MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

4 Sequences and graphs

4.1 Sequences

Guidance document | Key Stage 3

Making connections

The NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of 'core concepts'.

The fourth of these themes is *Sequences and graphs*, which covers the following interconnected core concepts:

4.1 **Sequences**

4.2 Graphical representations

This guidance document breaks down core concept *4.1 Sequences* into three statements of knowledge, skills and understanding:

4.1.1 Understand the features of a sequence

4.1.2 Recognise and describe arithmetic sequences

4.1.3 Recognise and describe other types of sequences (non-arithmetic)

Then, for each of these statements of knowledge, skills and understanding we offer a set of key ideas to help guide teacher planning.

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Overview

Students began to consider sequences in Key Stage 1, when step counting to learn times tables and when looking at the composition of numbers. In Key Stage 2, students were introduced to the use of symbols and letters to represent variables and unknowns in familiar mathematical situations and began to generalise number patterns.

Students will have explored non-numerical (shape) and numerical sequences, noticed a pattern, described the pattern in words and found the next term in the sequence from the previous term. They will primarily have focused on generating and describing linear number sequences, though they may have also experienced naturally occurring patterns in mathematics, such as square numbers.

The extent to which students have explored these concepts in depth may vary. Therefore, students should consolidate, secure and deepen their understanding of linear sequences and how to find and use term-to-term rules to generate the *next* term. Then, they can progress on to describing *any* term in the sequence directly in relation to its position in the sequence.

This core concept extends students' knowledge of sequences through exploration of the mathematical structure, not just by spotting the patterns that the structure creates. Algebraic notation is used to express the structure, and students should become familiar with finding and using the nth term of a linear sequence. It is important that students have time to develop a full understanding of the connection between the notation and the sequence, and come to see the nth term as a way of expressing the structure of every term in the sequence.

This learning has connections to other areas of algebra, particularly solving equations (when checking if a number is a term in a sequence) and straight line graphs. Work on sequences in Key Stage 3 provides the foundation for exploring quadratic sequences and simple geometric progressions in Key Stage 4.

Prior learning

Before beginning to teach *Sequences* at Key Stage 3, students should already have a secure understanding of the following from previous study:

Key stage	Learning outcome
Upper Key Stage 2	• Generate and describe linear number sequences • Use simple formulae • Describe positions on the full coordinate grid (all four quadrants)
Key Stage 3	• 1.2.1 Understand multiples • 1.2.2 Understand integer exponents and roots • 1.4.1 Understand and use the conventions and vocabulary of algebra, including forming and interpreting algebraic expressions and equations **Please note:** Numerical codes refer to statements of knowledge, skills and understanding in the NCETM breakdown of Key Stage 3 mathematics.

You may find it useful to speak to your partner schools to see how the above has been covered and the language used.

You can find further details regarding prior learning in the following segments of the NCETM primary mastery professional development materials[1]:

- Year 5: 2.21 Factors, multiples, prime numbers and composite numbers
- Year 6: 1.31 Problems with two unknowns
- Year 6: 2.27 Scale factors, ratio and proportional reasoning

Checking prior learning

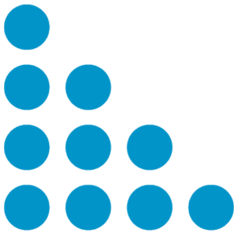
The following activities from the NCETM primary assessment materials[2] and the Standards & Testing Agency's past mathematics papers[3] offer useful ideas for assessment, which you can use in your classes to check whether prior learning is secure:

Reference	Activity
Year 6 page 27	*Ramesh is exploring two sequence-generating rules.* *Rule A is: 'Start at 2, and then add on 5, and another 5, and another 5, and so on.'* *Rule B is: 'Write out the numbers that are in the five times table, and then subtract 2 from each number.'* *What's the same and what's different about the sequences generated by these two rules?*
Year 6 page 27	*Roshni and Darren are using sequence-generating rules.* *Roshni's rule is: 'Start at 4, and then add on 5, and another 5, and another 5, and so on.'* *Darren's rule is: 'Write out the numbers that are multiples of 5, starting with 5, and then subtract 1 from each number.'* *Roshni and Darren notice that the first few numbers in the sequences generated by each of their rules are the same. They think that all the numbers in the sequences generated by each of their rules will be the same.* *Do you agree? Explain your decision.*
Year 6 page 27	*On New Year's Eve, Polly has £3.50 in her money box. On 1 January she puts 30p into her money box. On 2 January she puts another 30p into her money box. She continues putting in 30p every day.* *How much money is in the box on 10 January?* *How much money is in the box on 10 February?* *Write a sequence-generating rule for working out the amount of money in the money box on any day in January.*

2017 Key Stage 2 Mathematics Paper 3: reasoning Question 21	*The numbers in this sequence increase by the same amount each time.* *Write the missing numbers.* [ ] [*1*] [$1\frac{5}{8}$] [$2\frac{1}{4}$] [ ] Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0
2018 Key Stage 2 Mathematics Paper 2: reasoning Question 9	*The list below shows the years in which the Cricket World Cup was held since 1992:* *1992, 1996, 1999, 2003, 2007, 2011, 2015* *Adam says,*  *Adam is **not** correct.* *Explain how you know.* Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0
2018 Key Stage 2 Mathematics Paper 3: reasoning Question 1	*The numbers in this sequence increase by the same amount each time.* *Write the missing numbers.* [ ] [*42*] [*49*] [ ] [*63*] [ ] Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0

Key vocabulary

Term	Definition
arithmetic sequence	A sequence of numbers in which successive terms are generated by adding or subtracting a constant amount to/from the preceding term. Example 1: 3, 11, 19, 27, 35, …, where 8 is added. Example 2: 4, –1, –6, –11, …, where 5 is subtracted (or –5 has been added). The sequence can be generated by giving one term (usually the first term) and the constant that is added (or subtracted) to give the subsequent terms. Also called an 'arithmetic progression'.
cube number	A number that can be expressed as the product of three equal integers. Example: 27 = 3 × 3 × 3. Consequently, 27 is a cube number. It is the cube of 3 or 3-cubed. This is written compactly as $27 = 3^3$, using index (or power) notation.
geometric sequence	A series of terms in which each term is a constant multiple of the previous term (known as the common ratio) is called a geometric sequence, sometimes also called a 'geometric progression'. Example 1: 1, 5, 25, 125, 625, …, where the constant multiplier is 5. Example 2: 1, –3, 9, –27, 81, …, where the constant multiplier is –3. A geometric sequence may have a finite number of terms or it may go on forever, in which case it is an infinite geometric sequence. In an infinite geometric sequence with a common ratio strictly between zero and one, all the terms add to a finite sum.
*n*th term of a sequence	This is the name for the term that is in the *n*th position starting the count of terms from the first term. The *n*th term is sometimes represented by the symbol u_n.
prime number	A whole number greater than one that has exactly two factors: itself and one. Examples: 2 (factors 2, 1), 3 (factors 3, 1). 51 is not prime (factors 51, 17, 3, 1).
sequence	A succession of terms formed according to a rule. There is a definite relation between one term and the next and between each term and its position in the sequence. Example: 1, 4, 9, 16, 25, …
square number	A number that can be expressed as the product of two equal numbers. Example: 36 = 6 × 6 and so 36 is a square number or '6 squared'. A square number can be represented by dots in a square array.
term	A term is either a single number or variable, or the product of several numbers or variables. Terms are separated by a + or – sign in an overall expression. Example: In $3 + 4x + 5yzw$; 3, $4x$ and $5yzw$ are three separate terms.

triangular number	1. A number that can be represented by a triangular array of dots with the number of dots in each row from the base decreasing by one. Example:  The triangular number 10 represented as a triangular array of dots. 2. A number in the sequence 1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4, … Example: 55 is a triangular number since it can be expressed as 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10.

Collaborative planning

Below we break down each of the three statements within *Sequences* into a set of key ideas to support more detailed discussion and planning within your department. You may choose to break them down differently depending on the needs of your students and timetabling; however, we hope that our suggestions help you and your colleagues to focus your teaching on the key points and avoid conflating too many ideas.

Please note: We make no suggestion that each key idea represents a lesson. Rather, the 'fine-grained' distinctions we offer are intended to help you think about the learning journey irrespective of the number of lessons taught. Not all key ideas are equal in length and the amount of classroom time required for them to be mastered will vary, but each is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

The following letters draw attention to particular features:

D Suggested opportunities for **deepening** students' understanding through encouraging mathematical thinking.

L Examples of shared use of **language** that can help students to understand the structure of the mathematics. For example, sentences that all students might say together and be encouraged to use individually in their talk and their thinking to support their understanding (for example, *'The smaller the denominator, the bigger the fraction.'*).

R Suggestions for use of **representations** that support students in developing conceptual understanding as well as procedural fluency.

V Examples of the use of **variation** to draw students' attention to the important points and help them to see the mathematical structures and relationships.

PD Suggestions of questions and prompts that you can use to support a **professional development** session.

For selected key ideas, marked with an asterisk (*), we exemplify the common difficulties and misconceptions that students might have and include elements of what teaching for mastery may look like. We provide examples of possible student tasks and teaching approaches, together with suggestions and prompts to support professional development and collaborative planning. You can find these at the end of the set of key ideas.

Key ideas

4.1.1 Understand the features of a sequence

Students should be familiar with finding and using difference patterns in linear sequences to generate the terms of a sequence. This collection of key ideas provides an opportunity to secure and deepen understanding by applying this prior learning to other linear and non-linear sequences, including those containing negative and fractional terms.

4.1.1.1* Appreciate that a sequence is a succession of terms formed according to a rule

4.1.1.2 Understand that a sequence can be generated and described using term-to-term approaches

4.1.1.3 Understand that a sequence can be generated and described by a position-to-term rule

4.1.2 Recognise and describe arithmetic sequences

Students should realise that generating many terms in a sequence using the term-to-term rule is not an efficient method. By exploring the mathematical structure of a linear sequence, students can instead describe linear sequences using the *n*th term. Students should then experience a range of ascending and descending sequences, and those containing terms that are decimals and fractions, in order to develop a deep and secure understanding of the *n*th term and how to use it.

The *n*th term is new learning to Key Stage 3. It is crucial that students are given time to become fluent at describing linear sequences using the *n*th term rule, as well as reason with and apply it in order to solve mathematical problems, such as finding the 50th and 100th terms of a linear sequence.

4.1.2.1 Understand the features of an arithmetic sequence and be able to recognise one

4.1.2.2* Understand that any term in an arithmetic sequence can be expressed in terms of its position in the sequence (*n*th term)

4.1.2.3 Understand that the *n*th term allows for the calculation of any term

4.1.2.4 Determine whether a number is a term of a given arithmetic sequence

4.1.3 Recognise and describe other types of sequences (non-arithmetic)

Much of this core concept focuses on arithmetic sequences, but students should also experience other types of sequences, including special number sequences, that are connected to new learning in Key Stage 3 (for example, triangular numbers). While the Fibonacci sequence is not explicitly named until the Key Stage 4 programme of study, opportunities should be provided in Key Stage 3 to begin exploring other types of number sequences in preparation for this future work.

4.1.3.1 Understand the features of a geometric sequence and be able to recognise one

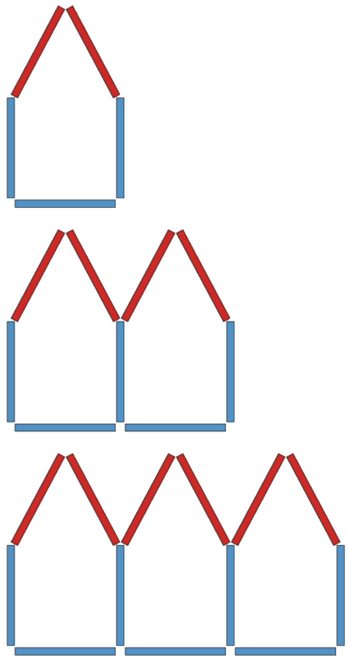
4.1.3.2 Understand the features of special number sequences, such as square, triangle and cube, and be able to recognise one

4.1.3.3 Appreciate that there are other number sequences

Exemplified key ideas

4.1.1.1 Appreciate that a sequence is a succession of terms formed according to a rule

Common difficulties and misconceptions
Students may have an intuitive sense that the terms in a sequence progress logically according to a rule, but may find it difficult to express clearly what that rule is. Students should, through discussion and sharing ideas, be able to clearly articulate a rule and should be encouraged to use mathematical language to describe sequences wherever possible. For example, when describing the sequence 3, 5, 7, 9, 11, … students may often say, 'It goes up in 2s'. Through discussion, this response can be refined so that students are more explicit regarding the starting number and the amount added each time. For example, 'The sequence begins with three, and two is added each time' or 'The 1st term is three, the 2nd term is three plus two, the 3rd term is three, plus two, plus two, etc.'. It is not uncommon for students to notice only additively increasing sequences (i.e. arithmetic sequences where the common difference is positive), so students should experience a varied collection of types of sequence. For example: • 23, 19, 15, 11, 7, 3, … (decreasing arithmetic sequences) • 3, 6, 12, 24, 48, 96, … (geometric sequences, where there is a constant multiple or ratio between successive terms) • 1, 4, 5, 9, 14, 23, … (Fibonacci-like sequences, where terms are generated by adding the two preceding terms) Also, sequences of squares (or cubes or multiples of a given number or odd numbers) are useful in that, while students may wish to describe them using differences, there is the opportunity to notice that, for example, 'the third number in the sequence is the third square (or cube or multiple of seven or odd number)'. The focus in this core concept is being aware that there is a consistent mathematical rule that generates terms, without necessarily describing that rule precisely. However, discussion of different sequences and how they progress, using increasingly sophisticated mathematical language, is an important precursor to finding term-to-term and position-to-term rules later. Students should also experience sequences where there are multiple ways in which the sequence could be extended. For example, the terms in the sequence 1, 2, 4, … could be generated by: • doubling each term to get the next (1, 2, 4, 8, 16, 32, …) • adding one, then two, then three, etc. (1, 2, 4, 7, 11, …)

What students need to understand	Guidance, discussion points and prompts
Understand the characteristics of sequences. *Example 1:* *Here is a sequence of shapes.*  *What is staying the same?* *What is changing?* *a) How many red sticks will the 4th shape of this sequence require?* *b) How many blue sticks will the 5th shape of this sequence require?* *c) What can you say about the 18th shape in this sequence?* *d) How many blue sticks are in the shape that has 16 red sticks?*	**R** Presenting sequences of 'growing shapes', such as those in *Example 1*, is helpful in supporting students to spot patterns and rules. Students may be more readily able to see (particularly if pictures are colour-coded, as in *Example 1*) both how the number of sticks is increasing and how each picture/term has the same structure, i.e. always having a number of pairs of red sticks (depending on the position number) and one more than that number of blue sticks.

Example 2:

Afsal thinks that the following statements about sequences are true:

a) Sequences always increase.

b) A sequence can have either positive or negative terms, but it cannot have both.

c) An arithmetic sequence always has a common difference that is an integer.

d) All sequences have terms that increase by the same amount each time.

For each statement, find an example of a sequence which shows that Afsal is not right.

D Asking students to generate examples of their own that fit certain criteria is an effective way of encouraging deeper thinking.

V An important use of variation is to encourage students to be aware of the full range of examples of a certain mathematical object (in this case, sequences) – both standard and non-standard examples.

Students often develop misconceptions as a result of being exposed to a limited range of examples. *Example 2* is designed to get students to think about such possible misconceptions and refine in their own minds what makes a list of numbers form a sequence.

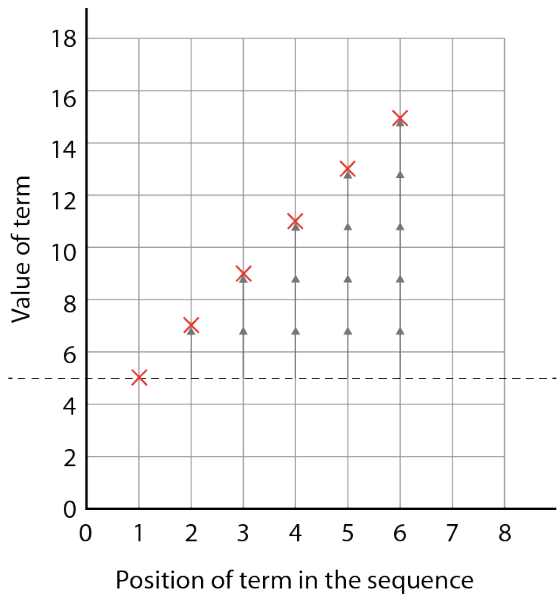
Example 3:

Consider the sequence: 5, 7, 9, 11, …

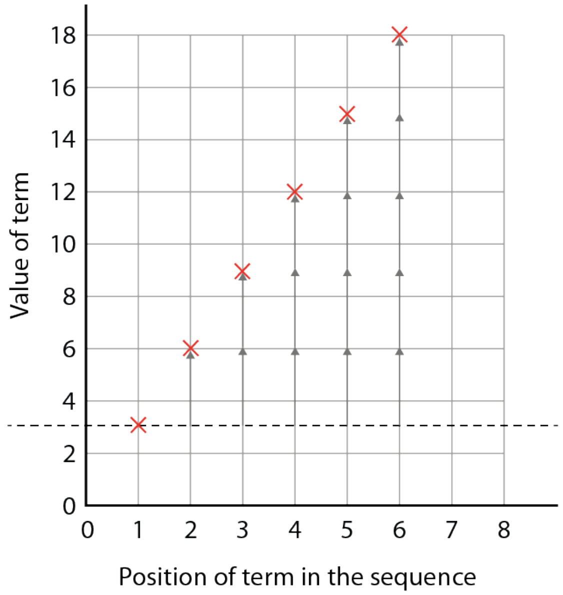
Charlie says that the 6th term is going to be twice the 3rd term. Is he right?

PD The idea that you can multiply a term to find terms further on in a sequence is a very common one (students asked for the 100th term will often find the 10th term and multiply it by ten). Do your students harbour this misconception? Why do you think this might be so? How might you address it?

R By representing the terms in the sequence as points on a graph, students may be better able to reason why, for example, simply doubling the 3rd term does not obtain the 6th term:



Similarly, students may be able to extend to the special case where multiplying terms to obtain others does work:



i.e. that it is only true if the set of points passes through (0,0) because the increase from the origin to the 1st term is equivalent to the other term-to-term increases.

Example 4:

Here is a sequence of numbers.

5, 10, 15, 20, 25, 30, …

a) Will the number 67 be in the sequence? Explain your answer.

b) What position would 55 be in the sequence? Give a reason for your answer.

Give some more examples of numbers that are (and are not) in this sequence and explain why.

D Students will often look to see how the sequence is progressing from term to term. *Example 4* has been designed to encourage students to think about what structure is common to each term in the sequence.

Being able to say that 'All of the numbers are multiples of five' (and not just that 'They go up in fives') is an important step as it prepares the ground for the important question *'Which multiple of five is each term?'*, which will support students later in being able to identify any term in the sequence.

Here are some other sequences that students could work on in a similar way:

- 5, 15, 25, 35, …
- 90, 80, 70, 60, …
- $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
- 1, −1, 1, −1, 1, …
- 10, 100, 1 000, 10 000, …
- 0.3, 0.6, 0.9, 1.2, …

Understand that some numeric sequences can be described by a non-mathematical rule. *Example 5:* *A sequence is generated by the rule 'The value of a term in this sequence is given by the number of letters in the position number'.* *The first five terms are 3, 3, 5, 4, 4, …* *What are the next three terms?*	**D**	Students could be invited to devise their own sequences, similar to *Example 5*, that cannot be described with a mathematical rule; for example, the number of days in consecutive months: 31, 28, 31, 30, …

4.1.2.2 Understand that any term in an arithmetic sequence can be expressed in terms of its position in the sequence (*n*th term)

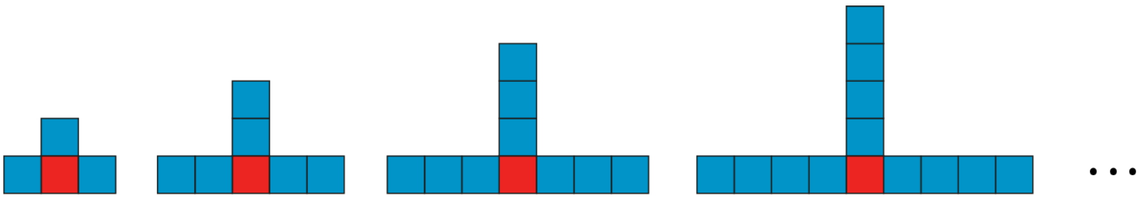
Common difficulties and misconceptions

When examining arithmetic sequences where each term is obtained by adding a fixed amount (constant difference) to the previous term, it is natural for students to express the rule in terms of this fixed amount. For example, students may see the sequence 4, 7, 10, 13, … as 'Add three' and think that, therefore, the nth term is $n + 3$ rather than the correct $3n + 1$.

This misconception can be explored by examining the three times table (i.e. 3 (3×1), 6 (3×2), 9 (3×3), 12 (3×4), 15 (3×5), …, $3n$) and a range of other sequences that consist of terms in the three times table with one, two, three, etc. added (or subtracted). Students should be encouraged to notice what is the same and what is different – i.e. it is the '$3n$' that determines the 'increasing by three'. It will then be helpful for students to experience substituting $n = 1, 2, 3$, etc. into the expression '$n + 3$' to realise that this will give a sequence which begins at four and increases by one, rather than beginning at four and increasing by three.

The fundamental awareness here is that, in the general statement $3n + 1$, the common difference is represented by the '3' (the coefficient of n) because as n increases, the value of the whole expression increases by three. It will be important to explore the potential confusion between multiples of three (for example) and numbers which increase by three, and to recognise that these are not necessarily the same thing.

R The use of growing shape patterns that generate such sequences of numbers can support students in seeing what is constant in each term and what varies. For example: '*How many squares are needed to make each shape in this sequence?*'



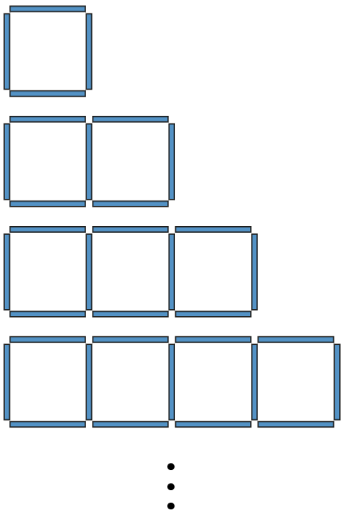
Rewriting each of the numerical terms to reveal the structure, e.g. $1 + (1 \times 3)$, $1 + (2 \times 3)$, $1 + (3 \times 3)$, $1 + (4 \times 3)$, … can also assist in developing this awareness.

What students need to understand

Appreciate that each term in an arithmetic sequence has the same structure and that the expression of that structure in terms of its position in the sequence is the *n*th term.

Example 1:

The following sequence of growing shapes is made up of sticks.



It generates the following sequence of numbers (the number of sticks):

4, 7, 10, 13, …

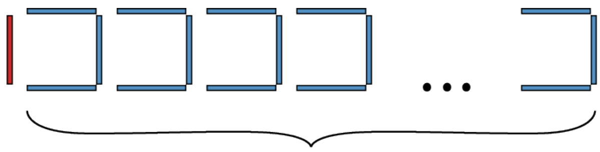
a) Draw the 17th diagram. Think about the structure of the diagram as you draw it.

b) How would you work out the number of sticks in the 17th diagram without counting them one by one? What calculation would you perform?

Guidance, discussion points and prompts

R The purpose of the diagrams in *Example 1* is to support students in seeing a common structure in each of the terms of the sequence.

D Asking students to draw the 17th diagram (or any diagram further along in the sequence) will help them to focus on the structure of the 17th term. It will be important to discuss the various answers that students come up with in order to draw their attention to the role that the '17' plays. For example, some students may draw one stick followed by 17 groups of three sticks, like this:



and this might give rise to them writing the calculation:

$1 + 17 \times 3$ or $17 \times 3 + 1$

In these calculations that students devise, the '17' is acting almost like an honorary variable.

Questions such as *'What might the calculation for the 25th (or 50th, or 100th, or 347th, etc.) term be?'* will help students to generalise and to see each term in the sequence as of the form $3n + 1$.

PD Consider other ways that students might draw the 17th diagram, what different calculations might arise and whether or not these all lead to the same algebraic expression for the *n*th term. You may like to try this activity together as a group of teachers to see what you come up with.

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Example 2:

The terms in each of these sequences are generated by adding 3 each time and the 17th term is circled.

a) 4, 7, 10, 13, 16, …, (52)

b) 5, 8, 11, 14, 17, …, (53)

c) 14, 17, 20, 23, 26, …, (62)

d) 27, 30, 33, 36, 39, …, (75)

For each sequence, write the 17th term as an expression involving 17.

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V By keeping the constant difference the same in all parts of *Example 2*, it is intended that students will see that three is the *multiplier* in the 17th term, because three is added each time. By the 17th term, seventeen 3s will have been added.

D It may help to draw students' attention to the fact that, as each term has been increased by three, it is possible to write each term in such a way to show the '3's, e.g.:

$4 = 1 + 3$

$7 = 1 + 3 + 3$

$10 = 1 + 3 + 3 + 3$, etc.

and to ask, *'How many 3s will have been added to generate the 17th term?'*

It would be beneficial to probe students' understanding a bit deeper by asking for the 25th (or 50th, or 100th, or 347th, etc.) term and then to generalise this to the *n*th term.

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Example 3:

The following are the nth terms of different sequences.

a) $2n + 1$

b) $2n + 5$

c) $2n + 7$

d) $2n + \frac{1}{2}$

e) $2n + \frac{3}{4}$

f) $2n + 0.7$

g) $-2n + 5$

h) $0.7 - 2n$

How do these sequences increase?

What do you notice about parts g) and h)?

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V By keeping the coefficient of *n* constant and only varying the constant term in *Example 3*, students' attention can be drawn to which aspect of the *n*th term is determining the difference between successive terms.

In order to address the possible misconception referred to in the overview, it will be useful to get students to think about the question, *'Why isn't the amount that is added telling us the common difference?'* and to ask, *'What is this added amount telling us?'*. In order to support students in fully generalising the idea of an arithmetic sequence, it will be important for students not to see decreasing sequences (parts g) and h)) as separate or different, but to see them as sequences formed by adding a *negative number* each time rather than a positive one.

R In addition to substituting successive values of $n = 1, 2, 3, 4, \dots$ etc. into the expression for the *n*th term, students could be encouraged to plot the sequences. For example, the sequence 1, 2, 3, … can be represented on this graph:

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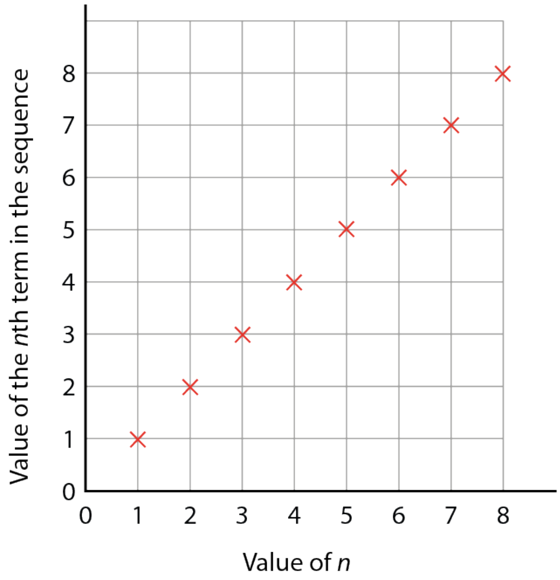


Students should notice that the common difference can be read from this graph and is represented by the steepness of the line joining the points (although note that it is not appropriate to draw the line, because points for non-integer values of *n* – the position number – do not have any meaning in terms of the sequence). Such graphs can also be used to show arithmetic sequences with negative common differences (i.e. decreasing sequences).

Example 4:

How do these sequences increase (or decrease)?

a) $5n + 3$

b) $-7n + 3$

c) $0.9n - 3$

d) $-n + 3$

How can you tell just by looking at the nth term?

L To support their developing understanding, it will be important for students to articulate clearly what happens to each term as *n* increases and to realise that the coefficient of *n* determines the increase value.

Sentences of the form *'As the value of n increases by one, the value of each term increases (or decreases) by _'* should be encouraged.

Example 5:

What's the same and what's different about these sequences?

a) 5 9 13 17 …

b) 5.2 9.2 13.2 17.2 …

c) $5\frac{1}{2}$ $9\frac{1}{2}$ $13\frac{1}{2}$ $17\frac{1}{2}$ …

V Working with sets of clearly related sequences will help students to discern between the coefficient of *n* and the constant term.

Example 6:

What sequences of numbers are produced from the following nth terms?

a) $3n$

b) $3n + 1$

c) $3n + 2$

d) $3n + 3$

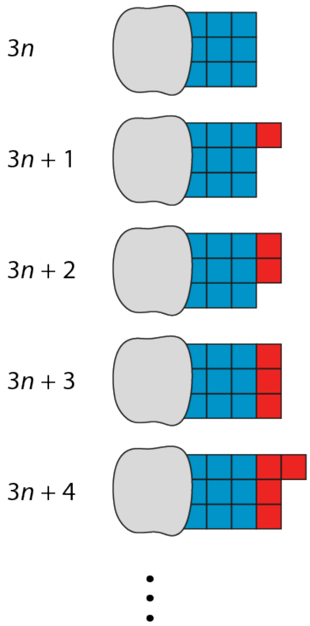
e) $3n + 4$

V It will be important to draw students' attention to what is the same and what is different in each of these sequences in order to help them become aware that, while multiples of three increase by three each time, sequences that increase by three do not necessarily contain multiples of three. Questions such as:

- *'Which sequences in this set are multiples of three?'*
- *'Why are some sequences not multiples of three even though they go up in 3s?'*
- *'Can you find some more nth terms that will result in generating multiples of three?'*

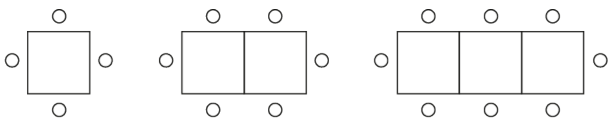
can help students to be aware of this important idea.

R Using representations such as this:



will help students to see that every third diagram gives the set of multiples of three.

While carefully crafted explanations from the teacher may be useful for some students, other students may listen and watch passively without engaging with the thinking that is necessary. Therefore, it is vital that students are given the opportunity to use these diagrams themselves to support their own reasoning through whole-class dialogue and discussion. Avoid merely presenting

	these diagrams as a form of teacher demonstration. **D** Students could be challenged to think of the *n*th term which might generate a *descending* sequence of multiples of three.
Example 7: a) *Mo thinks the nth term of the sequence $10, 6, 2, -2, \ldots$ is $4n + 6$.* *Do you agree? Explain your reasoning.* b) *Olivia thinks the nth term of the sequence $2, 7, 12, 17, \ldots$ is $n + 5$.* *Explain why Olivia is incorrect.* c) *$0, 5, 10, 15, 20, \ldots$* *$10, 15, 20, 25, \ldots.$* *$-5, 0, 5, 10, 15, \ldots$* *$-25, -20, -15, 10, -5, \ldots$* *Liam thinks all the sequences can be described using the nth term '5n' as they all 'increase by 5'. Do you agree? Explain your answer.* d) *True or false?* *The 10th term of the sequence $3, 7, 11, 15, 19, \ldots$ is 38.*	**V** An aspect of variation is to vary 'what it's not' (as well as what it is) to help students clarify the idea in their minds. *Example 7*, part a) is designed to encourage students to notice that although the difference between the terms is four, the term-to-term rule is 'subtract four' rather than 'add four'. The rule $4n + 6$ will generate the 1st term correctly but will not generate subsequent terms accurately. Parts b) and c) are designed to encourage students to pay attention to the position of each term in the sequence and not the term-to-term rule. Part d) is designed to highlight the misconception that the 10th term is twice the 5th term. **PD** Construct other 'what it's not' examples that highlight the importance of identifying the position of a term in a sequence.
Solve familiar and unfamiliar problems, including real-life applications. *Example 8:* *Each table seats 4 people. The diagram below shows how many people can be seated around different numbers of tables when they are put together.*  *How many chairs are needed for 15 tables?* *What is the largest number of tables needed for 37 chairs?*	Problems provide opportunities for students to intelligently practise their understanding of a concept (rather than mechanical repetition), to focus on relationships – not just the procedure – and make connections. **PD** What contexts are there where there is a genuine need to calculate the *n*th term of a linear sequence? To what extent does it matter if the contexts are genuine or contrived?

Solve problems where there is more than one answer and where there are elements of experimentation, investigation, checking, reasoning, proof, etc. *Example 9:* *Two sequences are given by the nth terms 4n + 6 and 6n – 3. Is the following statement true or false?* *'There is at least one number that is in both sequences.'* *Construct a convincing argument to support your answer.*	For students who have demonstrated a secure understanding of finding the *n*th term of a linear sequence, you could encourage them to go deeper by solving more complex problems, such as exploring all possibilities, creating their own examples and testing conjectures. **D** For problems such as *Example 9*, students could be encouraged to find a convincing argument or proof.
Example 10: *a) One term of a linear sequence is 3 and another term is 8. The nth term of a linear sequence is an ± b.* *Find possible values for a and b.* *b) One term of a linear sequence is –2 and another term is 1.5. The nth term of a linear sequence is xn + y.* *Find possible values for x and y.*	**D** For problems such as *Example 10*, students could be encouraged to find non-integer values and 'unique' solutions (i.e. ones that no one else in the class will find!).

Weblinks

1. NCETM primary mastery professional development materials
https://www.ncetm.org.uk/resources/50639
2. NCETM primary assessment materials
https://www.ncetm.org.uk/resources/46689
3. Standards & Testing Agency past mathematics papers
https://www.gov.uk/government/collections/national-curriculum-assessments-practice-materials#key-stage-2-past-papers