous layout, for example: 8 + 3 = 8 + 2 + 1 = 10 + 1 = 11	 8 + 3 = 8 + 2 + 1 = 10 + 1 = 11  3 + 8 = 3 + 7 + 1 = 10 + 1 = 11

<table>
<tr>
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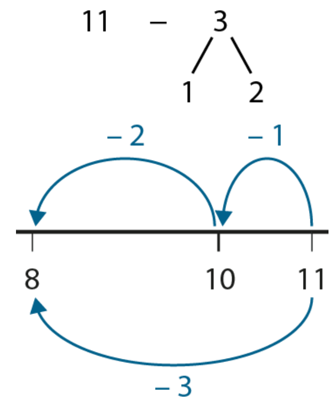
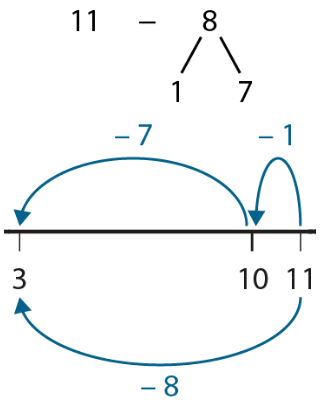
<td>Take care to always ensure that both you and the children use the equals sign correctly, to indicate equivalence.</td>
<td>


11 – 3 = 11 – 1 – 2
= 10 – 2
= 8



11 – 8 = 11 – 1 – 7
= 10 – 7
= 3
</td>
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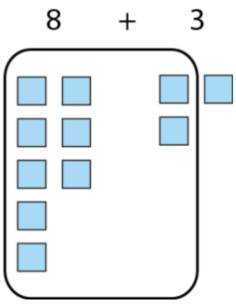
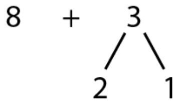
<tr>
<td>4:2</td>
<td>Now extend the ‘make ten’ strategy to ‘make a multiple of ten’, building on <i>Teaching point 3</i>, where children learnt that they can apply number bonds to <i>ten</i> to <i>multiples of ten</i>. In this strategy the single-digit addend is partitioned such that a multiple of ten can be made.

Begin by using calculations related to the one you used in step <i>4:1</i>, represented with Dienes and equations, before moving on to other examples. Show the related examples alongside one another so that children can make links between them. Give children enough practice representing this in the concrete, keeping the focus on reasoning <i>how</i> this strategy works, rather than counting the manipulatives.

Continue to describe the calculations in full sentences as described in step <i>4:1</i>.

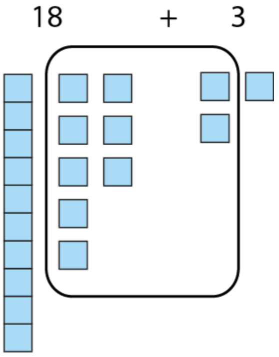
Ensure children can independently build up their own sequences (e.g. <i>6 + 7 = 13</i>, <i>16 + 7 = 23</i>, etc.), before moving on to the next step.</td>
<td>
Making ten:

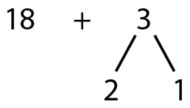




8 + 3 = 8 + 2 + 1
= 10 + 1
= 11

Making twenty:





18 + 3 = 18 + 2 + 1
= 20 + 1
= 21
</td>
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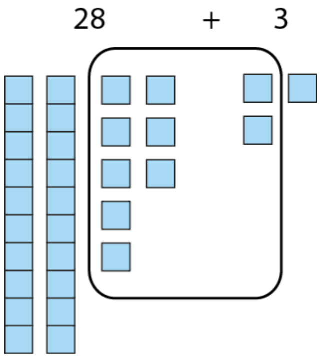
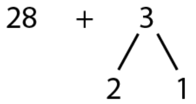
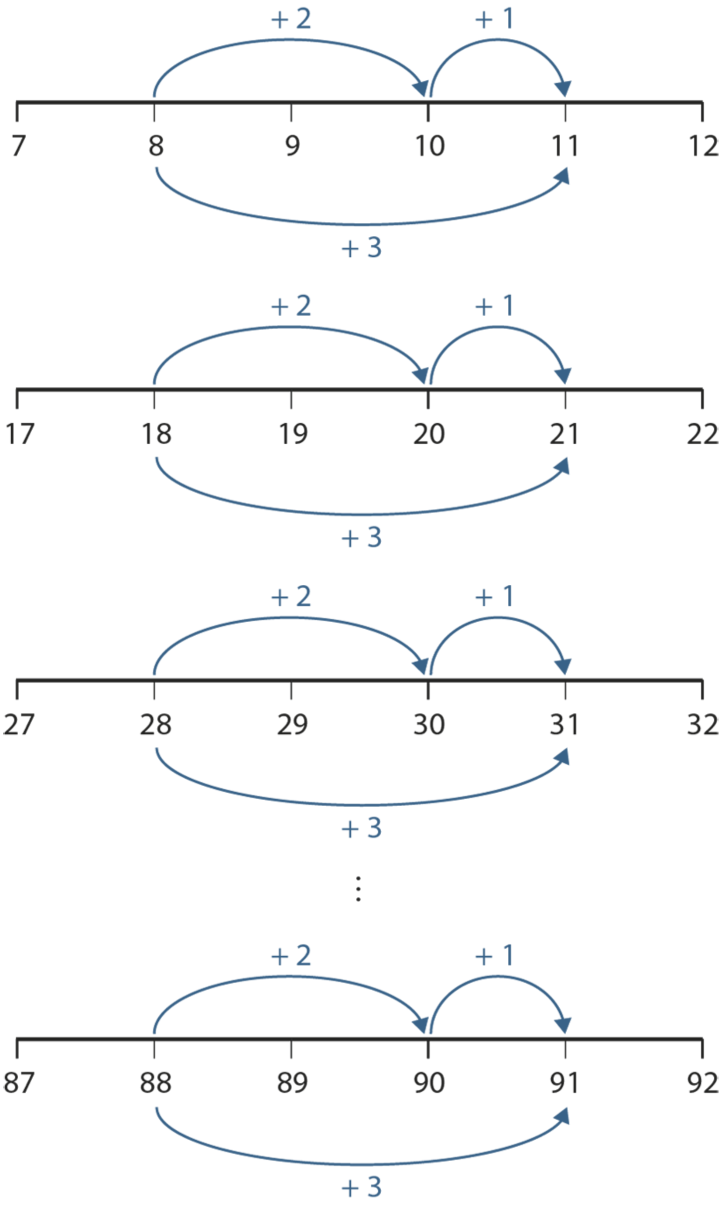
		Making thirty:   28 + 3 = 28 + 2 + 1 = 30 + 1 = 31
4:3	Next, show the calculations on a number line and write the equations one above the other to further draw attention to the link between the calculations. Ask children questions such as: - *'What's the same?'* (the ones digits are the same in all calculations for both the addends and the sum) - *'What's different?'* (the tens digits are different in each calculation) - *'What do you notice about the tens digit in the sum each time?'* (the tens digit of the sum is one more than the tens digit of the larger addend) - *'Why?'* (because the ones digits sum to ten or more; we've made another ten by adding the ones digits) Focus on the identification of related number facts by asking questions such as *'Which number fact can I use to help me add eight to fifty-four?'* To assess depth of understanding use dòng nǎo jīn questions of the form:	

- *'Is it true that if I add three to a number ending in eight, the sum will always end in one? Explain why / why not.'*
- *'What can you tell me about the tens number in the sum when we add three to a number ending in eight?'*
- *'Can you give me an example of some additions which require me to bridge a multiple of ten? And some which don't?'*

8 + 3 = 11
18 + 3 = 21
28 + 3 = 31
38 + 3 = 41

4:4

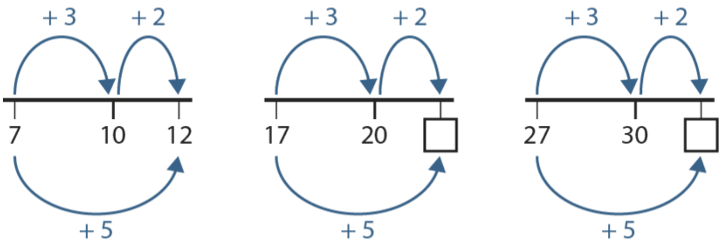
Now present varied practice in the form of missing number problems. Initially, scaffolding can be provided in the form of number lines and/or jottings that represent the appropriate partitioning (as shown opposite). Children should then progress to solving equations without scaffolding.

Finally, before moving onto the subtraction strategies, present a dòng nǎo jīn question, such as: 'Look at these two pairs of equations. What's the same within each pair and what's different?'

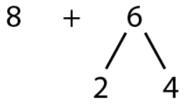
57 + 8 = 65   45 + 6 = 51
58 + 7 = 65   46 + 5 = 51

'Can you write some similar examples?'

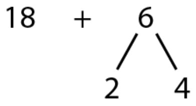
'Fill in the missing numbers.'



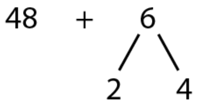
8 + 6 = □



18 + 6 = □



48 + 6 = □

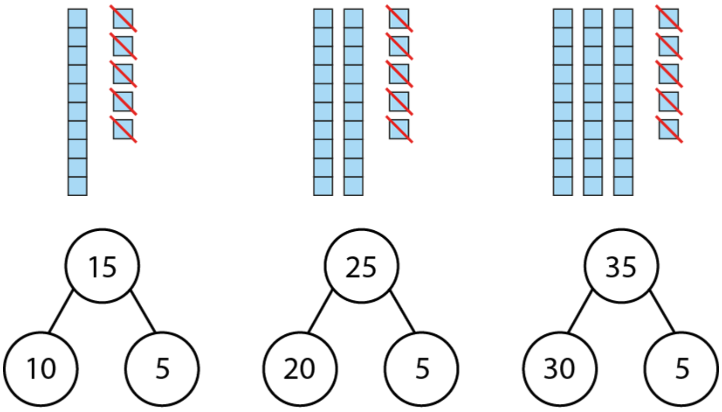
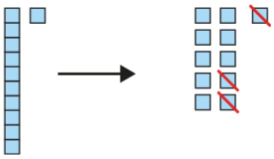
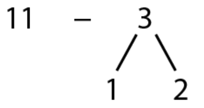
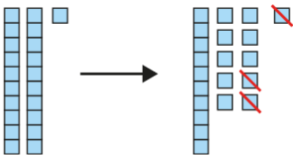
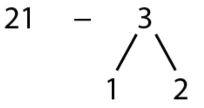


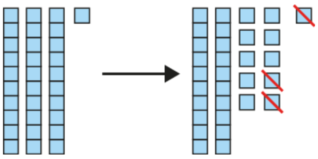
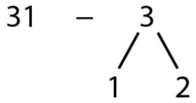
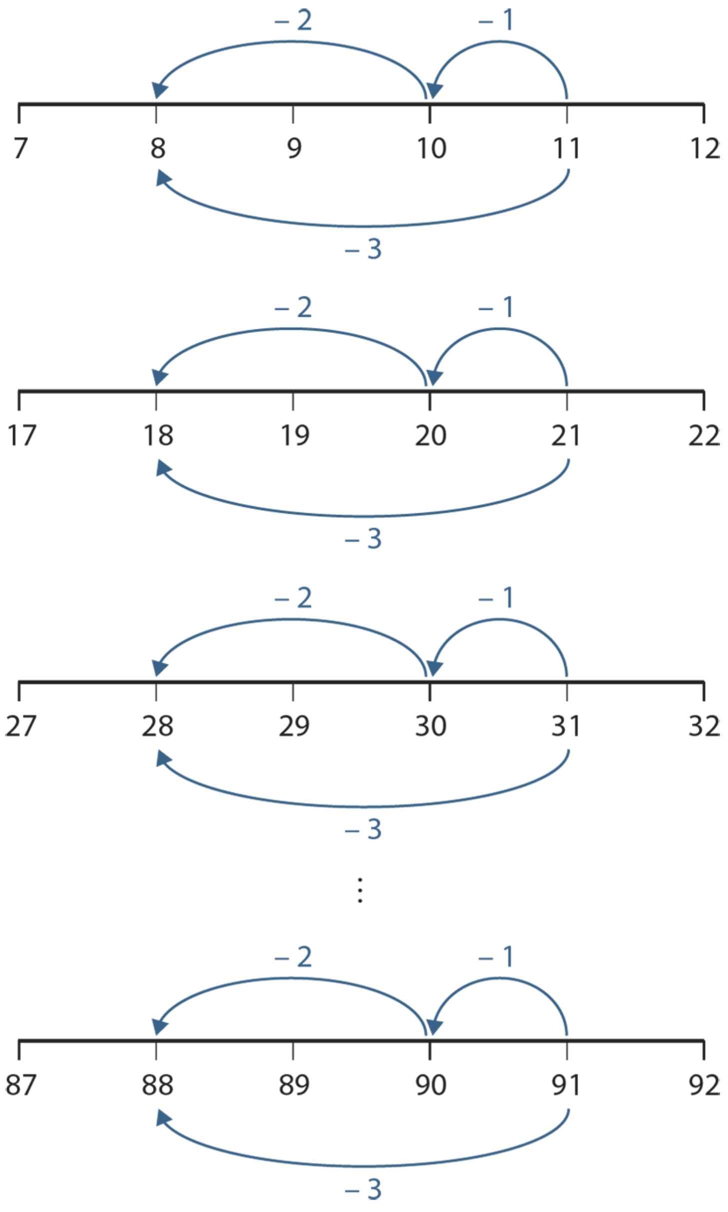
7 + 6 = 13

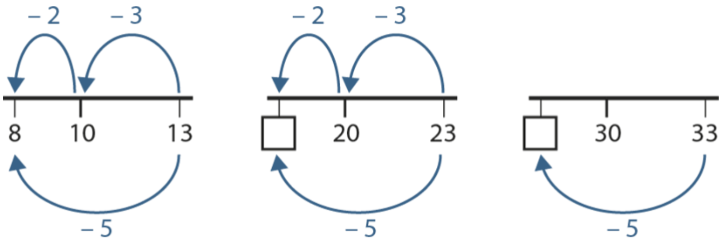
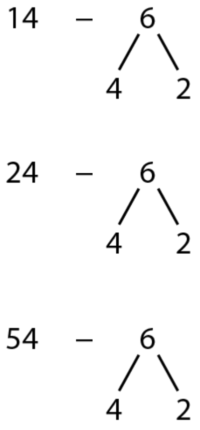
27 + 6 = □

47 + □ = 53

□ + 6 = 73

		82 = 9 + 73 $62 = 9 + \boxed{}$ $42 = \boxed{} + 33$ $\boxed{} = 9 + 13$
4:5	As a foundation for the next steps, revisit subtraction to make a multiple of ten (see segments *1.9 Composition of numbers: 20–100* and *1.10 Composition of numbers: 11–19*). This uses children's knowledge of partitioning two-digit numbers into tens and ones, with which they should already be confident. Present varied representations, including concrete, pictorial and abstract, as preparation for the next steps. Include part–part–whole models to link back to previous work on partitioning into tens and ones.	 15 – 5 = 10 25 – 5 = 20 35 – 5 = 30
4:6	Now extend the 'subtraction through ten' strategy (segment *1.11 Addition and subtraction: bridging 10)*, to subtraction through *multiples* of ten. This involves partitioning the single-digit subtrahend, so children must be confident in partitioning single-digit numbers in the way required to reach the previous multiple of ten, and fluent in subtracting a single-digit number from a multiple of ten (steps *3:4–3:6*). As with step *4:2*, use the calculations that link to those revised in step *4:1*. Give children the opportunity to work practically with Dienes – physically exchanging one ten for ten ones, to allow the subtraction to be completed, will emphasise the bridging-ten aspect. As before, keep the focus on reasoning *how* this strategy works, rather than counting the manipulatives.	Subtracting through ten:   11 – 3 = 11 – 1 – 2 = 10 – 2 = 8 Subtracting through twenty:   21 – 3 = 21 – 1 – 2 = 20 – 2 = 18

	Encourage children to describe the calculations in full sentences, for example: - *'First I partition the three into two and one.'* - *'Twenty-one minus one is twenty…'* - *'…and twenty minus two is equal to eighteen.'*	Subtracting through thirty:   31 – 3 = 31 – 1 – 2 = 30 – 2 = 28
4:7	Next, show the calculations on a number line and write the equations one above the other to further draw attention to the link between the calculations. Ask children questions such as: - *'What's the same?'* (the ones digits are the same in all calculations, for the minuend, the subtrahend and the difference) - *'What's different?'* (the tens digits are different in each calculation) - *'What do you notice about the tens digit in the difference each time?'* (the tens digit of the difference is one less than the tens digit of the minuend) - *'Why?'* (because the difference between the ones digits is greater than (or equal to) ten; we exchanged one of the tens for ten ones so we could subtract the ones) As before, focus on the identification of related number facts by asking questions such as *'Which number fact can I use to help me subtract six from fifty-four?'* To assess depth of understanding use dòng nǎo jīn questions of the form: - *'Is it true that if I subtract three from a number ending in one, the difference*	

	will always end in eight? Explain why / why not.' - *'What can you tell me about the tens number in the difference when we subtract three from a number ending in one?'* - *'Can you give me an example of some subtractions which require me to bridge a multiple of ten? And some which don't?'*	11 – 3 = 8 21 – 3 = 18 31 – 3 = 28 41 – 3 = 38
4:8	Now that children have spent time exploring different ways of partitioning numbers to bridge ten, and then generating sequences of equations using these bonds, present varied practice in the form of missing number problems. As in step *4:4*, scaffolding can initially be provided in the form of number lines and/or jottings that represent the appropriate partitioning. Children should then progress to solving equations without scaffolding. Present dòng nǎo jīn questions, such as the ones opposite and below: *'Look at these two pairs of equations.'* 57 + 8 = 65 45 + 6 = 51 58 + 7 = 65 46 + 5 = 51 *'Can you write pairs of subtraction equations which follow a similar pattern?'*	*'Fill in the missing numbers.'*  14 – 6 = ☐ 24 – 6 = ☐ 54 – 6 = ☐  12 – 3 = 9 22 – 3 = ☐ 32 – ☐ = 29 ☐ – 3 = 39

<table>
<tr>
<td></td>
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<td>
$72 - 4 = \square$
$52 - \square = 48$
Dòng nǎo jīn:
<i>‘Fill in the missing numbers.’</i>
$43 - \square = \square 7$
$\square 2 - \square = 36$
$\square 9 = 6 \square - 4$
</td>
</tr>
<tr>
<td>4:9</td>
<td>
Finally, once children are confident with the strategies and patterns for calculations bridging multiples of ten, provide a range of real-world problems, including measures contexts, such as those shown opposite and below:

<i>‘Megan and Hamza are going on holiday. Megan’s suitcase has a mass of eight kilograms and Hamza’s has a mass of seventeen kilograms. What is the total mass of their suitcases?’</i> (aggregation)
<i>‘David is using a measuring cylinder to measure rainfall. At first it contains forty-seven millilitres, and then eight millilitres more rain falls. How much water is in the measuring cylinder now?’</i> (augmentation)
<i>‘Jane’s summer holidays were forty-three days long. It rained on seven of the days. How many days did it not rain?’</i> (partitioning)
<i>‘Ella has collected seventy-four stickers. If she gives eight stickers to her friend, how many stickers will she have left?’</i> (reduction)

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<td>
Measures/data context:
<i>‘These are the times some children took to run a race.’</i>
<table>
<tr><th>Name</th><th>Time in seconds</th></tr>
<tr><td>Billy</td><td>47</td></tr>
<tr><td>Amina</td><td>52</td></tr>
<tr><td>Jen</td><td>45</td></tr>
<tr><td>Mo</td><td>48</td></tr>
<tr><td>Daisy</td><td>51</td></tr>
<tr><td>Jacob</td><td>39</td></tr>
</table>

<i>‘Who was six seconds slower than Jen?’</i>
<i>‘Who was five seconds faster than Billy?’</i>

</td>
</tr>
</table>

	Note that in the difference examples opposite, using the race times data, questions are posed that involve finding the difference between a two-digit number and a single-digit number (for example, ‘*Who was six seconds slower than Jen?*’, solved with the calculation *45 – 6 = 39)* rather than the difference between two two-digit numbers (for example, ‘*What was the difference between Jen and Jacob’s times?*’, solved with the calculation *45 – 39 = 6*). Problems of the latter form are covered in segment *1.16 Subtraction: two-digit and two-digit numbers.* To provide additional challenge and depth, use dòng nǎo jīn questions such as those shown opposite and below: - *‘A plant is nineteen centimetres tall and grows eight centimetres each week. How tall is it in three weeks?’* - *‘The teachers have eighty biscuits in the staff room. They eat eight each day. How many days until they have forty-eight left?’*	Dòng nǎo jīn:  *‘Someone spilled coffee on the price tag of the jeans! We know that the difference in price between the tracksuit and the jeans is eight pounds. How much do the jeans cost? Is there more than one possibility? Explain.’*