

#mathscpdchat 9 November 2021

Effective ways of using worked examples: what are they, and do you work with them? Hosted by <u>Catherine Edwards</u>

This is a brief summary of the discussion – to see all the tweets, follow the hashtag **#mathscpdchat** in Twitter



The links shared during this discussion were:

<u>Math Mistakes</u> which is a website by <u>Michael Pershan</u> who is a math(s) teacher in New York. The purpose of the site is to collect, organise and make sense of mistakes that students make while doing mathematics. Michael also hopes to use the mistakes to support the devising of worked examples that students can learn from. It was shared by <u>Catherine Edwards</u>



<u>GLT Book Club: Teaching Math with Examples</u> which is a Greenshaw Learning Trust Book Club video (June 2021) in which <u>Michael Pershan</u> talks to maths teachers about his work and his book <u>Teaching Math with Examples</u>. It was shared by <u>Catherine Edwards</u>

<u>Michael Pershan on Worked Examples</u> which is an ERRR podast interview with <u>Michael Pershan</u> in which he talks about principles that underlie learning from 'examples, self-explanation and feedback'. It was shared by <u>Catherine Edwards</u>

<u>Mr Barton Maths Podcast - Michael Pershan:Teaching with worked examples- part 2</u> which is a podcast interview where <u>Craig Barton</u> talks to <u>Michael Pershan</u> about his ideas and teaching approaches that are related to mistakes students make, and the use of worked examples in the teaching and learning of mathematics. It was shared by <u>Catherine Edwards</u>

<u>Algebra by Example</u> which are 'Example-based Problem Sets for Algebra' that are intended to give students 'practice solving problems and practice modelling, analysing, critiquing and articulating mathematical arguments'. It was shared by <u>Catherine Edwards</u>

<u>BerwickMaths</u> which is a BerwickMaths ('The Centre of Worked Examples') in which the author aims to clarify the meaning of, and help maths teachers 'deliver', 'instruction that involves prompt shifts progressively from highly-prompted examples to unprompted examples'. It was shared by <u>Catherine Edwards</u>

<u>Teaching Maths with Worked Examples</u> which is an online page created by <u>Karen Hancock</u>. It contains some of her thoughts about, and experiences of, working in mathematics with worked examples, together with some of the worked examples that she has herself designed/devised. It was shared by <u>Catherine Edwards</u>

<u>Tom Manners Online CPD</u> which is are online video sessions for maths teachers 'on different areas of representation, the importance of language that we use', and some about 'the NCETM's approach to Teaching for Mastery'. It was shared by <u>Tom Manners</u>

During this busy discussion many more tweets and 'conversations' were in response to the host's first question, rather than to any of her other three questions. The screenshots below show most of those replies and conversations, which focussed on various issues, such as: distinguishing between 'static' and 'live' ways of using worked examples, endeavouring to establish a common and correct use of mathematical language by all teachers and students in a school, whether learning from worked examples is enhanced when students use individual



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whiteboards, and the physical or IT materials that help some teachers use worked examples as effectively as possible. **Click on any of these screenshots-of-a-tweet to go to that actual tweet on Twitter.**

The following conversations were generated by this tweet from Catherine Edwards:



Catherine Edwards @Edwards08C · 15h Question 1

What is a worked example?

And perhaps more importantly what is NOT a worked example?

#mathsCPDchat

and included these 'first responses' from <u>Dattamatics</u>, <u>Catherine Edwards</u>, <u>Dani Quinn</u>, <u>Educator</u> <u>Supe</u>, and <u>Mary Pardoe</u>:

> Dattamatics @Dattamatics · 14h Replying to @Edwards08C

Similarly, I'm not sure many pupils know what a "model solution" is. I've also gone back and forth in my mind over just how helpful they really are for pupils. I think some just want to know how to solve the exact problem, and then when it changes, they can't adapt the solution.



Catherine Edwards @Edwards08C · 14h

I think in and of themselves model answers aren't that helpful. Students need to be taught how to read them so they can narrate the process and then apply to a different qu.

#mathscpdchat



Dani Quinn @danicquinn · 16h

Replying to @Edwards08C

I think of it as being the worked solution to a closed question with clear working where each line speaks for itself (although it could be a proof or answer to an always/sometimes/never). That wouldn't make it a good worked example per se, but I think they are typical features?



Educator Supe @ShakinthatChalk · 15h Replying to @Edwards08C

One that provokes further thought. Not always to clearly explain 🧐



Catherine Edwards @Edwards08C · 15h what sort of thought are you trying to promote?



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Educator Supe @ShakinthatChalk · 15h

Depends on the task at hand. Maybe boundary conditions, maybe as simple as where is the error here. The error in thinking (and one that gets levelled at text books) is to view the worked example(s) as a complete programme of understanding.



Catherine Edwards @Edwards08C · 15h as always context is king! what do you use to supplement worked examples? #mathsCPDchat



Mary Pardoe @PardoeMary · 15h Replying to @Edwards08C

A worked example might consist of a sequence of visual of prompts? Such as (in this sequence each one of the six single diagrams might be displayed separately) ...









... then these (left to right) ...



... then these (left to right) ...





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Catherine Edwards @Edwards08C · 15h looking at @danicquinn point- is that a worked example or is it a prompt which

leads to a worked example?

#mathsCPDchat



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Mary Pardoe @PardoeMary · 15h ··· Could be a worked example ... the diagrams are pre-drawn ... see my later tweet

#mathsCPDchat



Mary Pardoe @PardoeMary · 16h

Replying to @Edwards08C

By prompting/generating thinking as you go through it? e.g. ... show these one at a time ... each time a diagram is shown ask what knowledge they have that they can use to 'amplify' the diagram ... #mathsCPDchat

this conversation involving Sharon Malley, Catherine Edwards and Richard Dare:



Sharon Malley @mathsmumof2 · 15h Replying to @Edwards08C

It shows each stage of a mathematical calculation to produce an answer to a problem. I'm interested in the difference between live worked examples and static fully worked examples #mathscpdchat



Catherine Edwards @Edwards08C · 15h me too- I think there is a place for both, but which one when? #mathsCPDchat



Sharon Malley @mathsmumof2 · 15h Live first, then static to discuss and examine maybe 🤔 #mathsCPDchat



Catherine Edwards @Edwards08C · 16h Does it change for different topics/groups?

I like static then live for angle chasing

#mathsCPDchat



Richard Dare @dare_richard · 15h

I like @mrbartonmaths 'silent teacher' to reveal the example step-by-step and defer narrating it and discussing it until a little later.

#mathscpdchat



Sharon Malley @mathsmumof2 · 15h

It is such an effective strategy to ensure student focus is on the correct thing and of course it is a live worked example which you then as a static example you then narrate and explain #mathsCPDchat



these from Dani Quinn, Catherine Edwards, Richard Dare, Guy Carpenter and Peter Williams:



Dani Quinn @danicquinn · 16h Replying to @Edwards08C

I think of it as being the worked solution to a closed question with clear working where each line speaks for itself (although it could be a proof or answer to an always/sometimes/never). That wouldn't make it a good worked example per se, but I think they are typical features?



Catherine Edwards @Edwards08C · 16h

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I'd agree, I think the line by line is important. although is there an argument for missing/fading some of the lines to prompt thought?

#mathscpdchat



Richard Dare @dare_richard · 16h

Yes, that's interesting. Backfading blurs the line between a worked example and another pedagogic 'device' like scaffolding.



Dani Quinn @danicquinn · Nov 9

I agree. I think of a 'good' worked example as being able to speak for itself (i.e. that a pupil can read it in the way that we can read a crisp paragraph of explanation in another discipline). For me, fading is about building independence and muscle memory and is a teaching tool



Mr Carpenter MCCT @GuyCarpenter567 · Nov 9

An example should also exemplify the thought process of approaching a task. Learners cannot start to think about content and context without seeing the modelling of the metacognitive process that is crucial when tackling part of a problem/task



Catherine Edwards @Edwards08C · 15h HOw do we exemplify thought process?

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Live or static?





Peter Williams 🚗 @MathsImpact · 15h I find that live works much better.

I'm also a big fan of the "my turn" "your turn" approach.

I want students to be fully focused on what I'm doing when I model, not on recording it in their books.

Pens down, watching and listening.

#mathscpdchat



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Catherine Edwards @Edwards08C · 15h oh absolutely, it's really upset 9 set 1 this year, that they can't just copy down a pretty example and I expect them to listen to what I'm saying!

#mathsCPDchat

this long conversation started from one of <u>Guy Carpenter</u>'s comments, which tweet had also been part of a conversation shown above, and consisted of tweets from <u>Dani Quinn</u>, <u>Sharon</u> <u>Malley</u>, <u>Catherine Edwards</u>, <u>Richard Dare</u> and <u>Tom Manners</u>:



Mr Carpenter MCCT @GuyCarpenter567 · Nov 9

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An example should also exemplify the thought process of approaching a task. Learners cannot start to think about content and context without seeing the modelling of the metacognitive process that is crucial when tackling part of a problem/task



Dani Quinn @danicquinn · 16h

Replying to @GuyCarpenter567 @dare_richard and @Edwards08C

For sure. I love live modelling, whether silent or narrated (time and place for each). I feel sad when the whole working out is revealed with a click on PowerPoint. It's not what the experience of doing maths is like!



Mr Carpenter MCCT @GuyCarpenter567 · Nov 9 Replying to @danicquinn @dare_richard and @Edwards08C Agreed. Non engaging and boring.



Mr Carpenter MCCT @GuyCarpenter567 · Nov 9

I think it was @danicquinn that did a session on language and modelling responses from being specific to general. A real game changer for modelling the path to expertise and transferring key core knowledge and skills to similar problems.



Sharon Malley @mathsmumof2 · Nov 9

For my @leicesterscitt trainees I show 2 videos I recorded of myself where I show the exact same working but with different language in my explanation #mathsCPDchat



1	Dani Quinn @danicquinn · Nov 9	
Ì	How does the language differ? This is so interesting. What did they notice?	
-	Sharon Malley @mathsmumof2 · Nov 9	•••
	I solved an equation with an unknown on both sides, one with correct modelling using language like 'unknowns' 'inverse operation' 'maintain equality' etc the other where I 'change the side, change the sign' numbers 'disappear' etc #mathsCPDchat	g
-	Sharon Malley @mathsmumof2 · Nov 9	
	The correct language one is much longer, the 'incorrect' one is shorter so appe more efficient but it is much more difficult to understand why each stage is happening if you don't already know what to do #mathsCPDchat	ars
	Catherine Edwards @Edwards08C · Nov 9 "talk like a mathematician"	
	I've set myself a mini goal to be really strict about the correct use of language the year	nis
	#mathsCPDchat	
	Sharon Malley @mathsmumof2 · Nov 9	•••
	It pays massive dividends if you can do it consistently across all staff. I asked a student if she understood what I meant by 'maintain equality' in the context of solving equations she replied with 'yes like we do with fractions and multiplying 1' #mathsCPDchat	y9 (by
	Catherine Edwards @Edwards08C · Nov 9	
>	do you discuss and choose vocabulary in faculty time?	
	how do non specialists feel about using the language?	
	curious as we ahve started talking about some similar ideas	
	#mathsCPDchat	
1	Sharon Malley @mathsmumof2 · Nov 9	•••
	We plan examples centrally and in faculty time discuss how to model with a massive emphasis on correct language and definitions of terms. This reflects the whole school culture on vocabulary though. A small maths team so no non-specialists as yet. #mathsCPDchat	e
	Sharon Malley @mathsmumof2 · Nov 9	

We look at different examples depending on our current need but actually the language of it is defined at the start is then used across all topics e.g. commutativity or maintain equality it is the language of maths not a topic to topic vocabulary. #mathscpdchat



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Richard Dare @dare_richard · Nov 9

I like this idea of finding the deeper structures across Maths and highlighting the connections by consistency of language!

@mathsmumof2 I bet you could write a mean blog on this!

#mathscpdchat



Sharon Malley @mathsmumof2 · Nov 9

Not sure about a blog 😂 but I do have a 'Principles of Maths' document that I wrote for our faculty just 2 pages on what underlying structures we always go back to when we are teaching any topic #mathsCPDchat



Mr Carpenter MCCT @GuyCarpenter567 · 19h Replying to @Edwards08C @mathsmumof2 and 3 others @Mannermatics has a really nice vlog about this on his website



Tom Manners @Mannermatics · 19h Indeed I do :) tommanners.co.uk/cpd-videos/

First one down... although if you read in the notes, there is a link to the one I presented at #mathsconf23 which seemed to go down well!



Catherine Edwards @Edwards08C · 19h It was a very enjoyable presentation...may have inspired me!

#mathsCPDchat

this brief conversation 'branched off' from one of <u>Richard Dare</u>'s comments in a conversation shown above, and was with <u>Catherine Edwards</u>:



Richard Dare @dare_richard · 16h ··· Yes, that's interesting. Backfading blurs the line between a worked example and another pedagogic 'device' like scaffolding.



Catherine Edwards @Edwards08C · 15h

so is a worked example direct instruction and variations like backfading and mistake analysis something else?





Richard Dare @dare_richard · 15h

Oh, mistake analysis! Great stuff, but - yes - I'd say another strategy. One I'd use after using worked examples.

Whether worked examples are necessarily direct-instruction, I think is moot. @mrbartonmaths's use is d.i., whilst I'm not sure @mpershan's model is

#mathscpdchat

a tweet from <u>Guy Carpenter</u> was a reply to one of <u>Catherine Edwards</u>' comments in a conversation shown above:



Catherine Edwards @Edwards08C · 15h HOw do we exemplify thought process?

...

× ...

Live or static?

#mathsCPDchat



Mr Carpenter MCCT @GuyCarpenter567 · 16h ···· Replying to @EdwardsO8C @danicquinn and @dare_richard Both. 'Here I am going to do because' Also when looking at misconceptions that have happened 'Learner A has done..... is this right? Why do you think that? Well we can see that if I do this......' And so on.

this conversation started from one of <u>Peter Williams</u>' comments, which had been part of a conversation shown above, and consisted of tweets from <u>Dani Quinn</u>, <u>Nathalie Leighton</u>, <u>Richard</u> <u>Dare</u> and <u>Catherine Edwards</u>:



Peter Williams 🚗 @MathsImpact · 15h I find that live works much better. •••

I'm also a big fan of the "my turn" "your turn" approach.

I want students to be fully focused on what I'm doing when I model, not on recording it in their books.

Pens down, watching and listening.

#mathscpdchat



Dani Quinn @danicquinn · 17h Replying to @MathsImpact @Edwards08C and 2 others 100% I would love a nationwide moratorium on: clicky-clicky modelling (i.e. powerpoint instead of live) copying down asking 'do you understand?' going straight to independent work after a model



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Nathalie Leighton @LeightonM4ths · 19h ···· Replying to @danicquinn @MathsImpact and 3 others Definitely needs a mini whiteboard activity between modelling and independant practice #mathscpdchat



Richard Dare @dare_richard · 18h Replying to @LeightonM4ths @danicquinn and 3 others YES!



Catherine Edwards @Edwards08C · 18h ugh, I hate mini whiteboards

why would you want to rub your work out?

Mistakes are how we learn.

plus the pens always run out!

#mathsCPDchat



Richard Dare @dare_richard + 18h • We'd burn through books if students had to write on them big enough for me to see all students' responses!

They're wonderful!

Yes, mistakes are great - celebrate them.

It's the boards' display value which is immense.

#mathscpdchat



Nathalie Leighton @LeightonM4ths · 19h Replying to @Edwards08C @dare_richard and 3 others

I agree the pens are a pain,

but so worth it to see that they can solve the questions (not necessarily why 😳) and also to make sure that they all show their workings every time and that they present their work "my way"

...teaching solving linear equ to Y8 this week 🥺



Nathalie Leighton @LeightonM4ths · 19h ···· And rubbing your work out is what allows students to not be shy about making mistakes, I think.

I bought a2 size whiteboard for my Y12/13 classes and they work on those all the times in lesson - great for sketching . #mathscpdchat



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this conversation was prompted by an exchange between <u>Nathalie Leighton</u> and <u>Richard Dare</u> which was part of another conversation shown above, and included tweets from <u>Peter Williams</u>, <u>Karen Hancock</u> and <u>Catherine Edwards</u>:



Nathalie Leighton @LeightonM4ths · 19h ···· Replying to @danicquinn @MathsImpact and 3 others Definitely needs a mini whiteboard activity between modelling and independant practice #mathscpdchat



Richard Dare @dare_richard · 18h Replying to @LeightonM4ths @danicquinn and 3 others YES!



Peter Williams Replying to @dare_richard @LeightonM4ths and 3 others I do We do You do

Is a great structure for securing new skills.

#mathscpdchat



Karen @karenshancock · 19h Replying to @MathsImpact @dare_richard and 4 others I'm much more an "I do, we think, you do"



Date:





Karen @karenshancock · 19h Replying to @dare_richard @MathsImpact and 4 others This one. Read worked example. Explain to neighbour. Cold call to explain to class. Think, pair share on (1). (2) on mini whiteboards, or direct to paper if they prefer.

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Catherine Edwards @Edwards08C · 19h Is it substantially different for A-level than KS3?

#mathsCPDchat



Karen @karenshancock · 19h A Level:

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•••



M07 Differentiation (part 2)

Worked Example:

$$y = x^2 - \frac{16}{x}$$

has one local minimum and one root.

Find these two points and sketch the curve.

$$y = x^2 - \frac{16}{x}$$

 $y = x^2 - 16x^{-1}$
 $y' = 2x + 16x^{-2}$

At local menimum: y'=0

$$\begin{aligned} 2x + 16x^{-2} &= 0\\ 2x &= -16x^{-2}\\ x^3 &= -8\\ x &= -2 \end{aligned}$$

Local nuinimum (,)

Root:

$$y = 0$$

 $x^2 - \frac{16}{2} = 0$
 $x^2 = \frac{16}{2}$
 $x^3 = 16$
 $x = 16$

Date:





the following conversation developed from one of <u>Dani Quinn</u>'s tweets in another conversation, and included tweets from <u>RHMaths</u>, <u>Catherine Edwards</u> and <u>Peter Williams</u>:



	Dani Quinn @danicquinn · 17h Replying to @MathsImpact @Edwards08C and 2 others 100% I would love a nationwide moratorium on: clicky-clicky modelling (i.e. powerpoint instead of live) copying down asking 'do you understand?' going straight to independent work after a model	••••
1.00	RHMaths @MathsRh · 14h	
	On this note, what IT are you using for going through worked examples? I'm rea missing my IWB #mathscpdchat	lly
	Catherine Edwards @Edwards08C · 14h This year, Booklet and visualiser The use office lense to take a picture and put onto OneNote for students to loc at later. #mathscpdchat	 Ik
(Day)	Peter Williams 🚗 @MathsImpact · 17h	
	Replying to @MathsRh @danicquinn and 3 others	
	Mostly OneNote and a graphics tablet.	
	Sometimes my visualiser.	
	Dani Quinn @danicquinn · 17h Replying to @MathsRh @MathsImpact and 3 others	
and finall	y this conversation was generated by a question from Bryony Black, and included	
tweets fro	om <u>Catherine Edwards</u> and <u>Mary Pardoe</u> :	
	Bryony Black @bryony_black · 15h Does/can a worked example include annotations explaining the steps (like a written narration)?	
A	Catherine Edwards @Edwards08C · 15h	
1	Definitely I would argue the narration is the most important part #mathscpdcha	at
9	Karen @karenshancock · 14h	
6	We might have to disagree on this I think that it's important for the students try to do that rather than me doing it, at least initially. #mathscpdchat	to
A	Catherine Edwards @Edwards08C · 14h	
4	I think my classes need a bit more training before they could reliably narrate without causing misconceptions #mathscpdchat	





Karen @karenshancock · 14h I tend to think of it, if I'd be prepared to ask them what is the next step and confident they would get it correct, then I can definitely risk self explanation. #mathscpdchat

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Replies to Catherine's second question ...



Catherine Edwards @Edwards08C · 16h Question 2

...

What can we do to make our worked examples more effective?

#mathsCPDchat

... included:

- a reminder to think about possible misconceptions before creating a worked example;
- a suggestion that it is effective to provide a 'model solution' and challenge students to 'annotate' it;
- a comment that it is advisable to pre-plan the questions (about/related-to a worked example) that you intend to ask students, for example ...



Helen Scott @HelenScott88 · 16h ···· Depends on the worked example. Sometimes 'Why have I done XYZ?' or 'what did I do to get from here to here but I can't write down?' Sometimes I just want students to listen to me narrate my thought process. Then perhaps 'How would/could you do this differently?' #mathsCPDchat

Suggestions made in response to Catherine's third question ...



Catherine Edwards @Edwards08C · 17h Question 3 ...

How can we use worked examples to teach method selection?

#mathsCPDchat

... included:



- show part of a solution and ask students 'What do you notice?' 'What could/would you do next?';
- the teacher might 'speak-internal-reasoning-and-questioning' out-loud as she works through a problem in front of the class ... 'I model my thought process';
- talking to students about noticing:



Peter Williams 🦔 @MathsImpact · 18h Mostly it's just in my own language more regularly.

•••

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When I'm narrating I say "I noticed that..." very often.

It's also my go to prompt when students are stuck.

"What do you notice?"

"Do you notice anything familiar?"

"Have you noticed a connection?"

- creating 'model solutions' for 'minimally different questions' and 'same surface, different depth' questions;
- challenging students to create their own examples, for example ...



Karen @karenshancock · 16h Replying to @Edwards08C This type of thing or identifying topic? #mathscpdchat





Study the solution and answer these questions

- 1) Do you have a preferred method?
- 2) Can you write an equation where Method 2 would be more efficient?



Catherine Edwards @Edwards08C · 19h Replying to @karenshancock

I love the question about write a question where this method would be better.

I do find students get wedded to the "right" way of doing it

#mathsCPDchat

 challenge students to work (possibly in pairs/small groups) on method selection without actually solving the problems;

Comments in response to Catherine's last question ...



Catherine Edwards @Edwards08C · 19h Question 4

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How can we best match worked example types and structures to different groups and subjects?



... included:

- using 'probing' questions (possibly to find out if students have 'understood a 'compressed step' in a worked example') ... to which students might respond by writing/drawing on individual whiteboards;
- always creating fresh worked examples for a new class ... that getting to know students ('read the class') is important, but it cannot be rushed!