MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.10 Composition of numbers: 11–19

Teacher guide | Year 1

Teaching point 1:

The digits in the numbers 11–19 tell us about their value.

Teaching point 2:

The numbers 11–19 can be formed by combining a ten and ones, and can be partitioned into a ten and ones.

Teaching point 3:

A number is even if the ones digit is even; it *can* be made from groups of two. A number is odd if the ones digit is odd; it *can't* be made from groups of two.

Teaching point 4:

Doubling the numbers 6–9 (inclusive) gives an even teen number; halving an even teen number gives a number from six to nine (inclusive).

Teaching point 5:

Addition and subtraction facts within 10 can be applied to addition and subtraction within 20.

Spring 2018 pilot

Overview of learning

In this segment children will:

- develop an understanding that the numbers 11–19 are made up of 'ten and a bit'
- become confident in decomposing these numbers into their constituent tens and ones parts, and in combining tens and ones parts
- learn that these numbers follow the same mathematical patterns and rules as other two-digit numbers, despite their irregular names.

Through this segment, children will link names (for example, 'eighteen'), symbols (for example, 18), the 'ten and a bit' value of these numbers, and their place in the linear number system between 10 and 20. This group of numbers merits special attention since the names (for example, 'eighteen') express the ones part of the number first, whereas names of other two-digit numbers (for example, 'fifty-eight') express the tens part first. However, the way we write the numbers with digits, and the properties of the numbers, follow the same rules as all of the other two-digit numbers which children have already met.

Children will learn to see beyond the irregular names; they will come to understand that the regular structure of these numbers, revealed by the digits, allows them to transfer the mathematical knowledge they already have. They will explore odds and evens within the teen numbers, including learning that doubles of six, seven, eight and nine are even teen numbers. They will make connections between single-digit addition and subtraction (e.g. $3 + 2 = 5$, $6 - 1 = 5$) and addition or subtraction of a single-digit number to/from teen numbers (e.g. $13 + 2 = 15$, $16 - 1 = 15$).

The teaching of number and place value, as well as addition and subtraction, are embedded throughout the sequence. Children continue to use =, <, and >, as well as writing and interpreting equations and inequalities presented in a range of forms.

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An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks

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Teaching point 1:

The digits in the numbers 11–19 tell us about their value.

Steps in learning

Guidance | **Representations**

1:1

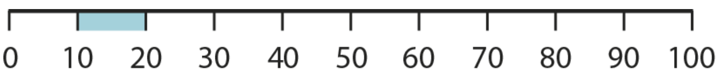
Introduce the teen numbers, within the context of numbers the children have already explored: we have already learnt about 0–10 and 20–100; now we will learn about the numbers *between* 10 and 20.

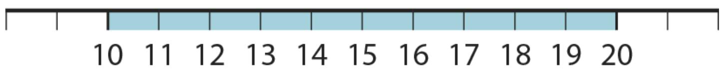
Show the teen numbers on a hundred square and on a number line.

Hundred square:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Number line:

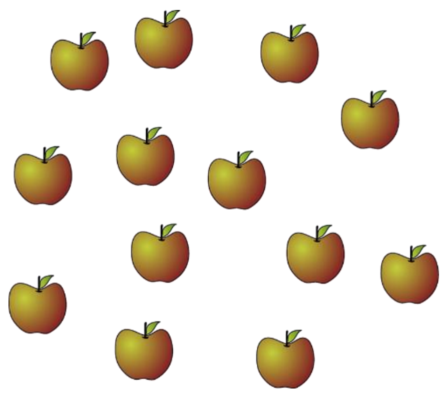




1:2

Begin the teaching sequence with the children thinking of a teen number as individual 'ones' – for example, 13 as '13 ones'. Count sets of 11–19 items and encourage children to draw round each group of ten – this will start to expose the 'ten and a bit' nature of these numbers. The names in English don't automatically draw attention to this, unlike some other languages

Individual ones:



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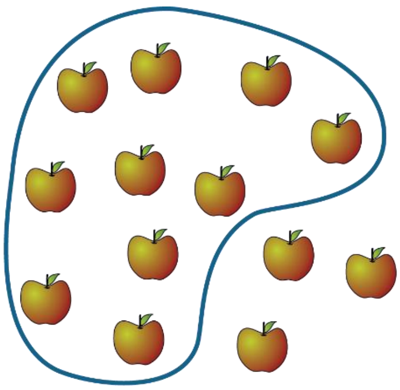
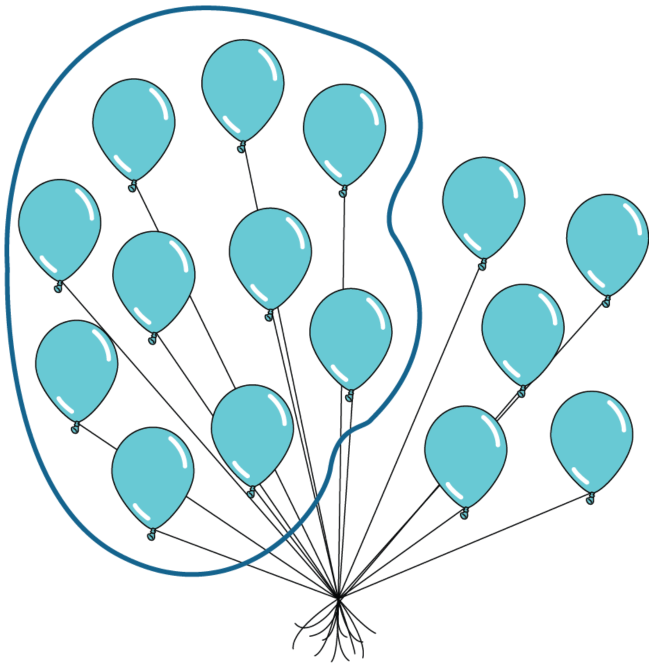
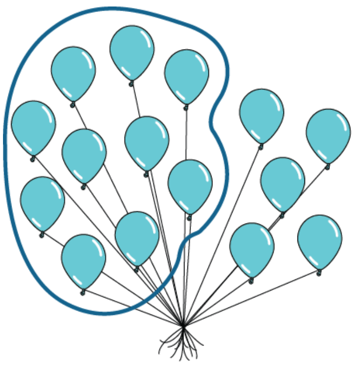
<table>
<tr>
<td></td>
<td>(eleven is ‘ten one’ in Chinese, and ‘one ten one’ in Welsh, for example).

Ask children to describe the number of items, using the following stem sentence: ‘There is one ten and ___ ones.’</td>
<td>Ten-and-a-bit:

‘There is one ten and three ones.’</td>
</tr>
<tr>
<td>1:3</td>
<td>Move on to showing children similar collections of objects, but now with ten of them already circled. Demonstrate and practise counting on from ten to see how many there are.</td>
<td></td>
</tr>
<tr>
<td>1:4</td>
<td>Now begin to record the quantities symbolically, relating the digits to the structure of the numbers.

Use a place-value chart (children should be familiar with this from segment 1.9 Composition of numbers: 20–100) to note down how many groups of ten and how many ones are in each picture.

Initially revisit pictures you used in step 1:3 (ten already circled) to help children make the conceptual step more easily.

Encourage children to describe the structure of the numbers in full</td>
<td>Ten ones are visible:

<table><tr><th>10s</th><th>1s</th></tr><tr><td>1</td><td>5</td></tr></table>
‘The 1 means one ten, and the 5 means five ones.’</td>
</tr>
</table>

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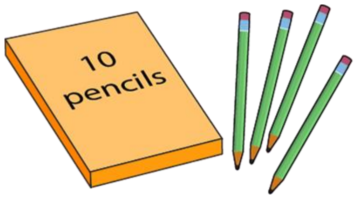
sentences, using a stem of the form:
'The 1 means one ten and the ___ means ___ one(s).'

Move on to examples where the group of ten can't be seen individually as ten ones. Use the 'counting on' technique again (from ten), rather than requiring children to rely on just the digits at this stage.

Ten ones are *not* visible:



10s	1s
1	4

'The 1 means one ten, and the 4 means four ones.'

1:5

Once children are confident with the 'ten and a bit' structure of the teen numbers, explore how these numbers are named. Begin with a representation from the previous step – for example, the representation of 14 pencils. Draw attention to the fact that we articulate the ones *first* and the tens *second* – 'fourteen' – and compare this to the way other two-digit numbers are named – for example, 'fifty-four'. Look at the names written down, clearly showing what each part of the name means: the 'four' relates to four ones and the 'teen' relates to one ten. Look back at the digits and introduce the idea that if we followed the naming rules for other two-digit numbers we would call fourteen 'one ten four'.

Examine the names of all the teen numbers, exploring how these do or don't relate to the structure of the number:

- 'Fourteen', 'sixteen', 'seventeen', 'eighteen' and 'nineteen': these numbers have the ones written as the children will recognise, along with 'teen'.
- 'Thirteen' and 'fifteen': the three and five can be seen, but aren't spelt in the normal way.
- 'Eleven' and 'twelve': these don't have 'teen' in the name, so particular attention should be paid to these.

Name	Digits	What it means
eleven	11	one ten one
twelve	12	one ten two
thirteen	13	one ten three
fourteen	14	one ten four
fifteen	15	one ten five
sixteen	16	one ten six
seventeen	17	one ten seven
eighteen	18	one ten eight
nineteen	19	one ten nine

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1:6

Use 'dual counting':

- *'Eleven, twelve, thirteen…'*
- *'One ten one, one ten two, one ten three…'*

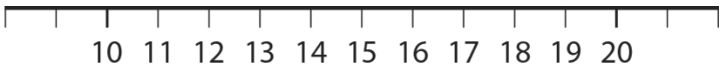
Alternatively:

- *'Eleven, twelve, thirteen…'*
- *'Onety-one, onety-two, onety-three…'* (this language may flow more easily, and links closely with *'Twenty-one, twenty-two, twenty-three…'*).

Counting in two ways will further reinforce the 'ten and a bit' structure.

Practise counting between ten and twenty in this way, but also between, for example, 0 and 40, so that children see the similarities between 'one ten one' (or 'onety one') and 'twenty-one' and 'thirty-one', etc. Tap the numbers on a number line or hundred square, whilst counting, to embed the link between the name and the digits. Similarly, tap the numbers on the Gattegno chart while counting, to further reinforce the 'ten and a bit' structure.

Number line:



Hundred square:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1:7

When counting in ones, periodically stop and ask children to identify one more and one less. Provide missing number problems for children to practise identifying missing numbers in the counting sequence.

One more and one less:

one less one more

11 ← 12 → 13

- *'Twelve is one more than eleven. Eleven is one less than twelve.'*
- *'Twelve is one less than thirteen. Thirteen is one more than twelve.'*

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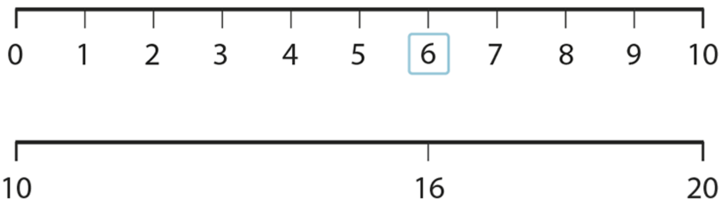
<table>
<tr>
<td></td>
<td></td>
<td>Sequences with one missing number:
<table><tr><td>11</td><td></td><td>13</td></tr></table><table><tr><td>20</td><td></td><td>18</td></tr></table>
'Twelve is between eleven and thirteen.'
'Nineteen is between eighteen and twenty.'</td>
</tr>
<tr>
<td>1:8</td>
<td>Now work on estimation of the position of teen numbers, relative to 10 and 20, on a blank number line. Initially scaffold the problems, by making a link between the position of, for example, 6 on a 0–10 number line, and 16 on a 10–20 number line – align one number line above the other as shown.

Progress to cases without the scaffolding described above. At first, children may well try to 'count up' from ten by making imaginary 'marks' on the blank number line. Teach them to move away from this additive way of thinking to a proportional way of thinking, where they are making a judgement about the position of 16 relative to both 10 and 20, rather than just to 10. Teaching them to first mark the midpoint will support them in this (though this alone is not sufficient), as will teacher modelling of the thinking, for example: 'Eighteen is a lot closer to twenty than it is to ten, so I think it sits about here… Maybe that's a bit too close…I think it is about here.'

Provide variation in the problems presented including:
asking children to place a given number onto the number line
asking children to estimate what number is being pointed to on the number line.</td>
<td>Scaffolded estimation – placing a number on the number line:


Unscaffolded estimation – estimating the value of a number on the number line:
'Fill in the missing number.'

</td>
</tr>
</table>

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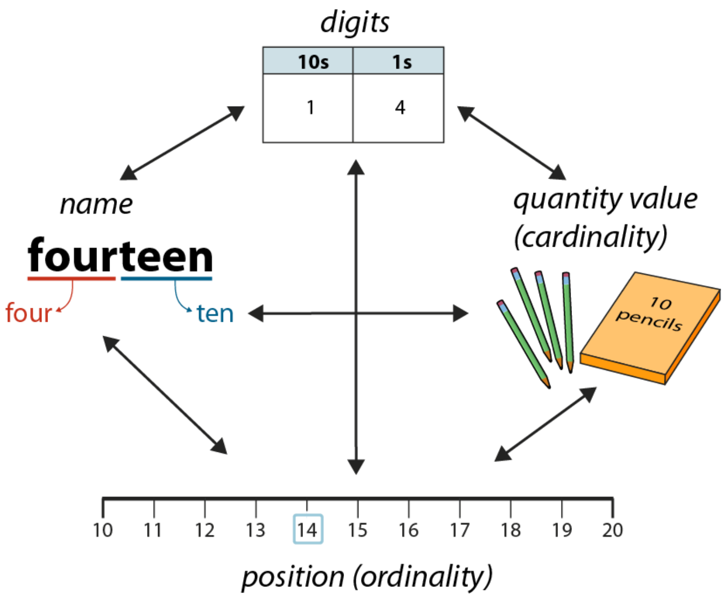
1:9	By this point children should be confident with the cardinality (quantity value), ordinality (position in the number system) and various representations (digits, number names) of the teen numbers. They should be able to link the different representations and confidently move between any pair of them.	

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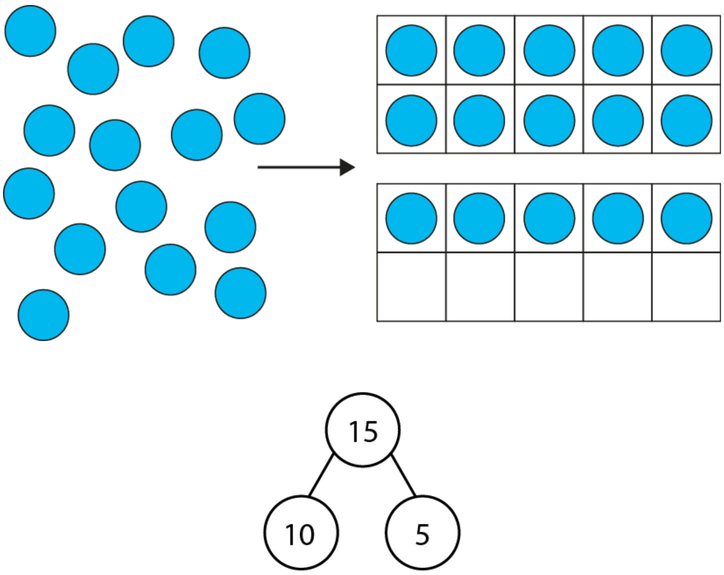
Teaching point 2:

The numbers 11–19 can be formed by combining a ten and ones, and can be partitioned into a ten and ones.

Steps in learning

	Guidance	**Representations**
2:1	Build on children's knowledge of the composition of teen numbers to support calculation. Remind children that these are the 'ten and a bit' numbers and demonstrate how we can work out how many objects there are in a group, without counting, by placing them onto two tens frames (filling one of the tens frames completely before moving onto the next). Record the partitioning into tens and ones on a part–part–whole diagram (cherry or bar model), and then record the corresponding equation. Provide children with a collection of between 11 and 19 objects and two tens frames, for practice. Using structured representations where the ten can be taken apart (for example, counters and tens frames, or straws and bands), rather than Dienes or place-value counters, helps children see clearly that the one ten is equivalent to ten ones.	 $15 = 10 + 5$
2:2	Now systematically show the structure of 11–20, using both structured representations (for example, counters and tens frames) and symbolic notation. In the representation shown here, the equations have been recorded in the form *11 = 10 + 1* to provide variation: it is important to show equations structured in a variety of ways, and recording as *11 = 10 + 1* rather than (or alongside) *10 + 1 = 11* will support this.	

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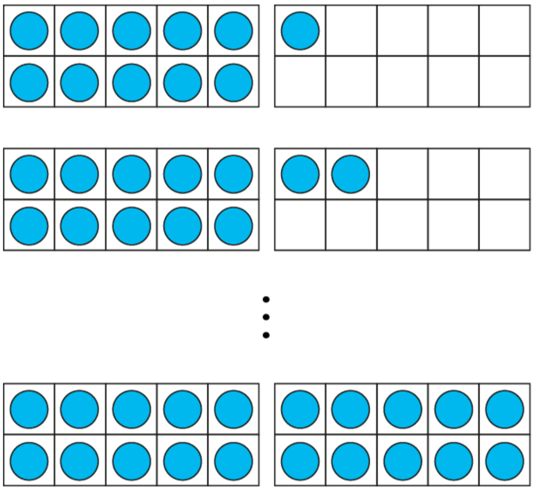
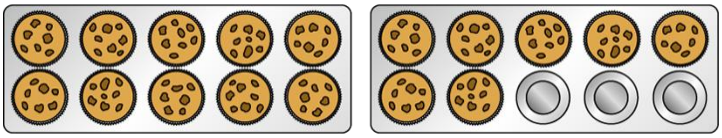
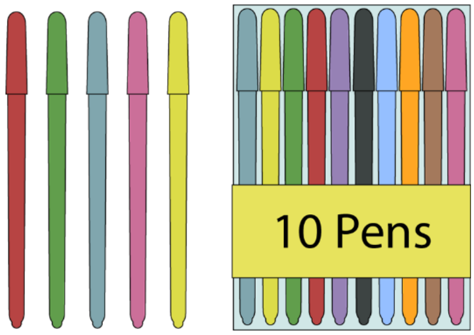
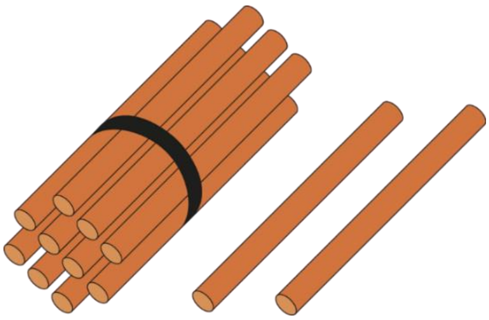
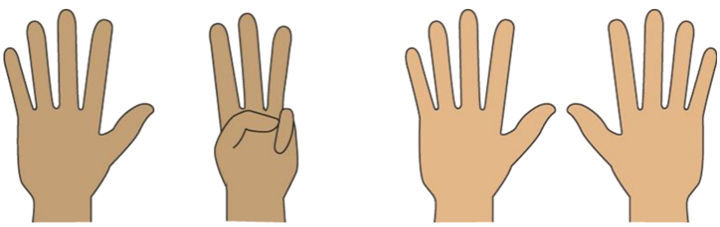
	Give the children practice recording the composition of teen numbers as equations like this, working towards fluency. By the end of this step, ensure that children are able to look at the tens frame representation of a given teen number and, without counting any individual counters: • write an equation to represent it • say the total number. Use the stem sentence: *'___ **is equal to ten plus** _____.'*	 11 = 10 + 1 12 = 10 + 2 ⋮ 20 = 10 + 10
2:3	Vary the 'ten and a bit' images used until children can quickly and confidently identify the quantity shown in a representation. As well as varying the objects being shown and their orientation, vary the arrangement so as not to always show the 'ten' on the left and 'a bit' on the right. When you present children with an image, check that that they can: • say the number of items shown • match the image to the number written as digits (for example, a number card) • write the number as digits themselves; if children write, for example, '60' instead '16', draw their attention back to the composition of the numbers and meaning of the digits (step *1:4*) • describe confidently, in full sentences, how they know the number, for example, *'I know there are twelve because twelve is equal to ten plus two.'*	Variation in pictorial representations:    

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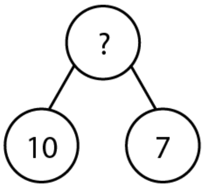
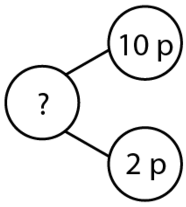
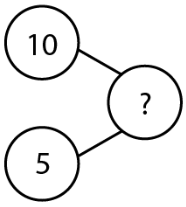
2:4	Progress to presenting addition problems in the context of 'ten and a bit': • using the generalised part–part–whole representation (cherry or bar model); initially, introduce this alongside the pictorial representations • completing 'missing sum' equations, paying attention to variation in construction of the equations • solving real-world 'ten and a bit' problems, with the support of part–part–whole diagrams (include both aggregation and augmentation contexts – see segments *1.5 Additive structures: introduction to aggregation and partitioning* and *1.6 Additive structures: introduction to augmentation and reduction*).	Part–part–whole diagram:  Missing sum equations: 10 + 4 = □ 6 + 10 = □ 10 + 7 = □ 1 + 10 = □ □ = 3 + 10 □ = 10 + 2 3 + 10 + 2 = □ □ = 1 + 1 + 1 + 1 + 10 Real-world problem – aggregation: *'Jaz has a two-pence coin and a ten-pence coin. How much does she have altogether?'*  Real-world problem – augmentation: *'Sam has ten marbles in his collection. Then his friend gives him five more. How many marbles does Sam have now?'* 

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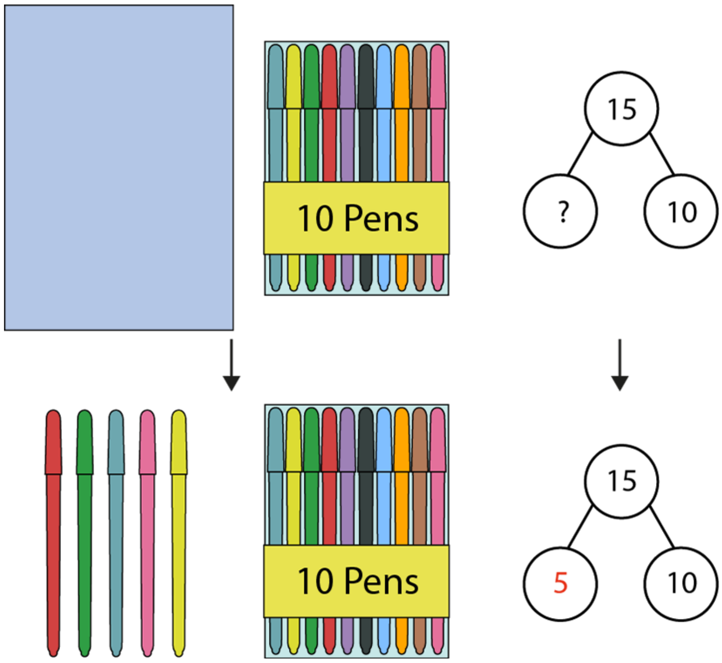
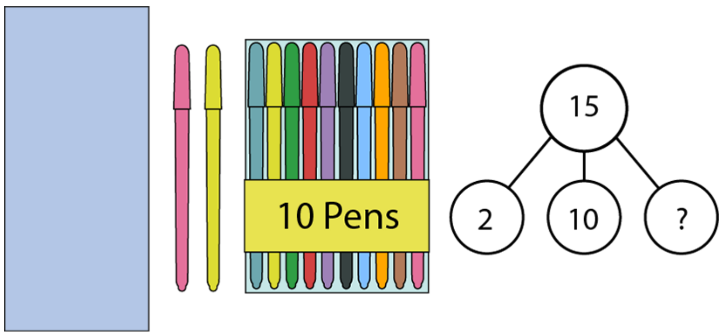
2:5	Once children have mastered addition in the 'ten and a bit' context, explore using missing part problems as a step towards subtraction. Include both: • problems with the ones part missing • problems with the tens part missing. Provide variation in terms of how these problems are presented, including real-world contexts, part–part–whole diagrams with a missing part', and 'missing addend' equations.	Real-world problem: *'Sara has fifteen pens. How many are hidden?'*  Missing addend equations: $\square + 10 = 15$ $19 = 10 + \square$ $\square + 3 = 13$ $11 = 1 + \square$
2:6	To provide additional challenge use dòng nǎo jīn questions that involve partitioning the tens part or the ones part in different ways (problems with three addends). Keep the focus on combining a ten with ones to make a teen number, making sure that the partitioning chosen always reinforces this strategy: $11 = 5 + 3 + \square$ does *not* support the 'ten and a bit' focus $11 = 5 + 1 + \square$ supports the 'ten and a bit' focus, because the missing part can be	Dòng nǎo jīn – real world problem: *'Sara has fifteen pens. How many are hidden?'* 

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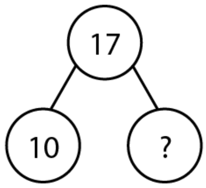
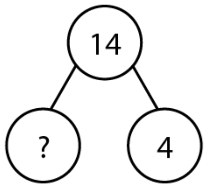
	found by thinking of 11 as 10 (i.e. *5 + 5*) and 1.	Dòng nǎo jīn – missing addend equations: $10 + 3 + 4 = \square + 10$ $19 = \square + 10 + 6$ $11 = 5 + 1 + \square$ $4 + \square + 7 = 17$
2:7	Once children have mastered the missing addend problems, introduce subtraction problems. Provide problems in the form of: • part–part–whole diagrams with a missing part • missing difference equations • real-world contexts, supported by part–part–whole diagrams (include both partitioning and reduction contexts – see segments *1.5 Additive structures: introduction to aggregation and partitioning* and *1.6 Additive structures: introduction to augmentation and reduction*).	Real-world problem – reduction structure: *'Jack has seventeen pounds. He spends ten pounds. How much is left?'*  Real-world problem – partitioning structure: *'There are fourteen children in the classroom. Four are painting. How many are not painting?'*  Missing difference equations: $14 - 4 = \square$ $\square = 16 - 6$ $19 - 10 = \square$ $\square = 12 - 10$
2:8	To provide additional challenge, use dòng nǎo jīn problems with two subtrahends, using a similar approach to that used in step *2:6*. Again, choose appropriate problems and teach children to solve them by attending to the 'ten and a bit' structure; for example, teach the children to solve *17 – 5 – 5* by seeing *17 – 10*, not by thinking *17 – 5 = 12* then *12 – 5 = 7*. Using the part–part–	Dòng nǎo jīn with two subtrahends – missing difference: *'Sally has red, blue and yellow marbles. There are seventeen marbles in total. Five marbles are blue and five marbles are red. How many marbles are yellow?'* $17 - 5 - 5 = \square$

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	whole model to mirror the structure of the equation will support this. For variation, you can also provide two-subtrahend problems with a missing subtrahend.	Dòng nǎo jīn with two subtrahends – missing subtrahend: *'A bag contained eighteen marbles; Sam took out three marbles and then Harry took out some marbles. There are now ten marbles in the bag. How many did Harry take out?'* 18 – 3 – □ = 10
2:9	Draw together children's understanding of the structure of the teen numbers (as 'ten and a bit') and their relative size, by presenting children with a range of missing inequality problems.	*'Fill in the missing numbers and symbols.'* 15 ◯ 17 15 ◯ 14 15 > □ 15 < □ 12 ◯ 10 + 2 10 + 3 ◯ 3 + 10 10 + 1 ◯ 15 2 + 10 ◯ 4 + 10

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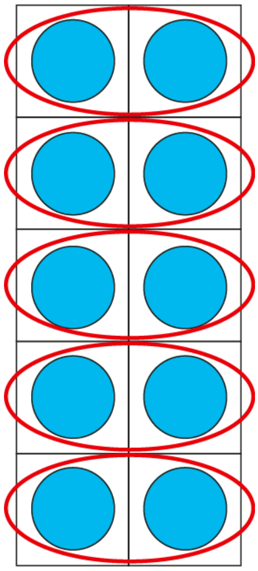
Teaching point 3:

A number is even if the ones digit is even; it *can* be made from groups of two. A number is odd if the ones digit is odd; it *can't* be made from groups of two.

Steps in learning

	Guidance	**Representations**
3:1	Children will already have learnt to identify odd and even numbers up to ten (see segment *1.4 Composition of numbers: 6–10*, *Teaching point 3*), and we now look at the numbers 11–19. Use tens frames and counters (twos-wise), or base-ten number boards, to remind children that ten is a multiple of two, and therefore an even number. Demonstrate that the ones digit alone will indicate whether a number is odd or even. Use the following stem sentences / generalised statements: • ***'We know the number ___ is odd/even because the ones digit is odd/even.'*** • ***'A number is odd if the ones digit is odd. It can't be made from groups of two.'*** • ***'A number is even if the ones digit is even. It can be made from groups of two.'***	Ten:  *'Ten is even because it can be made from groups of two.'*

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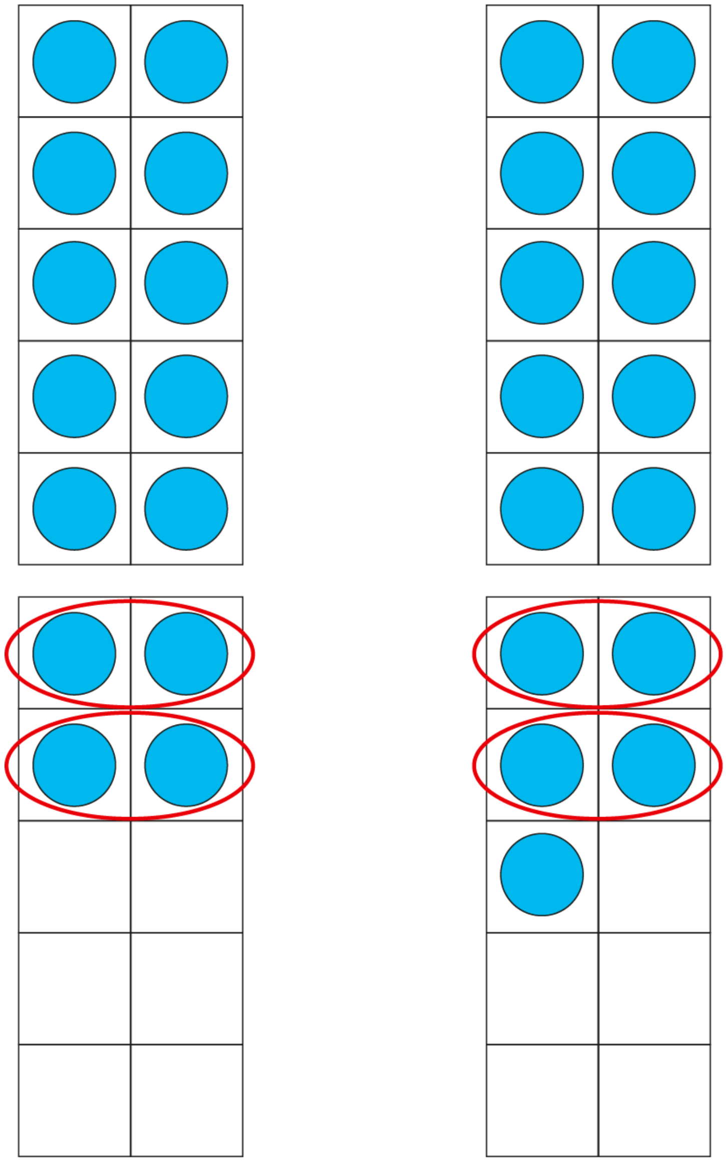
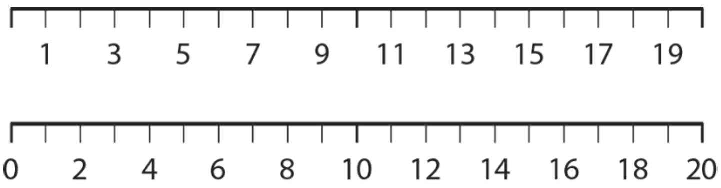
		Teen numbers:  14 even 15 odd • *'We know the number fourteen is even because the ones digit is even.* • *We know the number fifteen is odd because the ones digit is odd.'*
3:2	Use odd and even number lines to further emphasise the significance of the ones digit when identifying whether a number is odd or even: both odd and even numbers can have a one in the tens place.	Odd and even number lines: 

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Present children with the following tasks for practice:

- Sort numbers into odds and evens, repeating the stem sentence, ***'We know the number ___ is odd/even because the ones digit is odd/even.'***
- Complete odd/even missing number sequences.

Sorting activity:

'Put each number into the correct box according to whether it is odd or even.'

Odd	Even

5 6 10 9 1

15 16 19 11

Odd/even missing number sequences:

12		16

19		15

	11			17

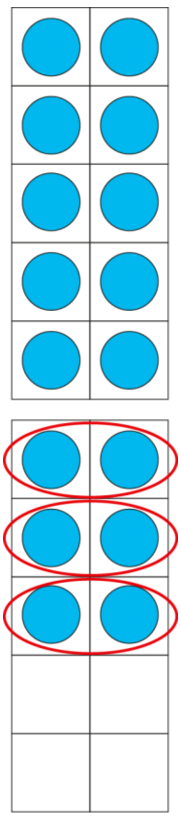
3:3

Use true or false questions to assess children's understanding, for example:

- *'Sixteen is an odd number.'* ✗

  Children may think this number is odd because one of the digits is one.
- *'Any number that ends in a zero is an even number.'* ✓
- *'Five is an odd number so fifteen is an odd number.'* ✓
- *'To tell whether a number is odd or even, I only need to look at the ones digit.'* ✓

True/false pictorial question:



This is an odd number. ✗

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- *'Nineteen can be made out of groups of two.'* ✗
- *'A number than can be made out of groups of two is an even number.'* ✓

To promote and assess depth of understanding, use a dòng nǎo jīn question: present a statement that is partly true, but is false overall, for example, *'Thirteen is an odd number because one is an odd number.'*

This example provides additional challenge because, while the first part of the statement is true (13 *is* an odd number), the explanation is incorrect.

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Now show how the generalisation that a number is odd/even because the ones digit is odd/even extends to other two-digit numbers. (The numbers 20–99 were introduced in segment *1.9 Composition of numbers: 20–100*).

The teaching of odds and evens for the numbers 20–100 should be left until now so that it follows the previous steps on teen numbers. This is so that children can focus on *one* ten being an even number, before they extend to more than one ten (i.e. tens digits greater than one). Remind children of the demonstration that ten is an even number (step *3:1*), then demonstrate that two tens is still even, as is three tens, four tens etc.; the number of tens is irrelevant and the ones digit alone will always indicate whether a number is odd or even. Expose this using a pattern such as that shown opposite.

Continue to use the stem sentence: ***'We know the number ___ is odd/even because the ones digit is odd/even.'***

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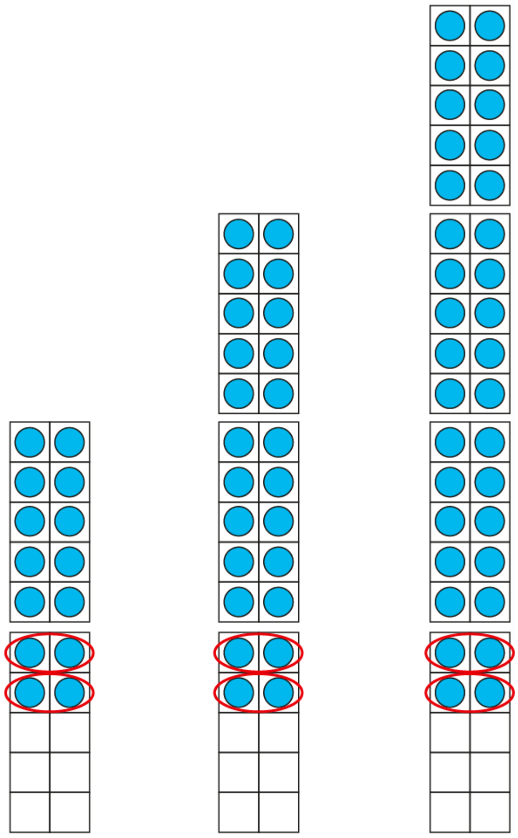

14 even | 24 even | 34 even

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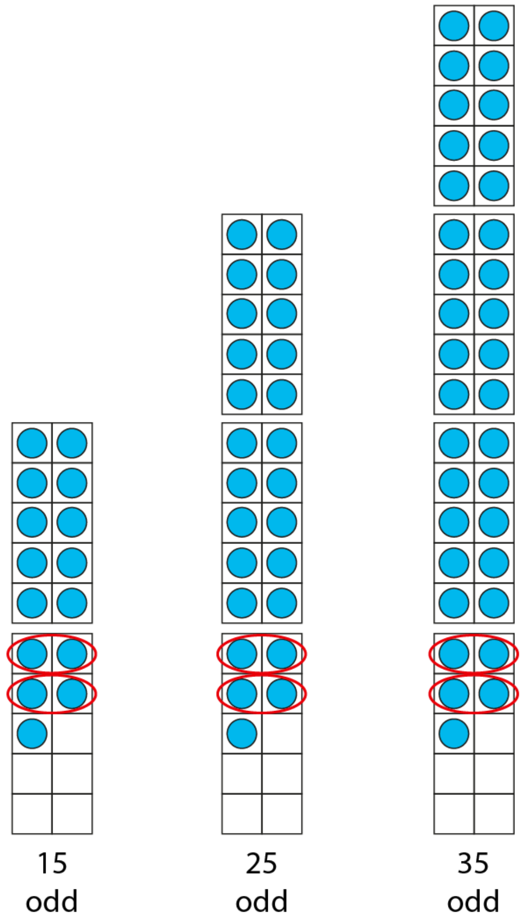
<td>
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<td>3:5</td>
<td>In segment 1.4 Composition of numbers: 6–10, children practised odds/evens counting within ten. Now extend to numbers up to 100. Initially this can be supported by pointing to numbers on the Gattegno chart as you count, in order to:

reinforce the pattern of the numbers
provide children with a prompt as to what the next number is.

Provide frequent practice until children can count in odds and evens without the scaffold of the Gattegno chart.

As in segment 1.4, present children with the following tasks for practice:

Sort numbers into odds and evens, repeating the stem sentence: 'We know the number ___ is odd/even because the ones digit is odd/even.'

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	• Complete odd/even missing number sequences. Now that you have extended odds/evens to the numbers 20–100, return to the main focus of this segment: the numbers 11–19.	

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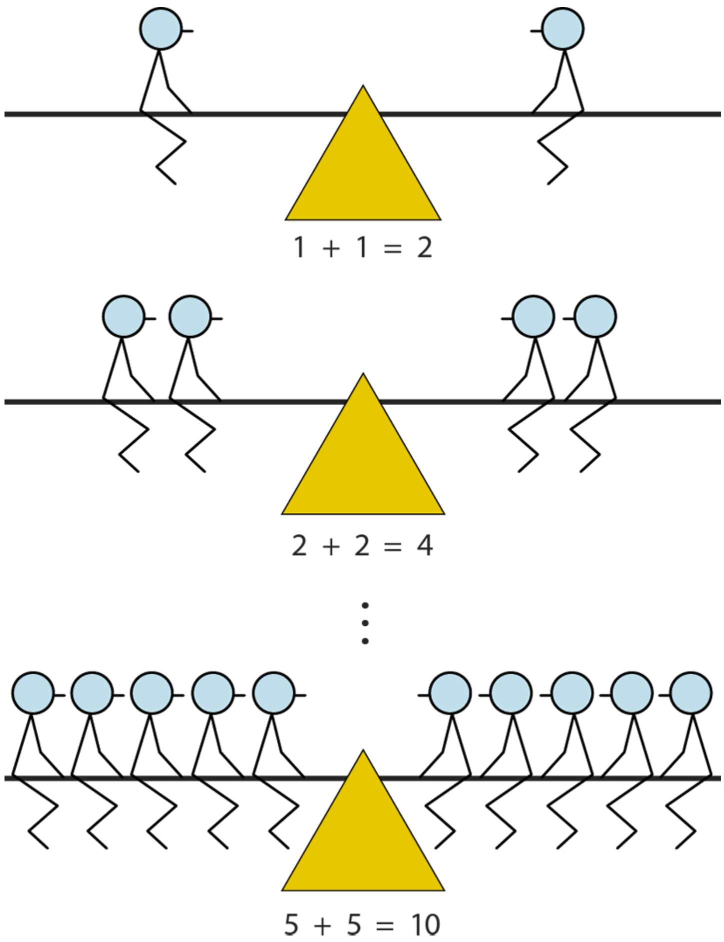
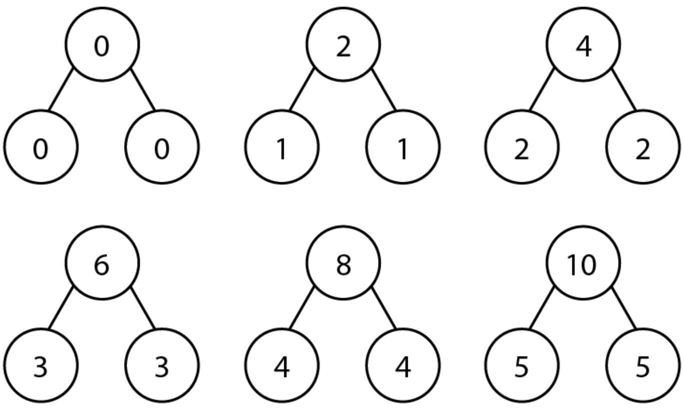
Teaching point 4:

Doubling the numbers 6–9 (inclusive) gives an even teen number; halving an even teen number gives a number from six to nine (inclusive).

Steps in learning

	Guidance	Representations
4:1	In segment *1.7 Addition and subtraction: strategies within 10*, children learnt that when both addends are the same, we can say we are doubling; for example, three plus three is double three. They also learnt that doubling a whole number always gives an even number. We now extend this to doubling six, seven, eight and nine. Start by reviewing the learning on doubles from segment *1.7*, using the seesaw images and part–part–whole diagrams. Remind children: • *'When both addends are the same, we are doubling.'* • *'If we have three plus three, we can say we are doubling three.'* • *'Doubling a whole number always gives an even number.'* • *'Halving is the inverse of doubling.'* Encourage children to use both addition/subtraction language and doubling/halving language to describe the calculations, for example: • *'Three plus three is equal to six; double three is six.'* • *'Six minus three is equal to three; half of six is equal to three.'*	

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4:2

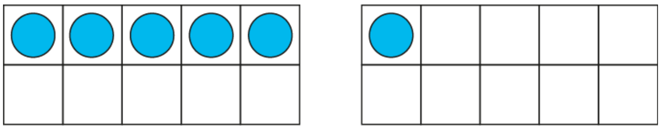
Now present a double-six context, such as: *'Yassin has a magic doubling-bag. He put six beans into his bag. When he emptied the bag, the number of beans had doubled. How many beans does he have now?'*

Represent the initial six on two tens frames, emphasising the five-and-a-bit structure (children should be familiar with describing the numbers six to nine as 'five and a bit' from segment *1.4 Composition of numbers: 6–10*).

Then ask the children to represent the doubling of the beans on the tens frames. Children can place the additional counters on the tens frames in any way they want, but spend some time looking at the arrangement opposite in particular; this makes the link between six being composed of five and one, and double six being composed of double five and double one.

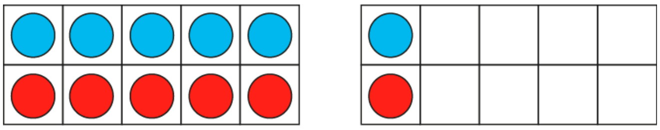
Show the equations and encourage children to describe, in full sentences, how we can double six using known facts, as shown opposite.

Record the calculation on a table showing the number of beans before (six) and after (twelve) (see step *4:3* below).



- *'Yassin has six beans at the start: five and one more.'*

6 = 5 + 1



- 'Six is five plus one more, so double-six is double-five plus double-one.'

5 + 5 = 10

1 + 1 = 2

10 + 2 = 12

- *'Yassin has twelve beans at the end.'*

4:3

Repeat as for step *4:2*, doubling seven, then eight, then nine, each time showing the starting number as 'five and a bit'. Complete the table with the results as you work through the numbers.

Look at the table and ask questions such as:

- *'Yassin put seven beans in the bag at the start; then the bag doubled the beans; how many beans are in the bag now?'*

Number of beans at the start	6	7	8	9
Number of beans at the end	12	14	16	18

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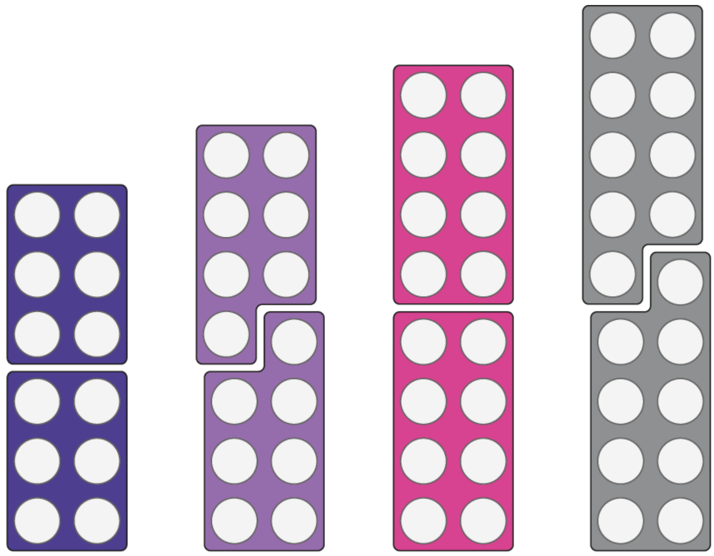
	• *'Yassin takes eighteen beans out of the bag; how many beans must he have put in the bag at the start?'* To reinforce the use of doubling/halving language, and the link between them, once children have answered correctly, reply in the following form: • *'Yes, that's right; double seven is fourteen.'* • *'Yes, that's right; half of eighteen is nine.'*	
4:4	Using the table from step *4:3*, draw children's attention to the fact that the double of each number is, of course, even, like the other doubles they already know. Because every 'one' (e.g. every one bean) is doubled, the double of the whole number must be even. Remind children of the generalised statement: ***'Doubling a whole number always gives an even number.'*** Use base-ten number boards to reinforce this relationship. The number boards can be manipulated by children without them losing the pair-wise structure of the starting numbers; however, unlike the tens frames, they are less useful for exposing the 'five and a bit' – 'double five and double the bit' relationship. Ask children to represent particular doubles using base-ten number boards, and to describe what they have shown using both addition and doubling language, for example: • *'Show me double seven.'* • *'Double seven is fourteen.'* • *'Seven plus seven is equal to fourteen.'* Again make the link with halving, asking questions such as *'Double seven is fourteen, so what is half of fourteen?'*	 6 + 6 = 12 7 + 7 = 14 8 + 8 = 16 9 + 9 = 18

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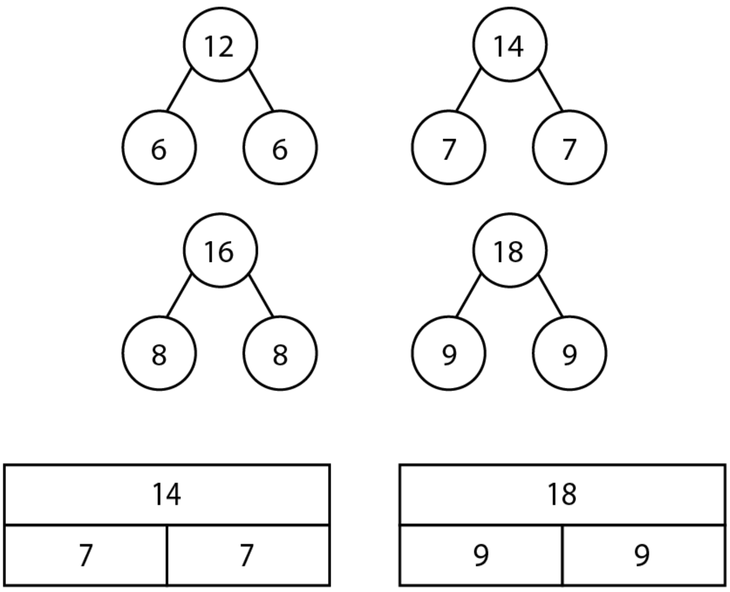
4:5	Summarise the pattern of halves and doubles, and also draw attention to the difference of two between consecutive doubles – for example double seven (14) is two more than double six (12) and vice versa. Then, provide children with missing number problems, and practice moving between doubling and halving. Note the inclusion of double five and double ten, which are known facts. This will support children in step *4:6*, where they memorise the doubles by making links between them, such as the fact that double nine (a fact they need to learn) is two less than double ten (a fact they already know).	*'Fill in the missing numbers.'* double 5 = ☐ half of 10 = ☐ double 6 = ☐ half of 12 = ☐ double 7 = ☐ half of 14 = ☐ double 8 = ☐ half of 16 = ☐ double 9 = ☐ half of 18 = ☐ double 10 = ☐ half of 20 = ☐ 5 double → ☐ / ← half ☐ double → 18 / ← half
4:6	Now work on making sure that all children have memorised these doubles facts. You can use the following strategies to help children learn them: • Link the position of 6 and 12 on the clock face: the number 6 is *half* way round, and 12 is at the top. • 7 and 14 can be linked using the word 'fortnight', which comes from the phrase 'fourteen nights'. Teach children that 'fortnight' means two weeks (this may be a new word for them) so they can equate two weeks of seven days (double seven) with a fortnight (fourteen). • Once children have learnt double six and double seven, double eight can be learnt as two more than 14 (two more than double seven). • Double nine can be learnt as two less than 20 (two less than double ten).	

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	During practice, pay particular attention to children sometimes giving odd numbers (e.g. 13) as the double of six, seven, eight or nine. Whilst children are likely to give incorrect *even* answers in the process of learning doubles (e.g. double 7 is 16), children who are giving odd-number answers have missed a very important teaching point and need to review the rule that doubles of whole numbers are always even. Build doubles facts into classroom routines and games, with opportunities for all children to regularly repeat the facts. Make sure the facts are displayed on the wall for children to refer to during the learning process (as part–part–whole representations, for example).	
4:7	Complete the teaching point by providing practice in the form of missing number problems (both addition and subtraction) and real-life contexts, such as: • *'I am making twelve cupcakes. I have put out six cupcake cases. How many more cases do I need?'* • *'A movie download costs eight pounds. I buy two of them. How much do I spend altogether?'*	*'Fill in the missing numbers.'* $7 + 7 = \square$ $\square = 8 + 8$ $12 = 6 + \square$ $18 = \square + 9$ $12 - 6 = \square$ $\square = 18 - 9$

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Teaching point 5:

Addition and subtraction facts within 10 can be applied to addition and subtraction within 20.

Steps in learning

	Guidance	Representations
5:1	see below	see below

5:1 Guidance

Introduce a context to make a link between an addition fact within ten and use of the same addition fact within twenty. For example:

- *'At first Darren has three conkers. Then he finds two more. Now he has five conkers.'*
- *'At first Piya has thirteen conkers. Then she finds two more. How many conkers does Piya have now?'*

Ask children how we can express the number of conkers that Darren has:

$3 + 2 = 5$

Then ask children how we can express the number of conkers Piya has, beginning by thinking about 13 as 'ten and a bit' ($13 = 10 + 3$):

$10 + 3 + 2$

Show that we can add the three and two first, then simply recombine that sum with the ten to make 15:

$10 + 3 + 2 = 10 + 5 = 15$

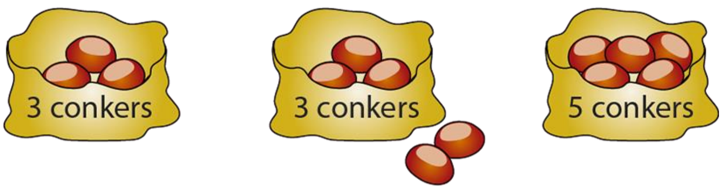
You can show this strategy using the part–part–whole cherry representation.

Now compare with $13 + 2 = 15$ as a representation of how many conkers Piya has, making the link to $10 + 3 + 2 = 15$.

5:1 Representations

'At first Darren has three conkers. Then he finds two more. Now he has five conkers.'

First Then Now

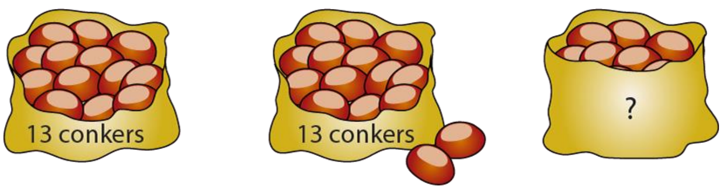


3 → +2 → 5

$3 + 2 = 5$

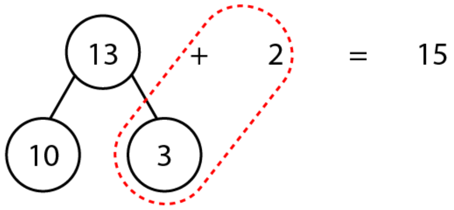
'At first Piya has thirteen conkers. Then she finds two more. How many conkers does Piya have now?'

First Then Now



13 → +2 → ?

$\underbrace{10 + 3 \quad + 2}_{+5} = ?$



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Use tens frames and counters as a generalised representation, showing a single-digit addition, then the corresponding teen addition, for example:

3 + 2 = 5 then 13 + 2 = 15
5 + 1 = 6 then 15 + 1 = 16
4 + 3 = 7 then 3 + 14 = 17

Make sure you include examples that show the link between making 10 and making 20, for example 8 + 2 = 10 and 18 + 2 = 20.

You can also use part–part–whole diagrams and number lines to represent these relationships. However, it is essential that the number line is not just used as a tool to calculate – for example, calculating 13 + 2 by jumping on two from 13; keep the focus on the connection between the single-digit calculation and the teen calculation. To reinforce these connections, ensure repeated use of sentences of the form 'Three plus two is equal to five, so thirteen plus two is equal to fifteen.'
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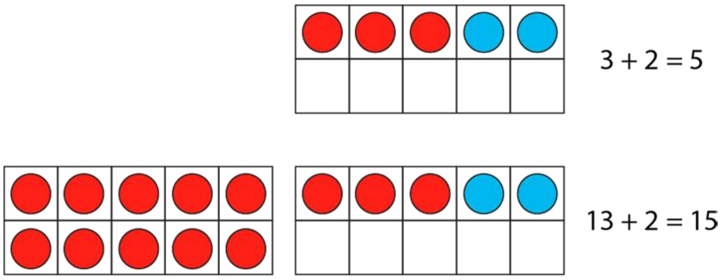
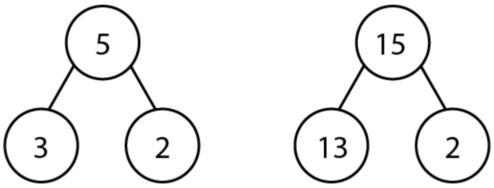
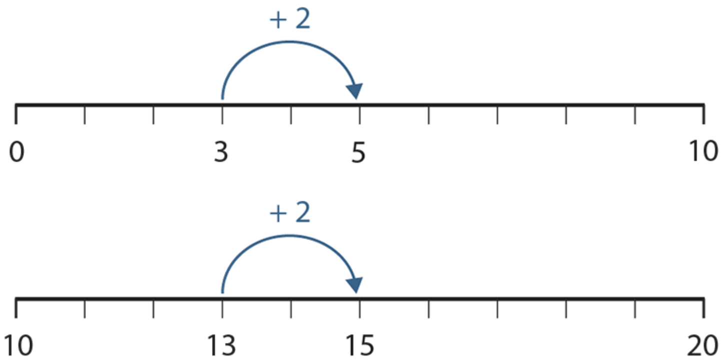
Tens frames and counters:


Part–part–whole cherry representation:


Number line:


'Three plus two is equal to five, so thirteen plus two is equal to fifteen.'
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<td>5:3</td>
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To provide extra challenge, use a dòng nǎo jīn question that presents a general statement, and asks children to state and reason whether it is always true, sometimes true or never true, for example:

'If I add two to a number ending in three, I will get a number ending in five.'
'If I add two to a number ending in nine, I will get a number ending in zero.'

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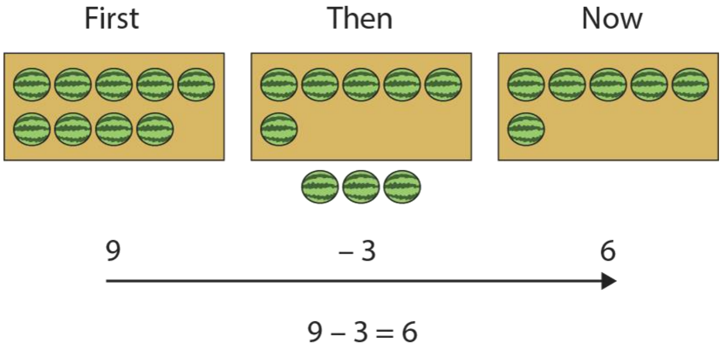
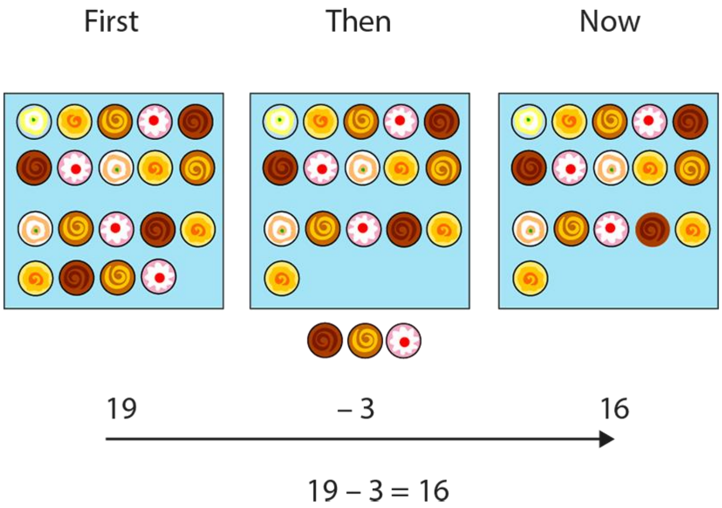
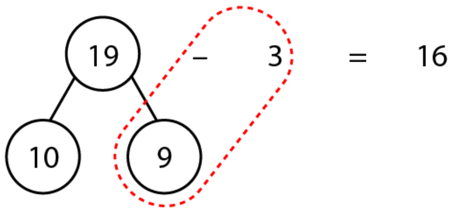
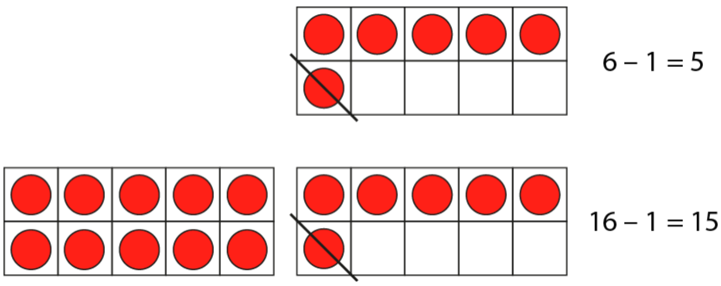
5:4	Using the same procedure as for addition facts (step *5:1*), introduce a context to make a link between subtraction within ten and subtraction of a single-digit number from a teen number. For example: • *'At first the grocer had nine watermelons. Then he sold three of them. Now he has six watermelons left.'* • *'At first the baker had nineteen cakes. Then she sold three of them. How many cakes does she have left?'* Ask children how we can express the number of watermelons that the grocer has: 9 – 3 = 6 Then ask children how we can express the number of cakes the baker has, again beginning by thinking about the teen number as 'ten and a bit': 10 + 9 – 3 = 16	*'At first the grocer had nine watermelons. Then he sold three of them. Now he has six watermelons left.'*  *'At first the baker had nineteen cakes. Then she sold three of them. How many cakes does she have left?'*  
5:5	As before, generalise using a tens frame and counters, showing several different subtraction facts within 10 alongside the related facts within 20, for example: • *9 – 1 = 8* then *19 – 1 = 18* • *9 – 2 = 7* then *19 – 2 = 17* • *9 – 3 = 6* then *19 – 3 = 16*	Tens frames and counters: 

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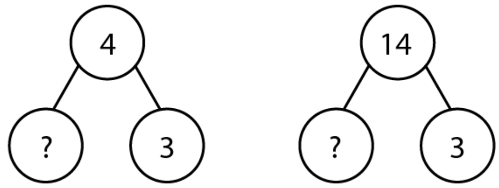
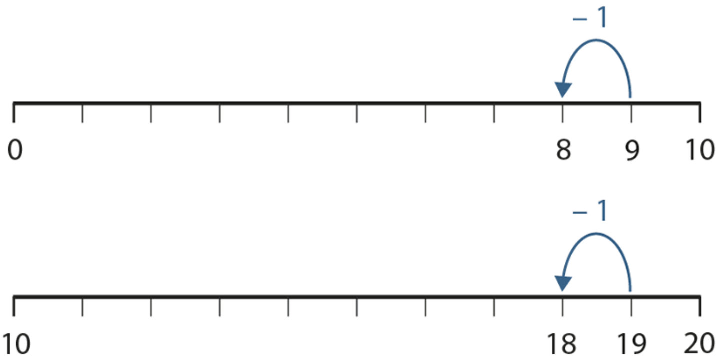
	You can also use part–part–whole models and number lines to represent these relationships. Again, the facts (e.g. *19 – 1 = 18*) are derived from what we already know (e.g. *9 – 1 = 8*). Use the number line as a tool to show the relationship between the single-digit calculation and the teen calculation, rather than as a tool for performing the calculation by 'counting back'. Continue to use sentences to describe the process, for example: *'Six minus one is equal to five, so sixteen minus one is equal to fifteen.'*	Part–part–whole cherry representation:  Number line: 
5:6	As for addition (step *5:3*), provide extra challenge by using a dòng nǎo jīn question – present a general statement, and ask children to state and reason whether it is always true, sometimes true or never true, for example: • *'If I subtract five from a number ending in seven, I will get a number ending in two.'* • *'If I subtract five from a number ending in zero, I will get a number ending in five.'* • *'If I subtract six from a number ending in zero, I will get a number ending in six.'*	
5:7	Provide children with the opportunity to apply their knowledge of both addition and subtraction relationships from the previous steps. Use missing number problems, providing variation in the position of the missing number and the location of the equals symbol (for example, *5 + 4 = 9* versus *16 = 3 + 13*), as well as problems with real-world contexts.	Missing number problems: $5 + 4 = \square$ $\quad$ $6 + 3 = \square$ $5 + 14 = \square$ $\quad$ $16 + 3 = \square$ $8 - 6 = \square$ $\quad$ $7 - 2 = \square$ $18 - 6 = \square$ $\quad$ $17 - 2 = \square$

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		5 + ☐ = 7 15 + ☐ = 17 2 = 9 – ☐ 12 = 19 – ☐

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